CHAPTER 4

Two-Layered Newtonian Model of Blood Flow in Inclined Asymmetric Stenosed Artery

4.1 Introduction

Circulatory disorders are well known to be responsible in most cases of death and stenosis or arteriosclerosis is one such case. An abnormal growth reducing the lumen of an artery is usually called a stenosis or atherosclerosis (Young 1968, Sankar and Lee 2009, Biswas and Chakravorty 2009, 2010) which is one of the most widespread diseases that can result in serious circulatory disorders, by reducing or occluding the blood supply. As for instance, stenosis in arteries supplying blood to brain, can cause cerebral strokes, likewise its presence in coronary arteries can bring about myocardial infarction, leading to a heart failure (Sinha and Singha, 1984).

It is indicated that hydrodynamic factors could play an important role in the initiation of stenosis. The complex geometry of arteries is also an important factor that affects the hydrodynamic factors (Guyton 1970, Puniyani and Nimi 1998). Besides, the inclination of an artery is a significant factor in the arterial network and the formation of a stenosis is along an inclined artery wall is equally significant, as both the factors non-uniformity and inclination may alter flow situation to a great extent.

Biswas and Chakraborty (2010) have considered the flow of blood represented by a two-layered Pulsatile blood flow in a stenosed artery with body acceleration and slip at wall. They studied that velocity and flow rate increases but effective viscosity decreases due to wall slip. Sankar and Ismail (2009) have analyzed two fluid mathematical models for blood flow in stenosed arteries taking blood in core region as Non-Newtonian fluid and the plasma in the peripheral layer as Newtonian fluid. They observed that the pressure drop, plug core radius, wall shear stress and the resistance to flow are significantly very low for the two fluid Casson model than that of the two fluid Herschel Bulkley model. It is reported by many authors (Chaturani and Kaloni, 1976; Chaturani and Upadhya, 1979; Shukla et al., 1980; Majhi and Usha,1984; Chaturani and Biswas,1983; Philip and Chandran,1996) have theoretically studied the flow of blood through uniform and stenosed tubes and analyzed the influence of slip velocity or peripheral plasma layer thickness on the flow variables.
such as velocity, wall shear stress and flow resistance. In these models, the peripheral layer thickness and slip velocity are assumed a priori based upon the experimental observations. To understand the flow patterns in stenosed arteries, Young (1968), Macdonald (1979), Desppande et al.(1979), Shankar and Hemalatha (2006) etc. have analyzed the flow of blood through an arterial stenosis. Shukla et al. (1980) have taken two-layered models and analyzed the influence of peripheral plasma viscosity on flow characteristics. Chaturani and Kaloni (1976), Chaturani and ponalagusamy (1982), Shankar and Lee (2007), Sankar and Ismail (2009), Sankar and Lee (2009) and Ponalagusamy (1986)) have considered the flow of blood represented by a two-layered model. In their models, the peripheral layer thickness is assumed as priori. They have studied that velocity and flow rate increases but effective viscosity decreases due to wall slip model. In the works of Chaturani and Biswas (1983), Misra and Kar (1989), a velocity slip at the constricted wall is adopted. Blood flow models with axially symmetric stenoses are considered by many authors (Young, 1968; Biswas and Chakraborty 2010). Singha et al. (2010) have dealt blood flow model through radially non-symmetric stenosed artery. Maruti Prasad and Radhakrishnamcharya (2008),Chakraborty et al. (2011) have proposed blood flow through an inclined tube with stenosis Recently Kumar et al. (2012) have proposed two-layered model of blood flow in presence of magnetic field. . Recently, Mathematical modeling of blood (Newtonian fluid) flow through inclined non-uniform stenosed artery, has been proposed by Biswas and Paul (2013).

However, among all the arteries present in a human body, generally many of them are uniform in shape. However, due to their non-uniformity owing to the presence of stenosis at the vessel wall, there arises a variation in the pressure gradient in blood flow. Besides, the inclination of an artery is a significant factor in the arterial network and the formation of a stenosis is along an inclined artery wall is equally significant, as both the factors non-uniformity and inclination may alter flow situation to a great extent. It is seen that the abnormal growth of stenosis at the arterial wall is mostly non-symmetrical.

With the above motivation, an attempt has been made to study the effects of slip (at the asymmetric stenosis) and the influence of flow variables (wall shear stress, velocity, flow rate, pressure gradient and apparent viscosity ) for 2-layered steady
laminar blood flow through an inclined asymmetric constricted vessel with velocity slip at interface.

4.2 Mathematical Formulation

We consider the steady flow of blood through an inclined artery, with an axially non-symmetrical but radially symmetrical stenosis or constriction. The constriction in the artery is developed due to the formation of stenosis in the lumen of the artery and is considered as mild and its gradual growths. In this study, we consider the shape of the stenosis as asymmetric. The artery length is assumed to be large enough as compared to its radius so that the entrance, exit and special wall effects can be neglected.

