CHAPTER 3

Two-Layered Newtonian Model of Blood Flow inside an Asymmetric Stenosed Artery

3.1 Introduction

It is already reported that hemodynamic factors could play an important role in the initiation of an atherosclerosis. The complex geometry of arteries (viz. uniform, bending, bifurcating, branching, tapering, discrete etc.) is also an important factor which obviously affects the local hemodynamics (Puniyani and Nimi, 1998; Guyton, 1970). The actual causes of stenosis are not well known but it has been suggested that the deposits of cholesterol on the arterial wall and proliferation of connective tissues may be responsible for the same (Chaturani and Ponalagusamy, 1986; Young, 1968; Shukla et al., 1980). The presence of stenosis in the cardiovascular system can cause circulatory disorders (Young, 1979; Caro, 1973; Fry, 1973). There is a large amount of evidence that hydrodynamic factors play a significant role in the formation, development and progression of arterial stenosis (Caro, 1973; Young, 1973). In recent years, there have been noticed a considerable interest and a good deal of enthusiasm in the viscometric studies and in theoretical modeling of blood flow as well as in the investigation of apparent viscosity, pressure-flow relationship, wall shear stress distribution and resistance to flow, in a vascular channel (Biswas, 2000; Fung, 1981; Puniyani and Nimi, 1998; Pedly, 1980). Quite a few analytical (Haynes, 1960; Cowin, 1974; Biswas and Nath, 2001) and a few experimental (Bugliarello and Hayden, 1962; Bugliarello and Sevilla, 1962; Young, 1973) investigations on blood flow with different perspectives, have already been performed over the years. Once stenosis develops in an artery, its most serious consequences are the increased resistance and the associated reduction of blood flow to the vascular bed supplied by the artery (Chaturani and Ponalagusamy, 1986; Sankar and Hemalatha, 2007). Hence, it is very useful to mathematically analyze the blood flow in stenosed arteries. Further, proper attention and rapidly growing enthusiasm of researchers, in this particular domain of Biomechanics, have increased quite a lot, with the advent of discovery that many cardiovascular diseases are closely related with the flow conditions in blood vessels which is clearly associated with cardiovascular diseases (Dintenfass, 1980; Dintenfass, 1981). However, all the arteries present in a human
body are not uniform in shape. Due to their non-uniformity, there arises a variation in the pressure gradient in blood flow.

Many investigators (Chaturani and Kaloni, 1976; Chaturani and Upadhyya, 1979; Shukla et al., 1980; Majhi and Usha, 1984; Chaturani and Biswas, 1983; Philip and Chandra, 1996) have theoretically studied the flow of blood through uniform and stenosed tubes and analyzed the influence of slip velocity or peripheral plasma layer thickness on the flow variables such as velocity, wall shear stress and flow resistance. In these models, the peripheral layer thickness and slip velocity are assumed a priori based upon the experimental observations. To understand the flow patterns in stenosed arteries, Young (1968), Macdonald (1979), Despande et al. (1979), Shankar and Hemalatha (2006) etc. have analyzed the flow of blood through an arterial stenosis. Shukla et al. (1980) have taken two-layered models and analyzed the influence of peripheral plasma viscosity on flow characteristics. Chaturani and Kaloni (1976), Chaturani and Ponalagusamy (1982), Sankar and Lee (2007), Sankar and Ismail (2009), Sankar and Lee (2009) and Ponalagusamy (1986) have considered the flow of blood represented by a two-layered model. In their models, the peripheral layer thickness is assumed a priori. Biswas and Chakraborty (2010) have considered the flow of blood represented by a two-layered pulsatile blood flow in a stenosed artery with body acceleration and slip at wall. They studied that velocity and flow rate would increase but effective viscosity decreases due to wall slip model. In the works of Chaturani and Biswas (1983); Misra and Kar (1989), a velocity slip at the constricted wall is adopted. Also Nath (2003) considered a slip condition at the stenotic wall.

