INTRODUCTION TO DIFFERENT APPROACHES IN POSITRON AND ELECTRON PENETRATION

- Introduction
- Range definition
- Experimental work
- Theoretical developments
1.1 Introduction

The information about the interaction of electrons and positrons through different materials is frequently required for the experimental methods in nuclear, atomic and solid state physics.

The transmission and penetration of charged particles through matter has a great importance for experimental as well as theoretical Physics. In the experimental study of the energy levels and transition of nuclei, there are many cases, which require the measurement of the kinetic energy of a charged particle by method, which depend upon absorption and scattering phenomena.

The study of the passage of electrons and positrons through matter has very useful applications in nuclear physics, semiconductor physics, health physics and several other related fields. It is also useful for the study of radiation damage, biological effect, and energy distribution at different depth in matter.

The mechanism by which a charged particle loses its kinetic energy, or is deflected from its original path, involve two principal types of interaction,

1. Inelastic interaction
2. Elastic interaction

Further we can divide these two into following

(a) Inelastic collision with atomic electrons.
(b) Inelastic collision with a nucleus.
(c) Elastic collision with atomic electrons.
(d) Elastic collision with a nucleus.
In an absorbing material, a moving particle is slowed down and finally brought to rest by the combined action of all four of these elastic and inelastic processes. A particle whose initial kinetic energy is say 1 MeV may have more than $10^4$ individual collision of each type.

Which type of interaction, if any will occur when a swift particle passes a particular atom is described only by the laws of chance. From collision theory, one can obtained the probabilities of any particular type of collision, of any particular energy loss, and of any particular change of direction of the motion of the incident particle. After the first collision, these probabilities can be applied to a second collision, then to a third, etc. This method is very complicated, but some reasonable results have been obtained, notably at Los Alamos, using electronic computing devices.

Electron bombardment of a sample is unique to microprobe analysis and produces a large number of effects from the target materials. It is critical to understand the volume of material from which these are produced and the effects themselves that includes: X-rays (both continuum and characteristic), backscattered and transmitted electrons, secondary electrons Auger electrons etc. (fig. 1.1). Ionisation, bremsstrahlung radiation and annihilation are the three mechanisms by which electron loses energy in medium. Coulomb interactions between fast moving electrons and molecular electrons excite and ionise the molecule producing ion pairs. When a fast moving electron is accelerated or decelerated, a photon is emitted, and such photons are called bremsstrahlung radiation (braking radiation).
Fig. 1.1 Effects produced by electron bombardment of a material
Although the positron is an anti particle of the electron, the cross section (probability) for direct annihilation is much smaller than that for elastic or inelastic scattering. For these latter processes, treating the positron as a positive electron and using the usual theoretical methods developed for electron scattering from atoms can carry out calculation. There are several important differences between electron and positron scattering. However since the positron is a distinct particle from the electron, exchange interactions with the bound atomic electrons do not occur. More interestingly, the formation of positronium (the bound state of an electron and a positron) can and dose occur in positron-atom scattering. In addition, the fact that the sign of the interaction between the positron and the atom is the opposite of that for electrons leads to rather different behaviour of the cross section.

As the electron passes through matter it loses its energy in ionising and radiative collision. In each of these it may suffer significant deflections. In addition there is a large number of deflection due to elastic scattering. The net result is that the electron’s path as it passes through the absorber is very tortuous. In practice, one finds the path length to be from 1.2 to 4 times the thickness of the absorber traversed, the ratio being largest for slow electrons in high Z materials.