The model basically consists of- a core of red blood cell suspension in the middle layer and the peripheral plasma layer in the outer region (as shown in Fig4.1). It is assume that both the core and the peripheral plasma layer are represented by a Newtonian fluid with different viscosities $\mu_1$ and $\mu_2$ respectively. Flow is steady, laminar and one-dimensional flow, obeying the constitutive equation for a Newtonian fluid.

Fluid velocity vector has the form $\bar{u} = (0, 0, u(r))$ in cylindrical polar system $(r, \theta, z)$ representing the radial, circumferential and axial coordinates respectively. The equations of motion governing the fluid flow in $(r, \theta, z)$ coordinate system (schlichting, 1968) are written as

\begin{equation}
\frac{\partial p}{\partial r} = 0 , \quad (4.2.1)
\end{equation}

\begin{equation}
\frac{\partial p}{\partial \theta} = 0 , \quad (4.2.2)
\end{equation}

\begin{equation}
\frac{\mu}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + \rho g \sin \psi = \frac{\partial p}{\partial z} , \quad (4.2.3)
\end{equation}

where $u = u(r)$ denotes the axial velocity, $\mu$ is the viscosity of blood and $p$ the pressure, $\psi$ is the inclination of the artery, $g$ is the gravity due to acceleration,
\[ c = -\frac{dp}{dz} \text{ is the pressure gradient.} \]

### 4.3 Boundary Conditions

The boundary conditions for the present problems are

(i) \( u_s = 0 \), at \( r = R \), (no-slip at the stenotic wall) \hspace{1cm} (4.3.a)

(ii) \( u_1 - u_2 = u_s \), at \( r = R \), (slip at the interface) \hspace{1cm} (4.3.b)

(iii) \( \frac{\partial u_1}{\partial r} = 0 \), at \( r = 0 \) \hspace{1cm} (symmetric condition) \hspace{1cm} (4.3.c)

(iv) \( \frac{\partial u_2}{\partial r} = 0 \), at \( r = 0 \) \hspace{1cm} (symmetric condition) \hspace{1cm} (4.3.d)

where \( u_s \) is the velocity slip at the interface of the stenotic wall.

### 4.4 Mathematical Analysis

Integrating the equation (4.2.3) twice, we get the velocity function for core region as

\[ u_1(r) = -\frac{cr^2}{4\mu_1} - \frac{r^2}{4} \frac{\rho g \sin \psi}{\mu_1} + A_1 \ln r + B_1 \hspace{1cm} (4.4.1) \]

and for the PPL

\[ u_2(r) = -\frac{cr^2}{4\mu_2} - \frac{r^2}{4} \frac{\rho g \sin \psi}{\mu_2} + A_2 \ln r + B_2 \hspace{1cm} (4.4.2) \]

where \( A_i, B_i \) (i=1-2) are constants to be determined with the help of boundary conditions (4.3.a-d).

Using the boundary conditions (4.3.a-d) above, the velocity functions for the core region \( (0 \leq r \leq R_i(z)) \) can be obtained from equation (4.4.1) as
\[ u_1(r) = u_s + \left( \frac{c + \rho g \sin \Psi}{4} \right) \left[ \frac{1}{\mu_1} \left( R_1^2(z) - r^2 \right) + \frac{1}{\mu_2} \left( R^2(z) - R_i^2(z) \right) \right], \]

\[ 0 \leq r \leq R_1^1(z) \quad (4.4.3) \]

and for the PPL

\[ u_2(r) = \left( \frac{c + \rho g \sin \Psi}{4\mu_2} \right) \left[ R^2(z) - r^2 \right], \quad R_1^1(z) \leq r \leq R(z) \quad (4.4.4) \]

The volumetric flow rate for the core region is evaluated as

\[ Q_1 = 2\pi \int_{r=0}^{R_1^1(z)} ru_1(r)dr \]

\[ = \pi u_s R_1^2(z) + \pi \left( \frac{c + \rho g \sin \Psi}{8} \right) 2R_i^2(z) \left[ \frac{R_1^2(z)}{2\mu_1} + \frac{1}{2} \left( R^2(z) - R_i^2(z) \right) \right] \]

\[ (4.4.5) \]

And, for the peripheral plasma layer

\[ Q_2 = 2\pi \int_{r=R_1^1(z)}^{R(z)} ru_2(r)dr \]

\[ = \pi \left( \frac{c + \rho g \sin \Psi}{8\mu_2} \right) \left( R^2(z) - R_i^2(z) \right)^2 \]

\[ (4.4.6) \]

The total flow rate \( Q \) is determined by using equations (4.4.5-4.4.6) as

\[ Q = Q_1 + Q_2 \]

\[ = \pi u_s R_1^2(z) + \pi \left( \frac{c + \rho g \sin \Psi}{8\mu_2} \right) \left[ R^4(z) - (1 - \mu_2') R_i^4(z) \right] \]

\[ (4.4.7) \]
where \( \mu'_2 = \frac{\mu_2}{\mu_1} \), \( c = \frac{dp}{dz} \).

