CHAPTER 2

LITERATURE SURVEY

2.1 INTRODUCTION

This chapter reviews the prior work relevant to the research problem and the technique used in this research to solve the problem.

2.2 STABILITY

A system is said to be bounded input bounded output stable (BIBO) if, and only if, every bounded input results in a bounded output. The output of such system does not diverge if the input does not diverge. From an engineering perspective, it is important that a system of interest remains stable under all possible operating conditions, only then the system is guaranteed to produce a bounded output for a bounded input.

The question of stability of dynamic systems arises naturally in certain very simple and ubiquitous mechanical systems. Torricelli (1644) formulated a criterion for the stability of equilibrium of mechanical systems under the influence of a gravitational field. It was this work that was generalized by Lagrange (1788) to hold for all conservative systems. Interest in the stability of these types of systems marks the starting point of the study of stability issues eventually leading to the present state of the art.

British Astronomer Airy (1840) developed a feedback device for pointing a telescope. His device was a speed control system, which turned the
telescope automatically to compensate for the earth’s rotation, affording the ability to study a given star for an extended time. Unfortunately, Airy discovered that by improper design of the feedback control loop, wild oscillations were introduced into the system. He was the first to discuss the instability of closed-loop systems, and the first to use differential equation in their analysis. Unstable systems are usually being avoided, unless some mechanism can be found to stabilize them.

2.3 STABILITY CRITERIA

Deriving and solving differential equations gives accurate solutions. However, the design and analysis of complex control system may become very tedious for a higher order system because of the many parameters it exhibits. The goal of being able to predict a system’s performance without having to solve differential equations drove the research and development in the subsequent years. To circumvent this difficulty various stability criteria have been developed. These criteria gave pertinent information regarding the stability of a system without directly applying the definitions for stability and without requiring complicated numerical procedures. For testing stability, various techniques had been proposed, the most popular technique for determining the stability of a discrete time system are Routh criterion, using Mobius transformation, Schur-Cohn criterion, Marden algorithm, Jury criterion, Lyapunov method, Root locus method, Bode and Nyquist criteria. Among the above, Marden algorithm and Jury criteria are the table form of stability tests. The table form is more suitable when computers are used for stability analysis and design purposes.
2.4 ONE DIMENSIONAL LINEAR TIME INVARIANT DISCRETE SYSTEM

The application of linear time invariant discrete time systems are widespread and include communication systems, audio systems such as CD players, instrumentation, processing of seismic and geo-physical signals, processing of biological signals and speech synthesis.

A one dimensional casual linear time invariant discrete system can be represented by the transfer function given in Equation (2.1).

\[ H_n(z) = \frac{W_m(z)}{F_n(z)} = \frac{w_0 z^m + w_1 z^{m-1} + \ldots + w_{m-1} z + w_m}{a_0 z^n + a_1 z^{n-1} + \ldots + a_{n-1} z + a_n} \] (2.1)

The discrete time transfer function is a rational function of Z with real coefficients, and for casual systems the degree of numerator polynomial is equal to or less than that of the denominator polynomial. i.e. \( n \geq m \). The transfer function can be used to find the response of the given system to an arbitrary time domain excitation, to find its frequency response, and ascertain whether the system is stable or unstable. Also the transfer function serves as the stepping stone between desire specifications and system design.

Digital signal processing is carried out by using discrete time systems. Various types of discrete time systems have emerged since invention of digital computer such as digital control, robotics and image processing systems. Two types of processes can be applied to discrete time systems, analysis can be used to deduce a mathematical representation for a discrete time system or to find the output signal produced by a given input signal. Design, on the other hand, is the process of obtaining through the use of mathematical principles a discrete time system that would produce a desired output signal when a specified signal is applied at the input. The most
common performance measure traditionally used by the design engineer involves various aspects of system responses to fairly simple deterministic input function. These measures include criteria known as stability, speed of response, peek over shoot, steady state response and frequency response. Among the above listed performance parameters stability is a prerequisite for system usefulness and must be assured while examining system performance relative to other design criteria. Stability is very important characteristic of the transient performance of the system. Almost every working system is designed to be stable. It is an important research area called for many scientists and engineers’ time, in the last two centuries. There are many contributions in the stability analysis of one dimensional linear time invariant discrete time systems.

Marden (1949) has given a tabular method similar to that of Routh criterion which provides a ready means of investigating the absolute stability of an nth order discrete system from its characteristic equation which has real coefficients. The determination of asymptotic stability of higher order systems is simple compared to that of Hurwitz criterion. The main drawback of this method is that the stability is ascertained only after completion of the entire table. Also in the case of unstable systems, the information about the root distribution cannot be obtained.

