PREFACE

The theory of submanifolds of a Kaehler manifold (or for that matter of any almost complex manifold with a Hermitian metric) presents an interesting geometry as its almost complex structure transforms a vector into a vector perpendicular to it, which naturally give rise to three special classes of submanifolds viz. holomorphic or invariant submanifolds (also known as almost complex submanifolds), totally real or anti-invariant submanifolds and slant submanifolds. In the first case the tangent space of submanifolds remain invariant under the action of $J$ and in the second case it goes to the normal space under the action of $J$.

In 1978 A. Bejancu introduced the notion of $CR$-submanifolds of a Kaehler manifold which generalizes holomorphic as well as totally real submanifolds in the sense that they become particular cases of $CR$-submanifolds. A real submanifold $M$ of an almost Hermitian manifold is called $CR$-submanifold if there exists a differentiable distribution $\mathcal{D}$ on $M$ satisfying (i) $JD_x = D_x$ and (ii) $J(D_x) \subseteq T^\perp_x(M)$ for each $x \in M$, where $\mathcal{D}^\perp$ is the complimentary orthogonal distribution to $\mathcal{D}$.

$CR$-submanifolds are an active area of research for the past thirty years and play an important role in many diverse areas of differential geometry, relativity as well as in the mechanics [3], [12]. Integrability of the distributions give rise to the notion of $CR$-product submanifolds which are those $CR$-submanifolds that are totally Riemannian product of leaves of $\mathcal{D}$ and $\mathcal{D}^\perp$. A lot of research has been done on $CR$-product submanifolds and characterizations are found for a $CR$-submanifold to become a $CR$-product submanifold (cf. [14], [18], [24]). Moreover, it is proved that there do not exist non trivial $CR$-products in hyperbolic spaces [46]. It was also found that $S^6$ does not admit non trivial $CR$-product spaces [46].

Bishop and O’Niell [6] in 1969 introduced warped product manifold as a generalization to Riemannian product manifolds. Easiest examples of warped product manifolds are surfaces of revolution. The study of warped products got impetus when B.Y.Chen studied warped product $CR$-submanifolds of a Kaehler manifold [15], [16]. After the impulse given by B.Y.Chen [15], [16], the study of warped product $CR$-submanifolds in Kaehler manifolds was extensively done only since 2001.

Singly warped products or simply warped products were first introduced
by Bishop and O’Neill [6] in 1969, in order to construct Riemannian manifolds with negative sectional curvature. In general, doubly warped products can be considered as generalization of singly warped products.

The warped and doubly warped product submanifolds form the main theme of this dissertation.

The dissertation comprises of four chapters and each chapter is divided into various sections. The mathematical relations obtained in the text have been labeled with double decimal numbering. The first figure denotes the chapter number, second represents the sections and the third point out the number of the definition, remark, equation, proposition, corollary or theorem, as the case may be. For example, Theorem 1.2.3 refers to third theorem of second section in the first chapter.

The first chapter is introductory and contains those definition and results which are needed for the subsequent chapters. Moreover, this serves the purpose of making the dissertation as the terminology for the forthcoming chapters.

Chapter 2 deals with the warped product CR-submanifolds of Kaehler and nearly Kaehler manifolds. The warped product CR-submanifolds can be defined in two ways (i) $N_\tau \times_f N_\perp$ and (ii) $N_\perp \times_f N_\tau$. In the first case, it was found that they are not different from the CR-product submanifolds [15]. We have a general inequality in CR-warped product submanifolds of Kaehler manifolds and we also discuss an example for the existence of CR-warped product submanifolds $N_\tau \times_f N_\perp$ in nearly Kaehler manifolds. In the last section, we discuss the generalization of results of B.Y. Chen [15], [16] and Sahin [47] given in previous sections of this chapter. Further K.A. Khan, Shahid Ali and Nargis have extended this study to generic warped product submanifolds in Kaehler manifolds.

In chapter 3 we pay our attention to warped product CR-submanifolds, CR-warped product submanifolds and doubly warped product CR-submanifolds in locally conformal Kaehler manifolds. We study a general inequality for CR-warped product submanifolds in an l.c.K. manifold, we see that some anti-holomorphic CR-warped product submanifolds satisfying a certain condition in an l.c.K. manifold, satisfy the equlity and in proper CR-warped product submanifolds, its holomorphic submanifold in an l.c.K. space form is also an l.c.K. space form and its totally real submanifold is a real space form. In this chapter, we discuss a lot of essential and interesting properties.
of these submanifolds.

The last chapter deals with the analogue of warped and doubly warped product contact CR-submanifolds in trans-Sasakian manifolds and also we study semi-slant submanifolds in trans-Sasakian manifolds. First we discuss the integrability conditions of the distributions on semi-slant submanifolds and study the geometry of the leaves of these submanifolds, where it was found that $\mathcal{D} \oplus \mathcal{D}^\perp$ is never integrable. We also discuss the existence of warped product contact CR-submanifolds and the non-existence of doubly warped product contact CR-submanifolds in trans-Sasakian manifolds. We have discussed an example in this chapter showing that the warped product contact CR-submanifold of Kenmotsu manifold do exist.

In the end we have given a bibliography which by no mean is exhaustive, but contains only those references which are referred in the text.