From equation (4.4.7), the pressure gradient term can be expressed as

\[
\frac{dp}{dz} = \rho g \sin \psi + \frac{8\mu_2}{\left[ R^4(z) - (1 - \mu'_2)R_1^4(z) \right]} \left( \frac{Q}{\pi} - u_s R_1^2(z) \right)
\]

(4.4.8)

Integrating between the limits \( p = p_0 \) at \( z = 0 \) and \( p = p_i \) at \( z = L \), where \( L \) is the length of the tube,

\[
p_i - p_0 = \int_{z=0}^{L} \left[ \frac{8\mu_2}{R^4(z) - (1 - \mu'_2)R_1^4(z)} \left( \frac{Q}{\pi} - u_s R_1^2(z) \right) - \rho g \sin \psi \right] dz
\]

(4.4.9)

Applying equation (4.4.9), the resistance to flow \( \lambda \) is defined as

\[
\lambda = \frac{p_i - p_0}{Q}, \quad \text{where } Q \text{ is given in equation } (4.4.7)
\]

\[
= \left[ 8\mu_2 \left( R_0^4 - \alpha^4 (1 - \mu'_2) \right)^{-1} \left( \frac{1}{\pi} - Q_1^{-1} (\alpha R_0)^2 u_s \right) - Q_1^{-1} \rho g \sin \psi \right] (L - L_0)
\]

\[
+ \int_{z=d}^{d+L_0} \left[ 8\mu_2 \left( \frac{1}{\pi} - Q_1^{-1} u_s R_1^2(z) \right) \left( R^4(z) - (1 - \mu'_2)R_1^4(z) \right)^{-1} - \frac{\rho g}{Q} \sin \psi \right] dz
\]

where \( Q_1 = Q \) at \( R = R_0 \) and \( R_1 = \alpha R_0 \)

The average pressure gradient in the axial direction is defined by the formula
\[
\frac{dp}{dz}_{av} = \frac{\int_{r=0}^{R(z)} r \left( \frac{dp}{dz} \right) dr}{\int_{r=0}^{R(z)} r dr}
\]

\[
\frac{dp}{dz} = \frac{dp}{dz}
\]  
(4.4.10)

Expressions for wall shear stress at the stenotic wall and at interface are obtained from the formula \( \tau_{R(z)} = -\mu_a \frac{\hat{\partial}u_2}{\hat{\partial}r} \big|_r = R(z) \) as follows:

\[
\tau_{R(z)} = \frac{R(z)}{2} \left( c + \rho g \sin \psi \right)
\]  
(4.4.11)

\[
\tau_{R_1(z)} = -\mu_1 \left( \frac{\hat{\partial}u_1}{\hat{\partial}r} \big|_r = R_1(z) \right)
\]

\[
= \frac{R_1(z)}{2} \left( c + \rho g \sin \psi \right)
\]  
(4.4.12)

Apparent viscosity can be expressed with the help of the formula

\[
\mu_a = \frac{\pi c R^4(z)}{8Q} \text{ as}
\]

\[
\mu_a = \left[ \frac{8u_s}{cR^2(z)} \left( \frac{R(z)}{R(z)} \right)^2 + \mu_2^{-1} (c + \rho g \sin \psi) \left( 1 - (1 - \mu_2') \left( \frac{R(z)}{R(z)} \right)^4 \right) \right]^{-1}
\]  
(4.4.13)

The non-dimensional form of the flow variables can be expressed by using the following non-dimensional variables:
\[
\bar{z} = \frac{z}{R_0}, \quad \bar{d} = \frac{d}{R_0}, \quad \bar{R} = \frac{R}{R_0}, \quad \bar{R}_1 = \frac{R_1}{R_0},
\]

\[
\bar{d} = \frac{\delta}{R_0}, \quad \mu'_2 = \frac{\mu_2}{\mu_1}, \quad \left( \frac{dp}{dz} \right), \quad \left( \frac{dp}{dz} \right)_0
\]

\[
\bar{A} = \frac{A}{R_0^{n-1}}, \quad \left( \bar{L}, \bar{L}_0 \right) = \left( \frac{L, L_0}{R_0} \right), \quad \bar{u} = \frac{u}{u_0}
\]

\[
\bar{\lambda} = \frac{\lambda}{\lambda_0}, \quad \bar{Q} = \frac{Q}{Q_0},
\]

\[
u_0 = \frac{cR_0^2}{4\mu_2}, \quad F = \frac{\rho g}{C}, \quad R_{(z)} = -\left( \frac{dp}{dz} \right)\frac{R_0}{2},
\]

\[
Q_0 = \frac{\pi cR_0^4}{8\mu_2}, \quad \left( \frac{dp}{dz} \right)_0 = -\frac{8\mu_2Q_0}{\pi R_0^4}, \quad \lambda_0 = \frac{8\mu_2L}{\pi R_0^4}, \quad \mu'_2 = \frac{\mu_2}{\mu_1}
\]