Jury (1961) discussed a simplified form of the Schur-Cohn criterion that can be readily applied to the stability test of linear discrete systems. In this the stability constraints for an nth order system is reduced to evaluation of (n-1) determinants and two auxiliary conditions, the total number being (n+1) or n if the auxiliary constraint is combined into one. However, by using the Lienard-Chipart or the modified Routh-Hurwitz criterion the number of these constraints could also be reduced to (n+1).
Jury (1962) discussed a simplified analytic test for testing stability of linear discrete systems. The necessary and sufficient condition for a real polynomial in the variable $Z$ to have all its roots inside the unit circle was derived from the test. The test was applied directly in the $Z$ plane with minimum number of constraint terms. General conditions on the number of roots inside the unit circle for $n$ even and odd were also presented.

Jury (1964) presented a slightly modified form of the table to obtain the Schur–Cohn determinants and stability constants $A_k \pm B_k$ in direct manner. The modified table requires only the solution of lower degree polynomials for the design purposes. In the case of stable systems, the bound on the maximum root within the unit circle can be determined for the purpose of determining the relative stability. This modified table provides information on the root distribution including the singular cases.

Mansour (1965) investigated sufficient conditions for instability whose inverse is necessary for stability. For checking instability of system ranges of coefficients of the polynomials were determined and for stability of sampled data system some restrictions on the magnitude of the characteristic polynomial were obtained. These conditions are very useful for stability investigation.

Jury (1967) described alternate coefficient constraint for stability of linear discrete time systems. These are obtained from the coefficients of characteristic polynomial. These are useful for stability test and also for determining the roots inside the unit circle. The disadvantage of this method is the information about the number of roots on and outside the unit circle cannot be determined.

Anderson et al (1973) formulated a new test for deciding whether a prescribed real polynomial of degree $n$ has all its roots inside the unit circle.
The test involves examination of sign of various linear combinations of the polynomial coefficients. It also examines the positive definiteness of symmetric matrix of different dimensions.

Raible (1974) proposed a modification of Jury’s procedure for the determination of the distribution of roots of polynomial with respect to the unit circle. Its advantage is the direct determination of root distribution in nonsingular cases. However Jury (1975) commented that this modification does not offer any computational simplification and utilized auxiliary constraints for obtaining the root distribution.

Schussler (1976) formulated a theorem dealing with properties of a real polynomial of degree n, the denominator of the transfer function of a stable discrete system and derived relationships to equivalent properties of a Hurwitz polynomial. It can be used as the basis for structures of non recursive filters requiring cascades of non recursive blocks of second order, which has its zeros on unit circle.

Rao et al (1976) presented a simple method and eliminated singularities encountered in the development of Jury’s table, by expanding and contracting the unit circle infinitesimally and also determined the number of roots on the unit circle. But in the non singular case the method may require few more computations. However, Bayoumi et.al, (1981) commented the method of expanding and contracting the unit circle infinitesimally dose not handle all singular cases.

Bistritz (1983) investigated the stability of linear discrete systems by using a new stability testing table. The methodology involves a significant reduction in size and computation. The main drawbacks of this method are, the coefficients of characteristic polynomial not directly substituted in the table and furnishes information about only absolute stability.
Bistritz (1984) described a method to determine the location of the zeros of a discrete system polynomial with respect to the $z$-plane unit circle. The analysis was carried out by a tabular formulation. The method is based on a sequence of symmetric polynomials. The table has comparable size and arithmetic as that of Routh table.

Edmund Jonckheere et al (1989) claimed that Raible’s table which was a simplified version of Jury’s stability test was none other than classical Schur algorithm performed on certain scattering function. As a main result the formula for the triangular factorization of the Schur-Cohn matrix in terms of the Raible table was derived and proved the Jury-Raible test for discrete stability was given as direct application of Schur algorithm.

Jury (1991) presented a missing link between the various stability criteria for linear discrete time systems as a note. The inner determinants of $(\Delta_{n-1} \pm)$ were obtained from the proper entries of the modified stability stable. This required expansion of only 2x2 determinants instead of higher determinants. Also other pertinent information was obtained from the table entries.

Premaratne et al (1993) proposed a polynomial array approach to tabular methods. For obtaining the critical stability constraints directly from the entries of the Bistritz table. When dealing with stability determination of two and multi-dimensional discrete time systems, the tabular methods together with polynomial array methods are indispensable.

Yuval Bistritz (1996) presented a modification to an efficient procedure to determine zero locations with respect to unit circle of polynomials with complex coefficients. It also bears more direct relation to the Schur-Cohn test. The test involves a comparable elementary arithmetic operation. For real polynomials, it has a slight advantage and preferable for
handling certain applications like feedback control of linear shift invariant systems.