In view of the above considerations, we are interested to study a two-layered Newtonian fluid for blood flow model in an asymmetric stenosed arteries and an investigation is carried out, on assuming blood to behave like a Newtonian fluid in the peripheral region (Schlichting, 1968; Streeter and Wyile, 1969) with viscosity $\mu_2$ and a non-Newtonian fluid in the core region (Fung, 1984; Kapur et al., 1982) with viscosity $\mu_1$, subject to the boundary conditions of zero-wall slip and the slip velocity conditions at interface. The motion of the fluid is laminar and steady. The inertia terms are neglected as the motion is slow. Nobody forces act on the fluid and there is no slip the wall.
3.2 Basic Equations

The basic equations governing the fluid flow consists of conservation equations of mass and momentum and, the constitutive equations of the fluid. The conservation equations of mass and momentum are the following (Biswa, 2000; Cowin, 1974; Schlichting, 1968):

Continuity equation:
\[ \dot{\rho} + \rho \nabla \cdot \vec{V} = 0 \]
(3.2.1)

Equation of motion:
\[ \nabla \cdot \vec{T} + \rho \vec{b} = \rho \ddot{\vec{V}} \]
(3.2.2)

where \( \rho \) is the density of the fluid, the superimposed dot indicates the material time derivative, \( \nabla \) (nebla) the vector differential operator, \( \vec{V} \) the usual velocity field, \( \vec{T} \) the stress tensor and \( \vec{b} \) denotes the body force per unit mass.

The constitutive equation of a Newtonian fluid (Biswa, 2000; Cowin, 1974; Schlichting, 1968) is

\[ T_{ij} = -\mu \partial_{ij} + D_{ijkl} \nabla_{kl} \]
\[ \Rightarrow T_{ij} = -p \partial_{ij} + \mu \left( \frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} \right) \]
\[ \Rightarrow T = -p I + 2 \mu D \]
(3.2.3)

where \( \mu \) is the shear viscosity, \( D \) the rate of strain tensor which is here the symmetrical part of the velocity gradient.

For an incompressible fluid, the equation of continuity (3.2.1) becomes

\[ \nabla \cdot \vec{V} = 0 \]
(3.2.4)

The equation of motion of a viscous fluid i.e., Navier stokes equation in vector form (from eq. 3.2.2 is
\[
\frac{d\vec{V}}{dt} = \vec{F} - \frac{1}{\rho} \nabla p + \mu \nabla^2 \vec{V}, \text{ where } v = \frac{\mu}{\rho}
\]

\[
\Rightarrow \rho \frac{d\vec{V}}{dt} = \rho \vec{F} - \nabla p + \mu \nabla^2 \vec{V}
\]

The conservation equation (3.2.5) reduces to the form

\[
\rho \left[ \frac{\partial \vec{V}}{\partial t} + (\nabla \cdot \vec{V}) \vec{V} \right] = \rho \vec{F} - \nabla p + \mu \nabla^2 \vec{V}, \quad (3.2.6)
\]

where \(\nabla\) (nebla) the vector differential operator and \(\nabla^2\) the Laplace's operator.

In absence of body force and the inertia terms, the equation of motion for a linear incompressible fluid for steady flow i.e., \(\frac{\partial \vec{V}}{\partial t} = \vec{0}\) is

\[
\mu \nabla^2 \vec{V} = \nabla p \quad ,
\]

\[\text{(3.2.7)}\]

### 3.3 Flow Geometry

A two-layered model for blood flow with an asymmetric stenosis at the vessel wall has been developed. The model basically consists of a core of red blood cell suspension in the middle layer and the peripheral plasma layer (PPL) in the outer (as shown in fig.3.1). It is assumed that both the core and the peripheral plasma layer are represented by a Newtonian fluid with different viscosities \(\mu_1\) and \(\mu_2\) respectively.

The geometry of the stenosis which is developed in the arterial wall in an asymmetric manner, is mathematically modeled in dimensionless form (Ponalagusamy, 1968) as

For PPL

\[
\frac{R(z)}{R_0} = 1 - A \left[ L_0^{-1}(z - d) - (z - d)^n \right], \quad d \leq z \leq d + L_0
\]

\[
= 1, \quad \text{Otherwise}
\]

\[\text{(3.3.1)}\]
The function \( R(z) \) which represent the shape of the central layer has been assumed to be of the form

\[
\frac{R(z)}{R_0} = \alpha - A_i \left[ L_0^{n-1} (z - d) - (z - d)^n \right], \quad d \leq z \leq d + L_0
\]

\[
= \alpha, \quad \text{Otherwise}
\]