Flow geometry

For PPL and core region

\[
\bar{R}(\bar{z}) = 1 - A \left[ \bar{L}_0^{n-1} (\bar{z} - \bar{d}) - (\bar{z} - \bar{d})^n \right], \quad \bar{d} \leq \bar{z} \leq \bar{d} + \bar{L}_0
\]

= 1, \quad \text{Otherwise}

\[
\bar{R}_1(\bar{z}) = \alpha - A \left[ \bar{L}_0^{n-1} (\bar{z} - \bar{d}) - (\bar{z} - \bar{d})^n \right], \quad \bar{d} \leq \bar{z} \leq \bar{d} + \bar{L}_0
\]

= \alpha, \quad \text{otherwise}

Velocity function:
\[ \bar{u}_1 = \bar{u}_s + \left( c + \frac{\sin \psi}{F} \right) \mu_2 \left[ \left( \bar{R}_1^2 - \bar{R}^2 \left( \frac{r}{R} \right)^2 \right) + \left( \bar{R}^2 - \bar{R}_1^2 \right) \right], \quad 0 \leq \frac{r}{R} \leq \frac{R_1}{R} \]

(4.4.16)

\[ \bar{u}_2 = \left( c + \frac{\sin \psi}{F} \right) \bar{R}^2 \left[ 1 - \left( \frac{r}{\bar{R}(z)} \right)^2 \right], \quad \frac{R_1}{R} \leq \frac{r}{R} \leq 1 \]

(4.4.17)

Flow rate

\[ \bar{Q} = \frac{Q}{Q_0} = 2 \bar{u}_s \bar{R}_1^2 (z) + \left( c + \frac{\sin \psi}{F} \right) \bar{R}^4 (z) \left\{ 1 - \left( 1 - \mu_2' \right) \left( \frac{\bar{R}_1(z)}{\bar{R}(z)} \right) \right\} \]

(4.4.18)

Pressure gradient:

\[ \frac{dp}{dz} = \frac{dp}{dz}_0 \]

\[ = \frac{\sin \psi}{F} - \left( \bar{Q} - \bar{u}_s \bar{R}_1^2 (z) \right) \left[ \bar{R}^4 (z) - \left( 1 - \mu_2' \right) \bar{R}_1^4 \right]^{-1} \]

\[ = \left( \frac{dp}{dz} \right)_{av} \]

(4.4.19)

Resistance to flow
\[
\bar{\tau} = \frac{\lambda}{\lambda_0}
\]
\[
= \left[ \left( 1 - \alpha^2 (1 - \mu'_2) \right)^{-1} \left( 1 - \frac{\alpha^2 \bar{\mu}_s}{\bar{Q}_1} \right) - \frac{F \sin \psi}{\bar{Q}_1} \right] \left( \frac{L - L_0}{L} \right) + \frac{1}{L} \int_{z=z_0}^{d+L_0} \left( \bar{R}^4 - (1 - \mu'_2) \bar{R}_1^4 \right)^{-1} \left( 1 - \frac{2\bar{\mu}_s \bar{R}^2}{\bar{Q}} \right) - \frac{F \sin \psi}{\bar{Q}} \right] dz
\]

\text{(4.4.20)}

Stresses at wall and interface:

\[
\bar{\tau}_{R(z)} = \frac{\tau_R}{(\tau_R)_0}
\]
\[
= \frac{R(z)}{2} \left( c + \rho g \sin \psi \right)
\]
\[
= \frac{cR_0}{2}
\]
\[
= \bar{R}(z) \left( c + \frac{\sin \psi}{F} \right)
\]

\text{(4.4.21)}

\[
\bar{\tau}_{R_1(z)} = \frac{\tau_{R_1}}{(\tau_R)_0}
\]
\[
= \frac{R_1(z)}{2} \left( c + \rho g \sin \psi \right)
\]
\[
= \frac{cR_0}{2}
\]
\[
= \bar{R}_1(z) \left( c + \frac{\sin \psi}{F} \right)
\]