Over the decades the Jury (1964) test has been considered as the most typical way of determining the Schur stability of real polynomials. The original proof of Jury test is based on Rouche theorem which is quiet complicated, and so some attempts have been made to give simple proofs of Jury test. Keel et al (1998) described a proof of Jury test for root distribution with respect to the unit circle using Raible’s simplified table under the assumption that characteristic polynomial has no root on the unit circle. The proof is based on an elementary property of root loci of associated family of polynomials. Song Zhaoqing et al (2000) discussed the nonregular case of Jury test. Younseok Choo (2011) carried out another elementary proof of Jury test directly for the original form of the Jury test.

Stability and performance of discrete time system is closely related to the root distribution its characteristic equation with respect to a certain shifted circle in the complex plane. Premaratne et al (2000) described an algorithm that checks root distribution of the given polynomial with respect to this stability boundary. It is based on a scaled version of the Marden-Jury table.

Park & Ikeda (2004) proposed a novel approach to stability analysis of linear time invariant discrete time systems based on experimental input and output data. Unlike conventional model based stability analysis, no mathematical model is employed. Data based stability conditions have been proposed for both open loop and closed loop systems.

Critical stability constraints are used to maintain the stability of a system when some parameters are perturbed from a nominal stable setting. Yuval Bistritz (2006) discussed the critical stability constraints for linear
discrete time systems. He introduced a new approach based on an efficient integer preserving variant of Bistritz Test (1996). Comparison with the modified Jury test, the constraints are obtained with less computation and appear as polynomials with degrees lower by a factor of two.

2.5 ORDER REDUCTION

For realization, control or computational purposes, it is usually desirable to be able to represent a higher order system by a lower order model. The fast development and use of small computers and processors in the design, analysis and implementation of dynamic systems increases the importance of model reduction procedure. Of course such procedures should approximate the original higher order system and also should be convenient to implement. Also the procedure should assure the stability of the original system once the reduction is complete. For one dimensional linear discrete time systems the following are the review of earlier research articles published several methods are available for model reduction.

Chen et al (1968) presented a technique for the model simplification problem. The reduced order model of a higher order system is obtained by expanding it into a continued fraction and ignoring some quotients if the higher system is represented by its transfer function model. In the case of state equation form the method is realized by partitioning the matrix and discarding some parts and gives the unified view point of the general analysis of linear systems. Shamash (1973) extended the continued fraction and time moments methods used for the order reduction of linear continuous time systems for use in reducing the order of linear discrete time systems. Continued fraction methods were used for the reduction in order of linear time invariant discrete time systems. Cauer type continued fractions were used to derive reduced order models which fit a combination of the
Markov parameters and time moments of the system. An application of the above methods of reduction is in the identification of discrete time systems.

Bistritz (1979) has given a solution for the problem of reduced order modeling of higher order linear time invariant single input, single output systems. The solution is based on manipulating two chebyshev polynomial series, one representing the frequency response characteristics of the higher order system and the other representing the approximate low-order modeling were formulated and both are based on expanding the transfer function of a given system into orthogonal polynomial series over a specified frequency interval.

Vaidyanathan (1985) developed a general theorem for the degree reduction of a bounded real digital filter transfer function. It is related to the extraction of a digital lossless two pair from a bounded real function in such a manner that the remainder function is reduced order bounded real function. The main application of this development is in the synthesis of low sensitivity digital filter structures that are free from zero input limit cycles.

Jury et al (1986) extended the one dimensional model reduction method to two dimensional discrete time systems. It is found by counter example that contrary to the one dimensional discrete systems, stability is not guaranteed for the reduced model in general. However stability is guaranteed for the reduced model if the original system is stable and also it is of separable form.

The design of feasible controllers for high dimensional multivariable systems can be greatly aided by a method of model reduction. In order for the design based on order reduction to include a guarantee of stability, it is sufficient to have a bound on the model error. Al-Saggaf et al
(1987) provided such a bound for discrete linear multivariable systems based on balancing.

Vijaykumar et al (1997) proposed a method of order reduction for linear discrete systems with real poles. Poles of the reduced order systems are synthesized using Eigen spectrum analysis and zeros are determined by exact matching of steady state parts of the transient response, minimization of error between the transient parts. By matching the two important properties of the system a reasonably acceptable reduced order model is obtained and the same is being extended to systems with imaginary poles.

Mukherjee et al (2004) presented a computationally simple approach for order reduction of linear time invariant discrete system. The dominant poles are retained in the lower order model. Steady state part of the transient response is exactly matched between the reduced order and original higher order system. By minimizing the error between transient parts of the responses of reduced and higher systems, the zeros of the reduced order system are determined. In this method the stability of the reduced system is assured.