(3.3.2)

where \( R(z) \) is the radius of the tube with stenosis, \( R_0 \) is the constant radius of the tube, \( R_1 \) is radius of the artery in the core region such that \( R_0 = R_1(z) \), \( L_0 \) is the maximum height of the stenosis in the PPL and that in the Core region such that the ratio of the stenotic height to the radius of the artery is much less than unity i.e. \( \frac{\delta}{R_0} \ll 1 \),
\[ A = \frac{\delta s}{R_0 L_0^n} \cdot \frac{n}{(n-1)}, \]

\[ A_1 = \frac{\delta i}{R_0 L_0^n} \cdot \frac{n}{(n-1)}, \]

Here, \( n \geq 2 \) is a parameter determining the shape of the stenosis.

It is of interest to note that an increase in the value of \( n \) leads to the change of stenosis shape. When \( n=2 \), the geometry of stenosis becomes symmetrical at \( z = d + \frac{L_0}{2} \) and \( \alpha = \frac{\delta i}{\delta s} \).

3.4 Mathematical Formulation

Let us consider a steady, laminar flow of blood through an axially non-symmetric but radially symmetric stenosed artery - a circular tube and one-dimensional flow, obeying the constitutive equation for a Newtonian fluid. Fluid velocity vector has the form \( \mathbf{u} = (0, 0, u(r)) \) in cylindrical polar system \((r, \theta, z)\) representing the radial, circumferential and axial coordinates respectively. The equations of motion governing the fluid flow in \((r, \theta, z)\) coordinate system (Schlichting, 1968) are written as

\[ \frac{\partial p}{\partial r} = 0, \quad (3.4.1) \]

\[ \frac{\partial p}{\partial \theta} = 0, \quad (3.4.2) \]

\[ \frac{\partial p}{\partial \theta} = 0, \frac{\mu}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) = \frac{\partial p}{\partial z}, \quad (3.4.3) \]
where \( \mathbf{u} = \mathbf{u}(r) \) denotes the axial velocity, \( \mu \) is the viscosity of blood and \( p \) the pressure.

As a result of equations (3.4.1-3.4.3), the governing equation of fluid flow for the core region is given by

\[
c + \frac{\mu}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_1}{\partial r} \right) = 0, \quad 0 \leq r \leq R(z),
\]  

(3.4.4)

where \( c = \left( -\frac{dp}{dz} \right) \) is the pressure gradient.

And, for the peripheral plasma layer (PPL) its integral is

\[
u_2(r) = -\frac{cr^2}{4\mu \mu_2} + A_2 \ln r + B_2, R_1(z) \leq r \leq R(z),
\]  

(3.4.5)

\( u_1, u_2 \) are the velocities in the core and the peripheral plasma layer regions and \( A_i, B_i \) (i=1-2) are the constants to be determined.

### 3.5 Boundary Conditions:

The boundary conditions for the present problems are

(i) \( u_2 = 0, \) at \( r = R(z), \) \( \) (no slip at the stenotic wall)  

(3.5.a)

(ii) \( u_1 - u_2 = u_s, \) at \( r = R_1(z), \) \( \) (slip at the interface)  

(3.5.b)

(iii) \( \frac{\partial u_1}{\partial r} = 0, \) at \( r = 0 \) \( \) (symmetric condition)  

(3.5.c)

(iv) \( \frac{\partial u_2}{\partial r} = 0, \) at \( r = 0 \) \( \) (symmetric condition)  

(3.5.d)

where \( u_s \) is the velocity slip at the interface of the stenotic regions.
3.6 Solutions

Integrating the Equation (3.4.4) twice, we get the velocity function for core region as

\[ u_1(r) = -\frac{cr^2}{4\mu_1} + A_1 \ln r + B_1, \quad 0 \leq r \leq R_1(z) \tag{3.6.1} \]

And for the peripheral plasma layer (PPL) is

\[ u_2(r) = -\frac{cr^2}{4\mu_2} + A_2 \ln r + B_2, \quad R_1(z) \leq r \leq R(z) \tag{3.6.2} \]