The range R of a particle is an experimental concept, relating to the thickness of an absorber, which the particle can just penetrate. Several distinct definitions of range, which depend upon the method employed to determine them, are in common uses.
1.2 Range Definitions

If a curve is plotted between the fraction of the incident Positrons and electrons which pass through a given thickness and those thickness, it is obtained that for high thickness of the absorber the curve passes into the background, which is due to cosmic and gamma rays. Schonland[1] and Flammersfield[2], define the point at which the extension of the linear portion of the transmission curve meets the background as the practical range, the point where the tail of the transmission curve meets the background is called the maximum range. The maximum range of electrons and positrons is a very important quantity but it is very difficult to explain theoretically, because the diffusion is a complicated phenomenon, which sets in at the end of transmission curve. The extrapolation of the linear portion of the transmission curve of the monoenergetic electrons to the thickness axis known as practical maximum range. A similar definition of range has given by some workers like Ebert et al[3]. Tabata et al[4]. They define the extrapolation range $R_{ex}$, as the point where the tangent at the steepest point on the almost straight descending portion of the transmission curve meets the thickness axis.

The practical range, the practical maximum range, and the extrapolation range are very useful experimental quantity, including the straggling effects hence can not be compared with theoretical straggling free range. The range of electrons theoretically, can be given as the limiting thickness of the absorber beyond which essentially none of the originally incident electrons emerge. But due to the straggling effects such a thickness practically does not exist.

It is not possible to compare the experimental measurements with the theoretical ranges.
Fig. 1.2 The range $R_m$ along with practical range $R_p$ and the maximum range $R_o$. 
The straggling free practical range \( R_{sf}^* \) for electrons and positrons can be defined as the point where the extrapolation of the linear portion of the transmission curve meets the straggling portion of the curve, when it is extrapolated in the backward direction. All the ranges, which have been defined here, are shown in fig. (1.2) But extrapolated range \( R_{ex} \) is not shown because it cannot be shown on semi log graph.

It has been proved experimentally that the range \( R_{sf}^* \) is not sensitive to the strength of the used source. But extrapolated range \( R_{ex} \) is very much sensitive to the source strength and percentage of transmission reached.

For light and medium atomic number absorbers the tail of transmission curve can be shown by a straight line this tail corresponds to straggling, and error involved in its extrapolation in the backward direction is very small. For high atomic number absorbers straggling part of the transmission curve cannot be represented by a straight line because some error is possible in its extrapolation in backward direction to find \( R_{sf}^* \). In the measurement that has been done by Gill \(^5\) et al. this error is found of the order of 2% for gold and lead.

Electron’s total path length is a quantity, which is completely different from its range. The distinction can be visualized from fig. (1.3) Where \( R \) is the range.

The total path length has been observed in a few experiments, using cloud chambers or photographic emulsions. Among these, one by E.J. Williams \(^6\) provides a direct comparison of total path length \( S \) to range \( R \). In this experiment, monoenergetic electrons were produced in a cloud chamber by the photoelectric absorption of monochromatic X-rays. The
The thickness of absorber is $R + dR$.

If the absorber had been of thickness $R$, the electron would have just penetrated it and would be said to have a range $R$. The total path length $S$ is measured along the actual path of the electron and is always considerably greater than $R$. 
total path length and the maximum distance reached in the initial direction were than measured.

Fig. (1.4) presents Williams observations on distribution of path length $S$ and range $R$ in oxygen for 145 individual electrons whose kinetic energy was 19.6 Kev. Several fundamental feature of the interaction of electrons with matter can be visualised from a study of these curves in this energy domain, radiative losses are negligible and curves represent the effects of ionisation losses, elastic scattering, and straggling of energy losses.

The curve $S$ shows the fraction $f$ of the electrons whose path length exceeds the distance $D$ cm. The slope of the curve $ds/dD$ is the fraction of electrons whose path lengths lie between $D$ and $D+dD$. These slope are sufficiently symmetric about $f = 0.5$, so that the mean path length $\bar{S}$ can be taken as the path length at $f = 0.5$, when one half the electrons have been stopped.