Satakshi et al (2005) developed a method of linear discrete system order reduction using a genetic algorithm (GA) to avoid the difficulties encountered in classical methods. The method gives better results in terms of $J$, for systems having different kinds of pole distribution. Improvement of the quality of reduced order model obtained using GA in case of linear discrete systems, but in order to use the approach further, for more verities of systems as well as for applications of reduced order model in different areas, specific systems have to be modeled.
Modeling practical systems usually results in system of higher order. Mukherjee et al (2007) proposed a model order reduction method for linear invariant systems based on the concept of dominant retention and exact matching of transient responses of lower and higher order models of the systems.

Reduction of single-input single-output (SISO) discrete systems into reduced order model using the conventional and bio-inspired evolutionary technique is used in controller design. Yadav et al (2009) presented both the techniques for model reduction of discrete systems. In the first method the original discrete system was first converted into equivalent continuous system by applying bilinear transformation and the order reduction was carried out by using the modified Cauer form and differentiation in continuous system. The reduced order model obtained was transformed back into discrete system using inverse bilinear transformation. In the second method the differential evolution optimization technique was used for getting the reduced order model based on the minimization of the integral squared error between the transient responses of the original higher order model and reduced order model pertaining to a unit step input.

Yadav et al (2010) proposed a model reduction method that uses the modern heuristic optimization technique in the procedure to derive the stable reduced order model for the discrete system. The algorithm has also been extended to the design of controller for the original discrete system. An error minimization technique is employed for both order reduction and controller design. Algorithm preserves more stability and avoids any error between the initial or final values of the responses of original and reduced model. This approach minimizes the complexity involves in the direct design of proportional integral derivative (PID) controller.
2.6 MULTI-DIMENSIONAL LINEAR TIME INVARIANT DISCRETE SYSTEM

Multi dimensional (n-D) digital filters find many applications in several practical areas such as geo-physics and processing of sonar and Radar data. The design of non recursive filters is relatively uninvolved. For a given response characteristic, recursive digital filters have less hardware requirements. But the stability constraints associated with the design of such filters is a major problem.

A casual n-D dimensional linear shift invariant digital filter is characterized by the rational transfer function

\[ T_1(z_1, z_2 \ldots z_n) = \frac{A(z_1, z_2 \ldots z_n)}{B(z_1, z_2 \ldots z_n)} \]

Where \( A \) and \( B \) are polynomials in the independent n complex variables.

2.6.1 Two Dimensional Discrete System

Two dimensional (2-D) digital filters find many applications, such as bio-medical electronics, image processing, and seismic record processing and in all applications involving two independent variables. For a given response characteristic, recursive filters have less hardware requirements and so wherever linear phase is not a requirement, recursive filters are preferred. But the problem of stability is associated with the design of digital recursive filters.

A casual 2-D dimensional linear shift invariant digital filter is characterized by the rational transfer function

\[ T_1(z_1, z_2) = \frac{A(z_1, z_2)}{B(z_1, z_2)} \]
Where A and B are polynomials in the independent complex variables $z_1$ and $z_2$

The reviews of related literature available in this topic are given below.

Shanks (1972) described two dimensional (2-D) recursive filters in terms of two dimensional Z-transforms. The designer of these filters faces two fundamental problems, their stability and synthesis. Stability is determined by the location of the zero valued region of the filter’s denominator polynomial. Two dimensional filters can be synthesized to approximate large varieties of two dimensional pulse responses. By using this general theorem, the stability can be computed for a 2x2 array and a mapping technique is used for the estimation of stability of larger denominator arrays. A given nonminimum phase denominator array can be stabilized by computing two successive planar least squares inverse filters. More work needs to be done on frequency domain synthesis and some other mapping techniques other than the simple rotation might be useful.

Haung (1972) discussed some aspects of stability problem in two dimensional recursive filtering. In particular derived a simplified version of stability theorem to test stability of two dimensional recursive filters in the frequency domain and demonstrated that its results are comparable to the theorem due to Ansell (1964) and Shanks (1972).

Anderson et al (1973) formulated a procedure for testing the stability of 2-D filter by checking whether or not a prescribed polynomial in the variables $z_1, z_2$ is non zero in the region of unit bidisc. The procedure does not involve the use of bilinear transformations. Key parts of the test involve the construction of a Schur-Cohn matrix and checking for positivity of the unit circle of a set of self inverse polynomials.
Maria et al (1973) proposed a method to check the stability of two dimensional recursive filters. The Jury table is modified and used to check the first condition of Haung’s theorem. In this, the stability of two dimensional digital filters with complex coefficients is checked by using an alternative procedure. The method has the advantage that all the determinants used in computation are of order two.