Using the boundary conditions (3.5.a-d) above in equations (3.6.1-3.6.2) we get the expressions of axial velocity for core region and PPL region as follows:

\[ u_1(r) = u_s + \frac{c}{4\mu_1} \left[ R_1^2(z) - r^2 \right] + \frac{c}{4\mu_2} \left[ R^2(z) - R_1^2(z) \right], \quad 0 \leq r \leq R_1(z) \tag{3.6.3} \]

\[ u_2(r) = \frac{c}{4\mu_2} \left[ R^2(z) - r^2 \right], \quad R_1(z) \leq r \leq R(z) \tag{3.6.4} \]

Using the expressions (3.6.3-3.6.4) for the velocities above, the volumetric flow rates \( Q_1 \) and \( Q_2 \) for the core and the peripheral regions can be obtained as

\[ Q_1 = 2\pi \int_{r=0}^{R_1(z)} ru_1(r)dr \]

\[ Q_1 = \pi u_s R_1^2(z) + \frac{\pi c}{8} R_1^2(z) \left[ \frac{R_1^2(z)}{\mu_1} + \frac{2}{\mu_2} \left \{ R^2(z) - R_1^2(z) \right \} \right] \tag{3.6.5} \]

Again, for Peripheral region

\[ Q_2 = 2\pi \int_{r=R_1(z)}^{R_2(z)} ru_2(r)dr \]

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\[
Q = \pi R_1^2(z)u_s + \frac{\pi c}{8\mu_2} \left[ R^4(z) - (1 - \mu_2') R_1^4(z) \right] \tag{3.6.6}
\]

The total flow rate \( Q \) is obtained as (using eqs. 3.6.5-3.6.6)

\[
Q = Q_1 + Q_2
\]

\[
= \pi R_1^2(z)u_s + \frac{\pi c}{8\mu_2} \left[ R^4(z) - (1 - \mu_2') R_1^4(z) \right]
\]

\[
Q = \pi R_1^2(z)u_s - \frac{\pi c}{8\mu_2} \frac{dp}{dz} \left[ R^4(z) - (1 - \mu_2') R_1^4(z) \right] \tag{3.6.7}
\]

where \( \mu_2' = \frac{\mu_2}{\mu_1} \) and \( c = -\frac{dp}{dz} \)

Applying equation (3.6.7) the pressure gradient term can be expressed as

\[
\frac{dp}{dz} = \frac{8\mu_2}{(1 - \mu_2') R_1^4(z) - R^4(z)} \left[ \frac{Q}{\pi} - u_s R_1^2(z) \right] \tag{3.6.8}
\]

Integrating between the limits \( p = p_i \) at \( z = 0 \) and \( p = p_0 \) at \( z = L \)

where \( L \) is the length of the tube,

we get the pressure drop as

\[
p_i - p_0 = 8\mu_2 \int_{z=0}^{L} \left[ R^4(z) - (1 - \mu_2') R_1^4(z) \right]^{-1} \left[ \frac{Q}{\pi} - u_s R_1^2(z) \right] dz
\]

\[
= 8\mu_2 \left[ R^4(z) - (1 - \mu_2') R_1^4(z) \right]^{-1} \left[ \frac{Q}{\pi} - u_s R_1^2(z) \right] (L - L_0)
\]

\[
+ 8\mu_2 \int_{z=d}^{d-L_0} \left[ R^4(z) - (1 - \mu_2') R_1^4(z) \right]^{-1} \left[ \frac{Q}{\pi} - u_s R_1^2(z) \right] dz \tag{3.6.9}
\]
The resistance to flow \( \lambda \) is defined as

\[
\lambda = \frac{p_1 - p_0}{Q}
\]

\[
= 8\mu_2 \left[ R_0^4(Z) \left(1 - \alpha^4 \left(1 - \mu_2'\right)\right) \right]^{-1} \left(\pi^{-1} - Q_1^{-1} (\alpha R_0)^2 U_s \right)_0 \left(L - L_0\right)
\]

\[
+ 8\mu_2 \frac{d+L_0}{z=d} \left[ R^4(z) - \left(1 - \mu_2'\right) R_1^4(z) \right]^{-1} \left(\frac{Q}{\pi} - u_s R_1^2(z) \right), \quad (3.6.10)
\]

where \( Q_1 = Q \) at \( R = R_0 \)