The broad distribution of path length is noteworthy and is due to the statistical distribution of energy losses, or straggling for each electron. Some have large losses, including one or more “branches” due to hard collision, and have short path lengths. Other suffers smaller and fewer losses per millimetre of path and have much longer path lengths. The theoretical value of $(dT/ds)_{\text{ion}}$ relates to the average energy losses, and thus its integral is $\bar{S}$, corresponding to the average path length, or mean path length.

The average path length of an electron of K.E. $T$ will be
The distribution of path lengths ($S$) and of range ($R$) for 19.6 Kev electrons in O$_2$ at 0° C and 1-atm pressure:

$\bar{R} = 0.32$ cm (mean range)

$R_0 = 0.52$ cm (extrapolated range)

$\bar{S} = 0.64$ cm (mean free path length)

$S_0 = 0.82$ cm (extrapolated path length)
\[ S = \int ds = \int \frac{dT}{r_i \left( \frac{dT}{ds} \right)} \]  

(1.1)

where \( \left( \frac{dT}{ds} \right) = \left( \frac{dT}{ds} \right)_{\text{ion}} + \left( \frac{dT}{ds} \right)_{\text{rad}} \) \n
(1.2)

equation (1) will not be valid as the lower limit of T approach zero (\( T_1 \rightarrow 0 \))

1.3 Experimental Work

Some experimental work with electrons and positrons had been reviewed by Katz and penfold\[^8\] in Aluminium. They did experiment only for electron interaction. Due to lack of positron sources no attempt was made for positron transmission.

Seliger\[^8-9\] and Gubernator\[^10-11\] did some work on the transmission of positron in the energy range of 180 Kev to 960 Kev, and 50 Kev to 160 Kev. Gubernater's experiment concludes that the ranges of positron in Al are less than those of the electron in the energy region below 160 Kev. These results are in qualitative agreements with the similar measurements of Seliger. In 1959, Gubernator and Flamammerfeld\[^11\] measured the ranges of 40 to 160 Kev electrons and positrons in Cu, Ag and Au. They reported that in Cu the range of positrons is less than that of electrons of the same energy while in Ag and Au positron has large ranges than electrons.

Some work of on transmission of electrons and positrons has been done by Gill\[^5\] et al. They studied experimentally the penetration of electrons of energy \( E_{\text{max}} = 0.25 \text{ Mev}, 0.77 \text{ Mev}, 1.53 \text{ Mev}, \text{ and } 1.71 \)
Mev through a large number of materials including some rare earth materials. Also they experimentally investigated the penetration of 1.88 Mev positrons in various materials including rare earth materials then they compared the results of electron and positron transmission.

1.4 Theoretical Developments

(a.) C.S.D.A. Ranges

The rate of energy loss of electrons and positrons is always subject to statistical fluctuations. For the simplified evaluation it is assume that during slowing down process, the rate of energy loss along the entire path is always equal to the mean rate of energy loss. This is called C.S.D.A. ranges, (CSDA means, continuous – slowing – down – approximation.). The CSDA range is the path length, which a particle travels in the course of slowing down, in a homogenous medium. Berger and Seltzer\textsuperscript{[12]} calculated these CSDA ranges with energy loss due to ionisation and excitation \textsuperscript{[13]}, and also bremsstrahlung process \textsuperscript{[14]}.

CSDA ranges are calculated by integrating the reciprocal of total stopping power:

\[ R^z_{\text{cSDA}}(T) = \int_{T_1}^{T} \left[ -\frac{1}{\rho \left( \frac{dE}{dx} \right)_{\text{total}}} \right] dT + R^z(T_1) \quad (1.3) \]

where \( T_1 \) is some lower limit of energy below which the calculation can not be possible

Normally \( T_1 = 1 \text{ KeV} \)
For intermediate and high-energy electrons $R \left( T_1 \right)$ is considered negligible. And we know:

\[ -\frac{1}{\rho} \left( \frac{dE}{dx} \right)_{\text{rad}} = -\frac{1}{\rho} \left( \frac{dE}{dx} \right)_{\text{coll}} + -\frac{1}{\rho} \left( \frac{dE}{dx} \right)_{\text{rad}} \]  \hspace{1cm} (1.4)

where + ve and - ve signs are for positron and electron respectively and $\rho$ is the density of materials.