\text{(4.4.22)}

Apparent viscosity:
\[ \bar{\mu}_a = \frac{\mu_a}{\mu_2} \]
\[ = \left[ \frac{2\bar{u}_s}{R^2} \left( \frac{R_1}{R} \right)^2 + \left( c + F \sin \psi \right) \left\{ 1 - \left( 1 - \frac{\mu_2}{\mu_1} \right) \left( \frac{R_1}{R} \right)^4 \right\} \right]^{-1} \]  

(4.4.23)

4.5 Results and Discussions

It is already reported that knowledge of rheological and fluid dynamic properties of blood and its flow, like velocity, pressure gradient, shear stress at wall, flow rate etc. might play an important role in the fundamental understanding, diagnosis and treatment of many cardiovascular (cvs), renal and arterial diseases (Difentass,1980,1981, Punder and Punder 2006, Cocklet,1972, Fung,1981,Puniyani and Niimi,1998). In view of this, analytical expressions of such flow variables and their variations are obtained as shown in fig. (4.2-4.32). It may be noticed that velocity is a function of shear viscosities(\( \mu_1, \mu_2 \)), radii \( R(z), R_1(z) \) of uniform inclined asymmetric constricted arterial segments at wall and interface, radial \( r \) and axial \( z \) coordinates, slip velocity \( u_s \), and \( L \), the artery length. Also, the non-uniform radii \( R(z) \) and \( R_1(z) \) in eqs. (4.4.14-4.4.15) depends upon axial distance \( z \), \( R_0 \) normal artery radius in an unobstructed tube.

The present model includes the following cases:

a) When \( R(z) = R_1(z), \hat{\delta}_i = 0 \), the present analysis results in one-layered model for blood flow through a uniform stenosed artery with cases of slip or no-slip \( u_s \geq 0 \). (b) when \( R(z) = R_1(z) = R_0, \hat{\delta}_i = 0 = \hat{\delta}_k \) and \( \alpha = 1 \), it reduces to Poiseuille flow with slip (\( u_s \geq 0 \)) or zero-slip (\( \geq \)) at blood vessel wall. (c) when \( R(z) = R_1(z) \neq R_0, \hat{\delta}_i \neq 0 \neq \hat{\delta}_k \) and \( \alpha \neq 1, (u_s \geq 0) \), it represents the present modeling for blood flow.

In the analysis, the combined influence of several parameters have been developed, in cases of uniform inclined arterial region. To analyze the quantitative effect of uniform inclined arterial region, maximum height of stenosis \( \hat{\delta}_i, \hat{\delta}_k \), slip velocities (\( u_s \geq 0 \)) at the interface, Newtonian behaviour of blood, two-layered flow etc., computer codes have been developed for the numerical evaluations of the analytic results, obtained for
velocity, flow rate, wall shear stress for parameter values \( \delta_s = 0.15, \delta_i = 0.12, u_s = 0.05, 1, 1.5, c = 0.5, 1.0, 1.5 \) (Verma and Parihar, 2009, 2010), \( F = 0.3 \) (Maruti Prasad and Radhakrishnachandramacharya, 2008), and viscosities \( \mu_1 = 1.2 \text{ cp}, \mu_2 = 2 \) \text{ cp}, \( \mu_2' = 0.62, \alpha = 0.82 \) \((< 1)\), \( n = 2, 6, 9 \) for a full scale location from \( z = d \) to \( d + L_0 \), and \( 0 \leq r \leq R_i(z), R_i(z) \leq r \leq R(z) \) for PPL and core regions have been used. In the foregoing analysis, an attempt is taken up to address the variations of velocity, flow rate characteristics etc., due to such parameters.

### 4.5.1 Variation of axial velocity

A comparison of velocity profiles that have been obtained from eqs. (4.4.16–4.4.17), for slip and no-slip cases, heights of stenosis and different axial locations for \( z = d + L_0/2, d + L_0/4, d + 3L_0/4 \) for shape parameter \( n = 2, 6, 9 \) and for other parameter values, is shown in Figs. (4.2–4.12). As tube radius \( r/R \) ranges from 0 (at tube axis) to 1 (at wall) on either side of axis, velocity decreases from a greater value at axis to a smaller one (slip velocity) at interface and, then to a minimum magnitude (zero-slip velocity) at boundary. As expected, velocity increases with slip at interface. Its values are higher for flows with slip \( (u_s > 0) \) than those with no-slip \( (u_s = 0) \). Also it is observed from Fig. that \( \bar{u}_1 |_{n=9} < \bar{u}_1 |_{n=2} < \bar{u}_1 |_{n=6} \) i.e. Velocity in the core region increases with a rise in \( R \). As slip increases, axial velocity increases accordingly. However, for all variations of shape parameter \( (n \geq 2) \), magnitude of axial velocity obtained with slip condition at the interface, is higher than that with non-slip velocity at interface.