The average pressure gradient term in the axial direction is defined as follows

\[
\left(\frac{dp}{dz}\right)_{av} = \int_{r=0}^{R(z)} \frac{dp}{dz} dr
\]

\[
= \int_{r=0}^{R(z)} r \frac{dp}{dz} dr
\]

\[
= \frac{dp}{dz} \quad (3.6.11)
\]

Expression for wall shear stress at the stenotic wall and at the interface obtained from the formula

\[
\tau_R(z) = -\mu_2 \frac{\partial u_2}{\partial r} \bigg|_{r=R(z)}
\]

\[
\tau_R(z) = \frac{c}{2} R(z) \quad (3.6.12)
\]

Also \( \tau_{R_1(z)} = -\mu_1 \frac{\partial u_1}{\partial r} \bigg|_{r=R_1(z)} \)

\[
= \frac{c}{2} R_1(z) \quad (3.6.13)
\]
Apparent Viscosity becomes

\[
\mu_a = \frac{\pi c R^4(z)}{8Q}
\]

\[
= \left[ \frac{8u_s}{c R^2(z)} \left( \frac{R_1(z)}{R(z)} \right)^2 + \mu_2^{-1} \left\{ 1 - \left(1 - \mu_2' \right) \left( \frac{R_1(z)}{R(z)} \right)^4 \right\} \right]^{-1}
\]  

(3.6.14)

To obtain the non-dimensional form of the flow variables we shall use the following non-dimensional variables:

\[
\bar{R} = \frac{R(z)}{R_0}, \quad \bar{R}_1 = \frac{R_1}{R_0}, \quad u_0 = \frac{cR_0^2}{4\mu_2}, \quad Q_0 = \frac{\pi c R_0^4}{8\mu_2}
\]

\[
\left( \frac{dp}{dz} \right)_0 = \left( -\frac{8\mu_2 Q_0}{\pi R_0^4} \right), \quad \bar{\mu}_a = \frac{\mu_a}{\mu_2}, \quad \mu_2' = \frac{\mu_2}{\mu_1}, \quad \bar{z} = \frac{z}{R_0}
\]

\[
\lambda_0 = \frac{8\mu_2 c L}{\pi R_0^4}, \quad \bar{u}_s = \frac{u_s}{u_0}, \quad \bar{u}_1 = \frac{u_1}{u_0}, \quad \bar{u}_2 = \frac{u_2}{u_0}
\]

Velocity function:

\[
\bar{u}_1(\bar{r}) = \bar{u}_s + \mu_2 \left[ \bar{R}_1^2(\bar{z}) - \left( \bar{R}(\bar{z}) \right)^2 \right] \left( \frac{r}{R(z)} \right)^2
\]

\[
+ \left[ \left( \bar{R}(\bar{z}) \right)^2 - \left( \bar{R}_1(\bar{z}) \right)^2 \right] \left( \frac{r}{R(z)} \right), \quad 0 \leq \frac{r}{R(z)} \leq \frac{R_1}{R(z)}
\]  

(3.6.15)

\[
\bar{u}_2(\bar{r}) = \left( \bar{R}(\bar{z}) \right)^2 \left\{ 1 - \left( \frac{r}{R(z)} \right)^2 \right\}, \quad \frac{R_1(z)}{R(z)} \leq \frac{r}{R(z)} \leq 1
\]  

(3.6.16)

Rate of flow:

\[
\bar{Q} = 2(\bar{R}_1)^2 \bar{u}_s + \left( \bar{R}(\bar{z}) \right)^4 - (1 - \mu_2')(\bar{R}_1(\bar{z}))^4
\]  

(3.6.17)
Pressure gradient term:

\[
\left( \frac{dp}{dz} \right) = \frac{dp}{dz} - \left( \frac{dp}{dz} \right)_0
\]

\[
\frac{dp}{dz} = \left[ \left( \frac{R(z)}{R(z)} \right) - (1 - \mu_2^2) \left( \frac{R_1(z)}{R(z)} \right)^4 \right]^{-1} \left[ Q - 2 \bar{\mu}_s \left( \frac{R_1(z)}{R(z)} \right)^2 \right]
\]

\[
= \left( \frac{dp}{dz} \right)_{av}
\]  