Siljak (1975) developed a recursive algebraic algorithm for testing various stability properties of two variable polynomials in finite number of steps. Stability is tested with respect to either the half plane or the unit circle. When compared with the existing stability criteria by Ansell (1964), Shanks et al (1972), Haung (1972) and Anderson et al (1973), this algorithm is simple.

There are sources of error in digital filters, analog to digital conversion error, error due to finite representation of filter coefficients and quantization error due to rounding of the result of multiplication of data with the filter coefficients. These nonlinearities give rise to nonlinear effects such as limit cycle oscillations. Maria et al (1975) studied these limit cycles from the deterministic point of view to determine its period, a bound for its amplitude and condition for its existence. Period and bounds for the amplitude for the limit cycles in the rows, columns and diagonals were found. Several sufficient and necessary conditions for the existence of limit cycles in the rows, columns and diagonals were found. A similar scheme was reported by Tien & Lin Chang (1977).

Bose (1977) introduced a stability test, by extending 1-D digital filters stability test to verify the stability of 2-D filters. The test and its implementation were based on the alteration of simple zeros of two polynomials over specified contours and the continuity property of zeros of a polynomial as a function of its coefficients. The presence or absence of real
zeros in a known interval was determined by Sturm’s theorem. It emphasized that knowledge of the exact location of zeros is not necessary.

Dennis Goodman (1977) presented a detailed discussion of stability of two dimensional linear digital filters and the differences between the one dimensional and two dimensional filters. It is shown that the fact that the impulse response trails of to zero or more stringently is square summable does not guarantee bounded input –bounded output(BIBO) stability. Necessary conditions for the impulse to be bounded and sufficient condition for it to be square summable and to approach zero geometrically along any fixed column or row are stated.

Karivaratha Rajan et al (1980) studied a class of two variable Hurwitz polynomial and its properties. Their application in the generation of two variable functions without non essential singularities of the second kind is indicated. Necessary and sufficient conditions on general and reactance one variable to two variable transformations without singularities of second kind are obtained.

Kamen (1980) developed a theory of asymptotic stability for large class of linear invariant half plane 2-D digital filters. Various necessary and sufficient conditions are derived for asymptotic stability. The stability theory is based on the treatment of linear shift invariant 2-D digital filters in terms of a 1-D difference equation whose coefficients belong to algebra of 1-D functions.

Reddy (1981) proposed an alternate approach for testing the stability of 2-D causal recursive digital filters which are devoid of non essential singularities of second kind. This method makes use of both Schur-Cohn Hermitian matrix formulation and bilinear transformation in a single variable. In this approach the Sturm’s test need not be performed. In addition
a set of simple necessary and sufficient conditions for a class of 2-D filters are derived.

Alexander et al (1982) presented two conditions for stability of 2-D digital filters in the presence of non essential singularities of second kind. The first is necessary condition, expressed in terms of tangent algebraic curve at a zero of the denominator polynomial and distinguished boundary of the unit poly disk. This necessary condition is preserved under parameter quantization in some cases. The second condition is sufficient condition for stability and is considerably weaker than imposed by Goodman.

Karan et al (1986) published a new stability test for two dimensional digital filters. The new procedure developed for testing stability is based on the new stability testing table reported by Bistritz (1983) for one dimensional discrete linear systems. The implementation procedure so developed results in simplification in testing stability. The presented test is based on the development of the table.

Roytman (1987) presented necessary and sufficient conditions for stability of a 2-D digital filter with simple non essential singularities of second kind on $T^2$, and the generalized form of the results for special cases were given.

During the last decade, several stability test for 2-D and N-D filters were presented. Jury (1988) contributed a modified stability table for checking the stability of 2-D digital filters. In this table, the appropriate entries of the first column are equivalent to the appropriate minors of the Schur-Cohn matrix. The modified stability table is for complex polynomials. It significantly reduces the computational effort for testing stability of 2-D discrete systems.
Karivaratha Rajan (1988) described a procedure to test given two variable polynomial to be a discrete scattering Hurwitz polynomial. In this procedure, Schur-Cohn matrices associated with the given polynomial are employed to test the set of necessary and sufficient conditions. This test procedure is based on setting up of an inner matrix for the polynomial. In this a procedure to test alternate set of necessary and sufficient conditions for a discrete strict Hurwitz polynomial is developed. This procedure is based on setting up of Schur-Cohn matrices associated with the (2V) two variables polynomial.

Xiaoning Nie et al (1989) formulated a theorem on a continued fraction expansion of a complex discrete reactance functions and derived an algorithm for the stability test of digital filter directly in the 2-domains. The algorithm is formulated in a table form can simply be programmed.