If we compare the CSDA ranges of electron and positron in any material with the corresponding experimental values, it is observed that CSDA range are always greater than the measured values.

**b.) Average penetration depth**

Rohrlich and Carlson \[^{[18]}\] calculated average penetration depth, $Z_d \pm$. Which is define as the amount of absorber thickness which when placed in the path of the beam of positrons and electrons, such that particles lose completely their initial orientation.

Mathematically the average penetration depth is given by the condition

\[ \langle \cos \theta \rangle_{\text{average}} = \frac{1}{e} \]  \hspace{1cm} (1.5)

where $\theta$ is the angle of multiple scattering.

This is the condition when the particle losses its memory of initial direction at average energy

\[ E_d = \gamma_d mc^2 \]  \hspace{1cm} (1.6)

The average penetration depth corresponding to $\gamma_d \pm$ is given by,
\[ Z_d^z = \int_{\gamma_0}^{\gamma} K_i^z(\gamma, \theta) \left[ \frac{dy}{ds} \right]^{z-1} dy \]  

(1.7)

where \( K_i^z(\gamma, \theta) = \langle \cos \theta \rangle_{\alpha}^z = \left[ \frac{G(\gamma)}{G(\gamma_0)} \right]^z \)  

(1.8)

And \( G(\gamma) = \left( \frac{\gamma+1}{\gamma-1} \right)^{\alpha} e^{\beta z / \beta} \)  

(1.9)

The constants \( a^z \) and \( b^z \) were considered to be approximately of the same order of magnitude for small and large values of atomic numbers. Using this expression Rohrlich and Carlson obtained the average penetration depths of positrons and electrons for energy ranges 0.1 to 2.04 MeV for Aluminium and lead. They found that the value \( Z_d^+ / Z_d^- \) first increase with energy upto 1.0 MeV and there after decrease with energy. While in the case of lead, this value of \( Z_d^+ / Z_d^- \) first increase with energy upto 1 MeV and then becomes constant. But they did not explain the reason of this behaviour.

Some other limitations of this method are as follows:

1. The constant \( a^z \) and \( b^z \) had been designed for \( Z_d^+ / Z_d^- \) in aluminium and lead only. It is very difficult to give separate constant for every element.

2. There is no indication has been given for the percentage error, in the final result.

3. In the expression for \( \langle \cos \theta \rangle \) and \( Z_d^z \), the rate the energy loss that is \( dT/ds \) is found. They used energy loss due to collision only; the energy loss due to radiation has been not taken into account. The estimated contribution due to radiation loss of 2 MeV electron in the case of lead is about 21% of total energy loss. And for energy
more than 2 MeV the radiation loss increase and it should be taken into account. Actually the average penetration depth given by the equation (1.5) is not experimental quantity. There is no known evidence, which could support some kind of relationship between the estimated average penetration depth and the measured ranges. Tomlin\textsuperscript{[15]} have reinvestigated the problem of electron penetration using lewis theory\textsuperscript{[16]} of multiple scattering and reported relatively simple expression for the first and second moments of the electron distribution in depth.

\textbf{(c.) Projected ranges}

Rohrlch and Carlson\textsuperscript{[13]} considered only the elastic scattering of electrons and positrons to calculate the value of $Z_d^\pm$. While the CSDA ranges\textsuperscript{[12]} were based on purely inelastic consideration. Both the approaches do not interpret the measured ranges. When electrons and positron passing through an absorber both elastic and inelastic interaction are possible. Both the process have been taken into account for calculating the projected range $R_p^\pm$ for positrons and electrons, by Batra and Sehgal.\textsuperscript{[17-20]}