(3.6.18)

Stress at the wall and interface:

\[
\overline{\tau}_{R(z)} = \left( \frac{dp}{dz} \right) \bar{R}(z)
\]  

(3.6.19)

Again

\[
\overline{\tau}_{R_1(z)} = \frac{\tau_{R_1}}{(\tau_R)_0}
\]

\[
= \left( \frac{dp}{dz} \right) \bar{R}_1(z)
\]  

(3.6.20)

Apparent viscosity:

\[
\bar{\mu}_a = \frac{\mu_a}{\mu_2}
\]

\[
= \left[ 1 + \frac{2 \bar{\mu}_s}{R(z)^2} \left( \frac{\bar{R}_1}{R(z)} \right)^2 + (\mu_2^2 - 1) \left( \frac{\bar{R}_1(z)}{R(z)} \right)^4 \right]^{-1}
\]  

(3.6.21)
Flow Geometry

For PPL

\[
\bar{R} (\bar{z}) = 1 - A \left[ \bar{L}_0^{-n+1} (\bar{z} - \bar{d}) - (\bar{z} - \bar{d})^n \right], \quad \bar{d} \leq \bar{z} \leq \bar{d} + L_0
\]

(3.6.22)

\[= 1, \quad \text{Otherwise} \]

For the Core region

\[
\bar{R}_1 (\bar{z}) = \alpha - A \left[ \bar{L}_0^{-n+1} (\bar{z} - \bar{d}) - (\bar{z} - \bar{d})^n \right], \quad \bar{d} \leq \bar{z} \leq \bar{d} + L_0
\]

(3.6.23)

\[= \alpha, \quad \text{otherwise} \]

3.6 Results and Discussions

It is already reported that knowledge of rheological and fluid dynamic properties of blood and its flow, like velocity, pressure gradient, shear stress at wall, flow rate, apparent viscosity etc., might play an important role in the fundamental understanding, diagnosis and treatment of many cardiovascular (cvs) ,renal and arterial diseases (Dientenfass, 1981; Punder and Punder, 2006; Cokelet, 1972). In view of this, analytical expressions of such flow variables and their variations are obtained and shown in figs. (3.2-3.38). It may be noticed that velocity is a function of shear viscosities \(\mu_1, \mu_2\), pressure gradient \(\frac{dp}{dz}\), radii \(R(z), R_1(z)\) of uniform constricted arterial segments at wall and interface, radial \(r\) and axial \(z\) coordinates, slip velocity \(u_s\) and \(L\), the artery length. Also, the non-uniform radii \(R(z)\) and \(R_1(z)\) in eqs. (3.6.22-3.6.23) depends on axial distance \(z\), \(R_0\) normal artery radius in an unobstructed tube.

The present model includes the following cases:

(a) When \(R(z) = R_1(z)\), \(\delta_1 = 0\), the present analysis results in one-layered model for blood flow through a uniform stenosed artery with cases of slip or no-slip \(u_s \geq 0\). (b) when \(R(z) = R_1(z) = R_0\), \(\delta_1 = 0 = \delta\) and \(\alpha = 1\), it reduces to Poiseuille flow with slip \(u_s\).
$\geq$ or zero-slip (\(\geq\)) at blood vessel wall. (c) when \(R(z) = R_1(z) \neq R_0, \hat{\delta}_1 \neq 0 \neq \hat{\delta}_s\) and \(\alpha \neq 1, (u_s \geq 0)\), it represents the present modeling for blood flow.