Kanellakis et al (1989) proposed a new stability testing algorithm for single-input single-output (SISO) two dimensional discrete time systems using a method in the z-domain exclusively. It is based on expanding the bilinear discrete reactance function into z-domain continuous fractions in a mixed fashion. The implementation has the form of a table which in the 1-D case is the bilinear image, in the z-domain, of the generalized Routh table of continuous time systems. The interpretation of the corresponding table of 2-D systems is analogous. A similar test was described by Antoniou (1990) based on state space stability criterion that reduces stability test for 2-D systems to stability test for 1-D systems.

When a two dimensional discrete system is designed by certain synthesis methodology, one often needs to know how robust it is in terms of stability under parameter variations due to model inaccuracy, measurement noise or in the content of digital filter, quantization error in implementation, it is therefore desire to define a quantitative measure of the perturbations that
will not cause instability and to develop a feasible procedure to compute it. 2-D stability under parameter variation, Lu (1989) proposed a method through the use of DeCarlo’s stability criterion and the Gastinel-Kahan theorem for the measure of stability robustness to the 2-D case. To accomplish the numerical computation an easy to use linear algebra type algorithm was developed, which makes it feasible to use the measure in practical design procedures. Shentov et al (1989) presented a procedure for checking the stability of 2-D digital filters with real coefficients. The key part of the test involves the construction of related all pass sections of reduced order, the stability of which was shown to be equivalent to the stability of the original function. Explicit formulas for checking the stability in terms of the filter coefficient are derived for some special cases. The procedure involves successive reduction of the order of certain alpha section, whose stability depends on the stability of original filter. The stability of the overall filter was assumed to depend on the denominator polynomial and hence the cases of reducible 2-D transfer function were not considered.

Zhiping Lin et al (1989) studied the open problem regarding the BIBO stability of inverse 2-D digital filters in the presence of nonessential singularities of the second kind on \( T^2 \). A necessary and sufficient condition has been derived for a class of 2-D functions. It was shown that there exist BIBO stable filter transfer functions having simple non essential singularities of the second kind on \( T^2 \), that also admit BIBO stable inverses.

Stability testing of 2-D recursive digital filters has been studied extensively in the past two decades and many results have been obtained. However the essential difficulties still remain. Guoxiang Gu (1990) proposed a numerical algorithm for stability testing of 2-D recursive filters. Differently from existing results, the algorithm consists of stable programs which are available in many software packages. It consists of a Cholesky factorization
and cosine discrete transform as well as table method for testing zero locations of 1-D polynomials.

Swamy et al (1990) derived the necessary and sufficient conditions for a very large class of functions for areal rational transfer function $G(z_1, z_2)$ and its inverse $G^{-1}(z_1, z_2)$ to be both bounded-input bounded-output (BIBO) stable in the presence of nonessential singularities of the second kind on $T^2$, the distinguished boundary of the unit bi disk. Necessary and sufficient conditions were obtained for the stability as well as the boundedness. Swamy et al (1992) presented a necessary condition in the 1-D domain for testing the stability of certain types of 2-D digital filters.

Whalley (1990) discussed the analysis of 2-D digital filters in terms of algebraic operations and boundary image maps. An algorithm for stability assessment was derived. In this method, much of tedium involved in the computation of the boundary images of filter models is removed by constructing an algorithmic test that represent the mapping of the extremity of the boundary image relative to the origin of the complex plane.

Xiheng Hu (1991) developed a method for 2-D complex polynomials by using polynomial array to overcome the difficulties in the implementation of algebraic methods for 2-D filter stability test. It reduces the task for constructing a polynomial array into computations on only a set of second order determinants with numerical entries and it is compatible to implement using computers regardless of the order of the polynomial under test.

Much interest has been shown during recent years in testing the stability of 2-D and N-D discrete systems. Kanellakis et al (1991) presented a method for testing stability of 2-D systems, the stability properties of Schwarz
matrix were used for testing the positivity of the Hermitian solution of the frequency dependent Lyapunov equation for discrete or continuous systems.

Bistritz (1999) described a method to test the stability of 2-D linear discrete system polynomials which provides a definite answer in finite number of arithmetic operations. The test consist of a sequence centro-symmetric matrices referred as a 2-D table that was constructed by a three-term recursion of 2-D polynomials and of a few accompanying conditions on 1-D polynomials. The algorithm for construction of the 2-D table as a simple recursive form that is readily implemented in a matrix oriented environment.

Bistritz (2001) developed a procedure for testing the stability of 2-D discrete time systems. The procedure determines whether the two variables polynomial has no zeros in the exterior. It is a simplified form of immittance type tabular stability test for 2-D discrete time systems. The test avoids construction of the table instead it bring forth the last 1-D polynomial of the 2-D table by interpolation. This replaces the construction of 2-D table by testing the stability of a finite number of 1-D polynomials using associated 1-D stability testing algorithm. It consists of a collection of 1-D stability test. The test requires an apparently low count of arithmetic operations.