The mean projected range can be define as the mean projection of the path of these particles on the direction of incident in the absorber. They considered that the inelastic scattering is statistically independent of energy loss fluctuations. Near the end of the range multiple scattering is large due to the small energy. At the large multiple scattering angle the electrons diffuse randomly this is called straggling. They assumed that the electrons first undergo a straight motion and their interaction with matter is only through inelastic process.
When electrons traverse a small thickness $x$ of the absorber the mean square angle of multiple scattering $\langle \theta^2 \rangle_x^\pm$ is required. The total stopping power $-\frac{1}{\rho} \left( \frac{dE}{dx} \right)_{ionol}$ is needed as input for calculating $\langle \theta^2 \rangle_x^\pm$.

A simple empirical relation of total stopping power found by Batra and Sehgal [17-20] this expression can easily be integrable.

In order to take into account the random motion of electrons and positrons by multiple scattering they given a definition of transport mean free path $\lambda_{tr}$, this transport mean free path can be define as the average distance a particle traverse before being scattered through an angle $\geq \pi/2$.

If $T_r^\pm$ is the energy and $\langle \theta^2 \rangle_x^\pm$ is the mean square projected angle corresponding to the instant when the motion of the particle become random, the projected range is given by:

$$R_p^\pm(T) = R_{code}^\pm(T) - R_{code}^\pm(T_r^\pm)$$  \hspace{1cm} (1.10)

These values of projected ranges [17-20] are comparable with the experimental values given by Gill et al [5]. For $6 \leq Z \leq 13$ materials the agreement is good. For intermediate and heavy elements the calculations by Batra and sehgal [17-20] give lower values of range as compared to the experimental values by Dr. Gill [5]. For intermediate $Z$ values the difference is small but goes on increasing with increasing value of $Z$. At $Z = 82$, the theoretical values are off by about 25% for particle energies of 1 MeV.

The difference in the theoretical and experimental values may be due to following reasons:
1. Batra and Sehgal \cite{17-20} had used Mott's expression \cite{21} for elastic scattering cross section, this expression is in the form of a power series in $\alpha Z$, where $\alpha = 1/137$. They used only the first term of this series and leaved the higher order terms. For small value of $Z$ the term $(\alpha Z)^2$ and $(\alpha Z)^3$ and so on can be ignored but for high $Z$ values these term can not be ignored.

2. In these calculations \cite{17-20}, the range coming from the diffusion part was not taken into account.

It is found from their reference 20, figs 5-6, that as the incident kinetic energy of electrons and positrons decreases and also with the increase of atomic number $Z$ of the absorber, the fraction of energy left with these particle increases.

If the diffusion part of the ranges is taken into account, the calculated values of straggling free practical ranges in very good agreement with experimental values.

Gill et al \cite{5} have developed a theoretical model for obtaining transmission curves of different energies electrons and positrons using the energy loss expression \cite{17-20} and Mott's scattering \cite{21} cross section for the scattering of these particles. The values of ranges and absorption coefficients thus obtained have been compared with the experimental data.

If a comparison is made between the experimental values of Dr. Gill and the theoretical values of Batra and Sehgal, \cite{18-20} the values are in good agreement for low $Z$- materials except the case of carbon. For the intermediate $Z$ the difference is about 16% and is about 30% in high $Z$ materials. If higher order term is taken into account in Mott's expression, or if the correction factor is used the values of range is
improved by about 6 to 12% in low Z and 10 to 18% in high Z materials.

If the contribution of diffusion part is also taken into account with correction factor or higher order term, the difference in experimental and theoretical values are – 0.01 % to 6% in low Z for low energy and about 6 to 9.5% for high Z at low energy. Similarly this difference is about – 0.50% to 5.3% in low Z for the energy 1.53 MeV and it is upto 8.4% in high Z, at 1.53 MeV energy.
References

7. L. Katz and A.S. Penfold, Rev. of Mod. Phys. 24, 28-44 (1952).