Further, it could be noticed that velocity in the core region \( (\bar{u}_1) \) attains the maximum value for the shape parameter \( n = 9 \) and the minimum magnitude at \( n = 2 \). Thus, due to non-symmetry in stenosis growth, velocity increases in magnitude than that velocity obtained for the formation in symmetric manner. It is also observed that axial velocity increases with an increase in artery inclination \( \psi = 0^0, 15^0, 30^0 \) and that \( \bar{u}_1 (\psi = 0^0) < \bar{u}_1 (\psi > 0^0) \) i.e. Velocity obtained with non-zero inclination (inclined artery) is higher than that with zero-inclination (horizontal artery). As stenosis size increases in size (from mild, moderate and severe), velocity decreases from the highest magnitude to the lowest magnitude. As the axial coordinate increases from the initiation of stenosis to its throat, velocity decreases from the maximum value to the minimum
magnitude. Velocity indicates a little deviation from parabolic profile, in the core region but in PPL, velocity shows a parabolic trend in case of all variations in parameters.

4.5.2 Variation of flow rate

In Figs. (4.13-4.19), we notice that the flow rate $\bar{Q}$ is minimum at minimum tubular area (at throat of stenosis) for symmetric and asymmetric stenoses and maximum at either end (at the beginning or, at the end of a constricted region). As $\tau$ increases in the constricted region $\bar{z}=\bar{d}$ to $\bar{z}=\bar{d}+\bar{L}_o$, flow rate decreases from the initiation to a maximum height of stenosis and there for, it reaches to almost the same magnitude at the termination of stenosis. As shape parameter $n$ increases from symmetric constriction ($n=2$) to as asymmetric stenosis ($n>2$), flow rate increases. However, in both the cases, flow rate attains the lowest magnitude at the throat of a stenosis. The magnitude of flow rate obtained with velocity slip is greater than that with zero-slip in axial velocity. As expected, flow rate increases with a rise in artery inclination. As artery inclination $\psi$ increases (from $\psi=0^0,15^0,30^0$), for a particular value of shape parameter $n$, flow rate increases in magnitude. As stenosis size develops from mild, moderate and severe forms, flow rate decreases from higher magnitude to a power one. However, flow rate increases with an increase in velocity slip and, it is minimum at the throat and the maximum at the ends of a stenosis. As parameter $c$ ($=0.5,1.0,1.5$) increases, flow rate increases throughout the stenotic region. Flow rate is the minimum at the throat of stenosis for $n=2$ (symmetric constriction) and in the neighbourhood of its initiation for $n>2$ ($n=6, 9$) in case of asymmetric stenosis.

4.5.3 Variation of wall shear stress

It could be noticed in the profile (Fig.4.20) drawn for $\bar{\tau}_{R1}(z)$ against axial distance $z$, for fixed values of $\delta_o, \delta_i$ (.15,.12) and $C=.5$ that as $n(=2,6,9)$ increases, wall shear stress increases in the half scale of $z$ ($d= 0$ to $d+L_o$); as inclination $\psi$ alters from $0^0-30^0$ degrees, and in next scale ($z=d+L_o/2$ to $d+2L_o$) wall shear stresses increases from a greater value to a lower one. These profiles show a non-linear trend all throughout the constricted region; the maximum magnitude is seen to attain at the throat of the constriction for stenosis at $n=2$ and at vary the termination of constriction for
asymmetric stenoses (n=6, 9) due to minimum tubular area. The Figs. (4.20-4.21) shows that as axial distance $\bar{z}$ increases, wall shear stress increases to a maximum value and then decreases to a minimum magnitudes. Also, as shape parameter value n increases, $\tau_{R_{1}(z)} |_{\bar{z}=2} < \tau_{R_{1}(z)} |_{\bar{z}=6} < \tau_{R_{1}(z)} |_{\bar{z}=9}$. As $\bar{Q}$, $\psi$ increases, $\tau_{R_{1}(z)}$ increases from a lower magnitude to a higher one for all variations in shape parameter n. Thus, wall shear stress is maximum at the throat of stenosis and the minimum at the initiation and termination of the constriction. In case of symmetric stenosis (n=2) wall shear s attains its greatest magnitude at $z=d+L_{a}/2$ (i.e., at the middle of the stenotic region) and that at the location $z=d+3L_{a}/4$ for an asymmetric constriction (n>2). As slip increases, shear stress decreases. In Figs. (4.22-4.24), wall shear stress increases as $Q (=.5,1,.1.5)$ increases for particular value of shape parameter n ($\geq 2$), attaining the greatest value at $z=d+L_{a}/2$ for n=2 and the same at $z=d+7L_{a}/8$ for n>2. It attains the minimum magnitude for mold constriction and the maximum value for severe stenosis. However, it increases with an increase in slip velocity and inclination.