The Schur & Cohn test plays an essential role in checking the stability of 1-D random processes such as auto regressive models by using reflection coefficients, partial correlations in the context to the 2-D random field modeling. Oliver Alata et al (2003) derived two necessary stability conditions for a 2-D auto regressive quarter plane model. One of these conditions is an extension of the Schur-Cohn stability criterion based on the 2-D reflection coefficients. However these stability conditions are necessary but not sufficient. Accordingly if they are satisfied, a test like Strintzis’ must also be used to decide upon the BIBO stability of the filter.
Bistritz (2005) proposed a set of new results; one is an improved form for the Maria- Fahmy 2-D stability table that deals with a tighter set of necessary and sufficient conditions for stability. The other is a new set of necessary and sufficient conditions for stability that holds for certain univariate polynomials that extend condition that hold for the reflection coefficient parameter in the Schur 1-D stability test.

Tamal Bose (2007) studied the stability of 2-D periodically shift varying filters formulated as the Givone-Rosser model. The applications of these filters include processing video signals with cyclostationary noise, image and video scrambling and design of multiplier less filters. Several sufficient conditions and one necessary condition were derived for stability. These conditions are compared for their relative computational complexities and restrictions. Based on the complexities of implementing these conditions, an algorithm was proposed to determine the stability of given 2-D periodically shift varying system.

2.6.2 n-Dimensional Linear Discrete System

Anderson et al (1974) obtained conditions and computational techniques for checking the stability of digital filter in three or more variables. The prescribed multi variable polynomial stability is determined by checking a set of multi variable polynomials with different number of variables using simpler stability conditions. This simplification lies in the fact that all but one variable in each of this set of polynomials lies on the unit circle, rather than either inside or on the unit circle. The checking of simpler stability conditions for each of these polynomials was carried out by checking the positivity of a number of polynomials which are readily computable.
Bose et al (1974) presented a systematic procedure to test for stability of discrete and continuous three dimensional filters. A test is based on repeated applications of an extended Hermit or Schur & Cohn formulation, and use of Sturm’s theorem to determine the content of a system of polynomial inequalities in a single indeterminate. The stability test of 3-D filters reduces to tests for sign variations of a set of real single variable polynomials. The stability test for higher than 3-D filters require tests for sign variations in a set of two variable polynomials.

Bose et al (1974) proposed an algorithm with a view towards computer implementation for stability test of filters of arbitrary dimension and complexity. The algorithm is based on the generation of number of multi variable polynomials, reduction of each of these into several single variable polynomials by a finite number of rational operations, and a scheme of repeated single variable polynomial factorization and back substitutions.

Strintzis (1977) established a stability criterion for multi dimensional digital and analog filters with rational transfer functions. The criteria generalizes and simplifies the stability test for 2-D digital filters developed by Haung (1972) and significantly simplifies the corresponding tests of stability of arbitrary multi dimensional filters established by Anderson et al (1973).

Bose (1979) demonstrated the availability of an alternate approach to n-D stability testing. The procedure dwells on the generation of a sequence of polynomials in an increasing number of real variables, which need to be tested for local positivity. It is an extension of 2-D digital filters stability test by the same author (1977).
Ezra Zeheb et al (1981) formulated a theorem to find the zero set of multi parameter rational function of a complex variable where the boundary of each parameters domain of definition, in the closed complex plane, is closed and composed of piecewise smooth Jordan curves. Based on the theorem simplified procedure for multi dimensional stability tests are derived for the continuous as well as for discrete case. The main computation burden of checking stability of a multi dimensional system is to check whether a multi variable polynomial has zeros on the distinguished boundary of certain region of analyticity. Zeheb (1984) formulated a procedure for reducing computational burden by using a transformation, by which the original polynomial is transformed into another expression which has zero on the distinguished boundary of the unit poly-disk of its variables if and only if the original polynomial has such a zero.

Walach et al (1982) presented procedure for computation of stability margins of n-D linear discrete systems. The stability margins are defined in terms of the singularities of the transfer function. It serves as a measure, in some sense for the tendency of a stable n-D system to become unstable.

Swamy et al (1985) examined some stability properties of three and higher dimensional linear digital filters and outlined differences between 2-D and higher dimensional cases. Also the authors presented necessary and sufficient conditions for stability for specific cases.

