CHAPTER 7

SUMMARY AND SUGGESTIONS FOR FUTURE WORK

The contributions made in the thesis are as follows:

In general, an introduction to stability and purpose of stabilization of 1-D LTIDS are represented in chapter 1, while the survey of the work related to thesis are presented in chapter 2.

In chapter 3, certain improved steps and procedures were suggested, which are the modified approaches obtained from the Marden Table as well as from Jury Table for 1-D LTIDS. Illustrations including stability, distribution of roots as well as design of single parameter in a control system were provided to depict the applications of the suggested methods.

In chapter 4, the lower order model of the higher order system was derived by using the steps and procedures suggested in chapter 3 and genetic algorithm was employed for the fine tuning of the parameters of the obtained model, the impulse response matching and ISE was used as performance measurers for making decision on tuning parameters.

In chapter 5, certain class of 2-D LTIDS represented in the form of characteristic polynomial was reduced into respective 1-D equivalent polynomial and the stability analysis are carried out extending the proposed method presented in chapter 3. Examples are included for illustrative purpose. The steps and procedures introduced in this chapter were extended suitably to
investigate certain class of multidimensional LTIDS and substantiated by illustrative examples.

In chapter 6, stabilization of certain class of unstable LTIDS was carried out by adopting the steps and procedures discussed in chapter 5. The output response of the stabilized 2-D system is depicted with help of unit impulse response along with the integral square error (ISE)

**FUTURE WORK**

The contributions made in the Thesis can be extended to handle the following domains:

1. Design of controllers and compensators
2. Characteristic polynomial containing complex coefficients.
3. Non linear systems.
4. Fuzzy systems.
5. Time interval systems.
6. Heuristic approaches particle swarm optimization, Ant colony algorithm and Honey Bee algorithm may be identified instead of Genetic algorithm for optimizing parameters for stabilization
7. Delay-Differential systems.
APPENDIX 1

Root Distribution of Linear Time Invariant Discrete Systems Using Marden Table

**Jury**

\[
F_n(z) = a_n z^n + a_{n-1} z^{n-1} + a_{n-2} z^{n-2} + \ldots + a_1 z + a_0 \quad (A1.1)
\]

\[
a_n \quad a_{n-1} \quad a_{n-2} \quad a_2 \quad a_1 \quad a_0
\]

\[
a_0 \quad a_1 \quad a_2 \quad a_{n-2} \quad a_{n-1} \quad a_n
\]

\[
b_0 = \frac{a_n^2 - a_0^2}{a_n}
\]

**Marden**

\[
F_n(z) = a_n z^n + a_{n-1} z^{n-1} + a_{n-2} z^{n-2} + \ldots + a_1 z + a_0
\]

\[
a_0 \quad a_1 \quad a_2 \quad a_{n-2} \quad a_{n-1} \quad a_n
\]

\[
a_n \quad a_{n-1} \quad a_{n-2} \quad a_2 \quad a_1 \quad a_0
\]

\[
b_0 = a_n^2 - a_0^2 = \delta
\]

**Relationship between Marden and Jury Coefficients:**

**Jury:**

\[
b_0 = \frac{a_n^2 - a_0^2}{a_n}
\]

**Marden:**

\[
b_0 = a_n^2 - a_0^2 = \delta
\]  

(A1.2)
\[ b_0 = \frac{1}{a_n} \delta \]

\[ \text{Jury Coefficient} = \frac{\text{Marden Coefficient}}{\text{Leading Coefficient}} \]

\[ b_0 = \frac{\delta_k}{b_{k-1}} \quad \text{(A1.3)} \]

Example: \[ b_0 = \frac{\delta}{a_n} \]

Sign relationship to assess IUC / OUC

**Case (i):** \( a_n > 0 \) that is Positive

Sign of Jury Coefficient = Sign of Marden Coefficient

Jury Coefficient = Scaled value of Marden

JURY TABLE = MARDEN TABLE

**Case (ii):** \( a_n < 0 \) that is negative

Sign of Jury Coefficient = (Sign of Marden Coefficient) * (-)

\[ = - \text{(Sign of Marden Coefficient)} \]

<table>
<thead>
<tr>
<th>Jury Coefficient</th>
<th>Marden Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>(+) IUC</td>
<td>(-) OUC</td>
</tr>
<tr>
<td>(-) OUC</td>
<td>(+) IUC</td>
</tr>
</tbody>
</table>

Interpreting Root Distribution by using Marden (i.e.) equating it to Jury test

**Method 1:**

\[ \text{Sign of} \quad \delta_k = \text{[Sign of} \quad \delta_k \text{]} \times \text{[Sign of} \quad \delta_{k-1} \text{]} \quad \text{(A1.4)} \]

\[ \delta_k = a_0 \delta_1 \]
\[ \delta_2^* = \delta_1^* \cdot \delta_2 \]

\[ \delta_2^* = \delta_1^* \cdot \delta_2 \]

\[ \delta_k^* = \delta_{k-1}^* \cdot \delta_k \]

**Method 2:**

If leading coefficient \( a_n > 0 \) that is positive, then use Marden table formula.

\[ \delta_1 = a_n^2 - a_0^2 \quad \text{here jury coefficient is} \quad b_0 = \frac{a_n^2 - a_0^2}{a_n} \]

sign of Marden coefficient = sign of jury coefficient

If leading coefficient \( a_n < 0 \) that is negative, then

\[ \delta_1 = a_n^2 - a_0^2 \quad \text{here jury coefficient is} \quad b_0 = \frac{a_n^2 - a_0^2}{a_n} = \frac{\delta}{a_n} \quad (A1.5) \]

If \( \delta_1 \) is positive then \( b_0 \) is (-) Negative.

If \( \delta_1 \) is (-) Negative then \( b_0 \) is (+) positive.

Sign of Jury Coefficient = (Sign of Marden Coefficient) * (-)

= - (Sign of Marden Coefficient)

= - \((a_n^2 - a_0^2)\)

\[ = \frac{1}{a_n a_0} \begin{bmatrix} a_0 & a_n \\ a_n & a_0 \end{bmatrix} = \delta = \text{scaled [Marden]} \quad (A1.6) \]
From the above observations we can conclude that Marden table itself will give information about the root distribution of real polynomials by making the following change in the determination of coefficients the reduced order polynomials.

i) If the previous $\delta$ value is negative (-) then calculate (-)$\delta$.
   (Interchange the rows and proceed by using Marden formula.

ii) Else, proceed with the Marden criteria.