4.5.4 Variation of pressure gradient

The variation of pressure gradient with flow parameter are presented in Figs. (4.25-30). It increases as $c (=.5,1.0,1.5)$ increases, attains the greatest value at $z=d+L_{a}/2$ for n=2 and the same at $z=d+7L_{a}/8$ for n>2. As artery inclination increases ($\psi=0^0,15^0,30^0$), shear stress increases but decreases with a rise in slip velocity. As stenosis size develops from mild to severe constriction, pressure gradient increases from lower to higher values. The trend remains the same for stenotic sizes as artery inclination increases.

4.5.5 Variation of apparent viscosity

The profiles for apparent viscosity (Figs. 4.31-4.32) indicate that $\mu_{a}$ (apparent viscosity) increases as stenosis growth develops from mild to severe forms. As expected, $\mu_{a}$ decreases with velocity slip for all three forms of constriction and shape parameter.
4.6 Conclusion

In the present investigation, steady flow of blood (Newtonian fluid) through a uniform inclined artery with the presence of an axially non-symmetrical but radially symmetric mild stenosis, is considered (Fig. 4.1). Analytic expressions of flow variables are obtained and variations of axial velocity, wall shear stress, flow rate, pressure gradient and apparent viscosity are shown graphically (Figs. 4.2-4.32). The present analysis includes Poiseuille flow of blood, 1-dimentional blood flow models through uniform vessel, with or without constriction, with slip or zero-slip cases as its special cases.

In Figs. (4.20-4.24) it is revealed that wall shear stress in the stenotic region increases rapidly from its approached value at \( z=d \) (at initiation) and attains its maximum at the throat of stenosis (at \( z=d+L_0/2 \) or \( d+3L_0/4 \)) for symmetric stenosis at \( n=2 \) and for asymmetric constriction (at \( n=6, 9 \)) and, thereafter, it reduces to a lower magnitude at the termination, the maximum value of \( \bar{\tau}_R(z) \) is reached near the middle of stenotic region (for \( n=2 \)) and a little away from the end (\( z=d+L_0 \)) of stenosis, there from, it decreases rapidly in the downstream, attaining the minimum value at the termination (\( z=d+L_0 \)) of constriction. Also, wall shear stress increases with a rise in stenosis size which is in agreement with Young's model (Young, 1986). As expected, it will reduce due to an employment of velocity at interface. \( \bar{\tau}_R(z) \) increases as \( n \) increases. Therefore, \( \bar{\tau}_{R(z)} \mid_0^0 < \bar{\tau}_{R(z)} \mid_W^0 \). Axial velocity and flow rate attains the greatest magnitude for mild stenosis case and the lowest one of flow variables indicated a deviation in the profile for symmetric (\( n=2 \)) stenosis than that in case of asymmetric (\( n>2 \)) stenosis. Apparent viscosity reduces due to slip velocity and artery inclination both.

In this investigation, it is easily noticed that the consideration of a 2-layered blood flow with asymmetric stenosis in a uniform inclined blood vessel, could provide better results than 1-layered axi-symmetric stenosed uniform tubes. Further it is noticed that employment of velocity slip at interface, could reduce wall shear stresses. It is seen that speed and rate of flow can be accelerated and wall shear stress can be reduced due to an employment velocity slip. Apart from this, damage to the vessel
wall could be lowered at one hand and normal blood flow can be restored on the other as it is said, the presence of stenosis at the vessel wall or in the neighbourhood and its gradual growth, could be a risk factor for several cardiovascular and arterial diseases, an employment of velocity slip at the constricted wall could play a vital role in reducing the size of atherosclerosis on one hand and restoration of regular blood flow on the other. Further, as slip might play a significant role in blood flow modeling, its accurate and exact measure in accordance with physiological situations, is quite important. The present mathematical model may be used as a device or tool in such cases.
Fig. 4.3: Variation of axial velocity against radial distance for $\bar{u}_i = .1$
for different angle of inclination $\psi$ and $\delta_i = .15, \delta_i = .12$

Fig. 4.4: Variation of axial velocity against radial distance for $\bar{u}_i = 0$, different angle of inclination $\psi$
and $\delta_i = .15, \delta_i = .12$
Fig. 4.5: Variation of axial velocity against radial distance for \( n = 2, 6, 9, \bar{u}_s = 0 \) and \( \psi = 0^\circ, 15^\circ, 30^\circ \)

Fig. 4.6: Variation of axial velocities against radial distance for \( \varepsilon = 1, \psi = 30^\circ, \bar{u}_s = 0 \) and \( n = 2 \)
Fig. 4.7: Variation of axial velocity with radial distance for different values of $\delta_s, \delta_l$ and $\bar{u}_s = 0.05$

\[ z = d + \frac{L_0}{8} \]
\[ z = d + \frac{L_0}{4} \]
\[ z = d + \frac{L_0}{2} \]