The problem of bounded-input bounded-output (BIBO) stability is important for the design of recursive digital filters. Roytman et al (1987) gave the sufficiency conditions for the stability of certain class of 3-D function with non essential singularities of the second kind.
Bauer et al (1991) derived the stability conditions for m-D shift varying discrete with zero initial conditions. Also the issues of robustness and margin of stability of the shift varying systems were discussed. The basic philosophy behind the methods developed is the construction of a linear shift invariant system, the states of which constitutes an upper bound to the shift varying system states. The test for one particular shift varying m-D system involves one m-D linear shift invariant stability test.

Bauer (1992) demonstrated that an m-D digital filter with singularities on the distinguished boundary is asymptotically stable for a finite extent input signal. Further a BIBO unstable m-D digital filter might behave asymptotically stable under finite word length conditions.

Tianguang Chu et al (2003) analyzed the stability of a class of discrete linear multi dimensional systems whose solutions are path dependent and may not be uniquely specified by initial conditions. Based on the concept of solvable Lie algebra and comparison principle, it presents a simple necessary and sufficient condition for exponential stability of the multi dimensional systems in terms of the spectral radius of the system matrices.

Mastorakis et al (2003) applied genetic algorithm (GA) for the stability analysis of m-D discrete systems. The stability of m-D systems is considered as minimization problem. The m-D stability problem is reduced to an appropriate minimization problem by using the last condition of Decarlo-Strintzis theorem and stability was analyzed using GA.

Ioana Serban et al (2007) contributed a multi dimensional Schur & Cohn type stability criterion and a procedure for testing this criterion. The implementation technique in the 2-D case was discussed. This criterion involves the use of functional Schur coefficients and leads to multi dimensional extension of the Schur-Cohn algorithm. Ioana Serban et al (2007)
also presented an algorithm for checking the stability condition of a 3-D filter, based on the slice functions mechanism and the functional Schur coefficients.

2.7 STABILIZATION

Jury et al (1977) verified Shanks’ conjecture that is used for stabilization of 1-D recursive filters. Then extended the technique to a class of polynomials of higher degrees in the same variables and investigated the condition under which the conjecture is valid for the special class of polynomials. Though the conjecture is known to be false in general, explored some conditions under which the conjecture is valid.

Swamy (1985) given a theorem based on planar least squares inverse (PLSI) polynomial for an n-D discrete system said to be practical BIBO stable compared the statement of the main theorem with the necessary and sufficient conditions for practical BIBO stability and proved that they are the same.

Mauro Bisiacco (1985) introduced controller theory for multivariable 2-D systems. As a consequence in 2-D systems theory, proposed a stabilization technique for stabilizing intrinsically unstable 2-D systems based on dynamic feedback law which preserves the quarter plane casualty.

Minzhi Zhang et al (1994) investigated the problem of local asymptotic stabilization of a class of two dimensional systems and developed a criterion based on continuous state feedback for local asymptotic stabilization of a class of two dimensional systems.
Engelo Borghs et al (2001) described the limitations of certain stabilization methods for time-delay systems. The class of methods considered was implemented the control law through a Volterra integral equation of the second kind. The analysis provided computable limitations to stability and a maximum allowable of the delay.

Tantaris et al (2003) presented a computationally effective procedure to determine all first order controllers that stabilize the given discrete linear time invariant system. The complete set of stabilizing controllers was determined in the controller parameter space. The solution involves Chebyshev representation of the characteristic equation on the unit circle. By adding more equations, the solution was found.

Xiaodong et al (2004) addressed the robust stabilization problem by using ortho-normal rational functions constructed from the Jury table and compressed Hankel matrix. The solution to the optimal and suboptimal Nehari problems was given via the compressed Hankel matrix.

Huijun Gao et al (2005) investigated the problem of stability analysis and stabilization of two dimensional discrete systems with stochastic perturbation. By means of linear matrix inequality (LMI) techniques, he addressed the robust stabilization problem for 2-D systems with both deterministic and stochastic uncertainties. Both stability and controller existence conditions were expressed as LMI condition.

Shipei Huang et al (2013) proposed a method by using a state feedback controller to guarantee the exponential stability for a class of uncertain 2-D discrete switched systems with state delays under asynchronous switching. The parameter uncertainties were assumed to be norm-bounded. The dwell time approach was utilized for the stability and controller design.
2.8 SUMMARY

In this chapter the basics of the techniques applied in the research were reviewed. It also gave a short walk through of the research works in stability analysis, model reduction and stabilization. It can be observed that the wide research had been made on the stability analysis of one dimensional, two dimensional and multi dimensional linear discrete time systems. A number of model reduction techniques based on integral squared error, matching of transient responses and using evolutionary algorithms had been experimented for the design of linear discrete time systems. It is also learned that stabilization algorithms had been developed to stabilize a variety of unstable linear discrete time systems. A survey has also been made on the usage of tabular methods for the performance analysis, model reduction, design and stabilization of linear discrete time systems.