Fig. 4.8: Variation of axial velocities against radial distance for $\bar{c} = 0.5, \psi = 15^0, \bar{u}_s = 0.1$ and $n = 6$

$\mu'_2 = 0.62, \alpha = 0.8, \bar{c} = 0.5, \psi = 15^0, \bar{u}_s = 0.1$, mild, moderate, severe

$\bar{c} = 0.5, \psi = 15^0, \bar{u}_s = 0.1$ and $n = 6$
Fig. 4.9: Variation of axial velocities against radial distance for $\bar{c} = 0.5, \psi = 15^0, \bar{u}_s = 0$ and $n=9$

Fig. 4.10: Variation of axial velocities against radial distance for $\bar{c} = 1, \psi = 30^0, \bar{u}_s = 0$ and $n=9$
Fig. 4.11: Variation of axial velocities against radial distance
for $C = .5, \psi = 30^\circ, u_s = 0$ and n=9

Fig. 4.12: Variation of axial velocities against radial distance
for $C = .5, \psi = 30^\circ, u_s = .1$ and n=9
Fig. 4.13: Variation of flow rate against axial distance for $u_i = 0.05$ different angle of inclination $\psi$ and $\delta_i = 0.15, \delta_i = 0.12$

Fig. 4.14: Variation of flow rate against axial distance for different inclining angles $\psi = 0^\circ, 15^\circ, 30^\circ$
Fig. 4.15: Variation of flow rate against axial distance for $u_s = .1$, different angle of inclination $\psi$ and $\delta_s = .15, \delta_s = .12$

$\mu_s' = .62, \alpha = .8, \psi = 0^\circ, u_s = 0, 0.05, 1$

Fig. 4.16: Variation of flow rate against axial distance for mild, moderate, severe stenoses and $n=2, u_s = 0, 1, 0.05$
Fig. 4.17: Variation of flow rate against axial distance for $\bar{c} = .5, \bar{c} = 1, \bar{c} = 1.5$ and $n = 2$

Fig. 4.18: Variation of flow rate against axial distance for $\bar{C} = .5, 1, 1.5, \psi = 0^\circ, \bar{u}_s = 0$ and $n = 6$
Fig. 4.19: Variation of flow rate against axial distance for $\psi=0^\circ, 15^\circ, 30^\circ, \overline{u}_s = 0, n = 6$

Fig. 4.20: Variation of wall shear stress against axial distance for $\overline{u}_s = 0, \overline{C}_s = 0.15, \overline{G}_s = 0.12$. 

$\mu'_2 = 0.62, \alpha = 0.8, F = 3$
Fig. 4.21: Variation of wall shear stress against axial distance for $\bar{u}_i = .1, \bar{\sigma}_i = .15, \bar{\sigma}_\varphi = .12$.

Fig. 4.22: Variation of wall shear stress against axial distance for different values of $Q = .5, 1.0, 1.5$. 

$\sigma = .5, \mu'_i = .62, \alpha = .8, \bar{u}_i = .1$.
Fig. 4.23: Variation of wall shear stress against axial distance for $\bar{u}_s = 0, 1, Fr = .3, \psi = 0^\circ, n = 2$

Fig. 4.24: Variation of wall shear stress against axial distance for $\bar{u}_s = 0, F = .3, \psi = 0^\circ, 30^\circ, n = 2$
Fig. 4.25: Variation of pressure gradient against axial distance for $c=.5,1,1.5$ and $n=2,6,9, \psi=0^\circ, F=.3$

Fig. 4.26: Variation of pressure gradient against axial distance for $u_i=0.1, F=.3, \psi=0^\circ, 15^\circ, 30^\circ$ and $n=2,6,9$
Fig. 4.27: Variation of pressure gradient against axial distance for $\bar{u}_s = 0, .05, .1, F = .3, \psi = 0^\circ, n = 2, 6, 9$

Fig. 4.28: Variation of pressure gradient against axial distance for $\bar{u}_s = 0, .1, F = .3, \psi = 0^\circ$ and $n = 2$
Fig. 4.29: Variation of pressure gradient against axial distance for \( \psi = 0^\circ, 30^\circ, F = 3, \bar{u}_s = 0, \bar{c} = 0.5 \) and \( n = 6 \)

Fig. 4.30: Variation of pressure gradient against axial distance for \( \bar{u}_s = 0, F = 3, \psi = 0^\circ, 30^\circ \) and \( n = 2 \)
Fig. 4.31: Variation of apparent viscosity against axial distance for $\bar{u}_z = 0, 0.05, 0.1, \psi = 0^\circ$, $F = 0.3$ and $n = 2$

Fig. 4.32: Variation of apparent viscosity against axial distance for $\bar{u}_z = 0, \psi = 0^\circ, 15^\circ, 30^\circ$ and $n = 2$