In the analysis, the combined influences of several parameters have been developed, in cases of uniform region. To analyze the quantitative effect of uniform artery, maximum height of stenosis \(\hat{\delta}_1, \hat{\delta}_s\), slip velocities \((u_s \geq 0)\) at the interface, Newtonian behaviour of blood, two-layered flow etc., computer codes have been developed for the numerical evaluations of the analytic results obtained for velocity, flow rate, wall shear stress and pressure gradient for parameter values \(\hat{\delta}_s = .15, \hat{\delta}_i = .12, u_s = .00, .05, .1, Q = .5, 1, 1.5\) (Verma and Parihar2009, 2010) and viscosities \(\mu_1 = 1.2 \text{ cp}, \mu_2 = 2 \text{ cp}, \alpha = .82 (< 1)\) for a full scale location from \(z=d\) to \(d+L_0\), and \(0 \leq r \leq R_1(z), R_1(z) \leq r \leq R(z)\) for PPL and core regions have been used. In the foregoing analysis, an attempt is taken up to address the variations of velocity, flow rate characteristics, shear stress etc., due to such parameters.

### 3.6.1 Velocity profiles

A comparison of velocity profiles that have been obtained from eqs. (3.6.15-3.6.16), for slip and no-slip cases, maximum heights of stenosis and different axial locations for \(z=d+L_0/2, d+L_0/4, d+3L_0/4\) for shape parameter \(n=2, 6, 9\) and for other parameter values, is shown in Figs. (3.2-3.13). As tube radius \(r/R\) ranges from 0 (at tube axis) to 1 (at wall) on either side of axis, velocity decreases from a greater value at axis to a smaller one slip velocity at interface and, then to a minimum magnitude zero-slip velocity at boundary. As expected, velocity increases with slip at interface. Its values are higher for flows with slip \((u_s>0)\) than those with no-slip \((u_s=0)\). Also it is observed from fig.(3.2-3.13) that \(u_i|_{n=9} < u_i|_{n=2} < u_i|_{n=6}\). As slip velocity increases, axial velocity increases. Velocity shows a decline in magnitude as stenosis size varies from mild case to severe forms. In all three forms of stenosis for motion, velocity increases as slip velocity increases. This behaviour is reflected in symmetric \((n=2)\) and asymmetric stenosis \((n>2)\). Although, a little deviation from parabolic trend of velocity is noticed in the core layer, but in the PPL region, it shows a parabolic trend.
3.6.2 Flow rate

Variation of flow rate $\overline{Q}$ (in eq. 3.6.17) against axial distance $z$, for stenosis heights, slip or no-slip and stenotic region, is drawn in Figs. (3.14-3.25).

In Figs.(3.14-3.25), we notice that the flow rate $\overline{Q}$ is minimum at minimum tubular area (at throat of stenosis) for symmetric and asymmetric stenoses and maximum at either end (at the beginning or at the end of a constricted region). As $z$ increases in the constricted region $z=d$ to $d+L_0$, flow rate decreases to a maximum height of stenosis. As slip velocity increases, flow rate $\overline{Q}$ increases. However, values of $\overline{Q}$ obtained with velocity slip are greater than those with no-slip at interface. Flow rate decreases as stenosis size increases from mild form to moderate stage. As slip velocity increases, flow rate increases. This behaviour is seen in all values of $n=2$ and $n>2$. For any magnitude of slip ($u_s \geq 0$) employment at three kinds of stenotic arteries, it is found that

$\overline{Q}_{\text{mild}} > \overline{Q}_{\text{moderate}} > \overline{Q}_{\text{severe}}$

Also, magnitude of flow rate at the throat of stenosis is seen to be appreciably lower in the case of moderate and severe stenosis.

3.6.3 Pressure gradient

Variation of pressure gradient (obtained from eq.3.6.18) with slip or, no-slip at interface and zero-slip at boundary and, maximum heights of stenosis is plotted against axial coordinate in Figs. (3.30-3.35). It is observed from figures that, in the presence of stenosis pressure gradient is lower with an axial slip at interface than that with no-slip. As slip increases in magnitude, it decreases further. In mild stenosis, it is almost constant for slip or no-slip, employed at interface. But this behaviour in other forms, as it changes with locations in moderate and severe stenoses and attains a maximum value at minimum tubular area of constricted region. As the value of $\overline{Q}$ increases keeping other variables fixed, pressure gradient increases.
3.6.4 Wall shear stress

Wall shear stress and its variation with slip and stenosis heights have been computed from eqs. (3.6.19-3.6.20) and plotted against axial distance $z$ ($d \leq z \leq d+L_0$) in Figs.(3.26-3.29).

From Figs. (3.26-3.29) we have observed that as axial distance $\bar{z}$ increases, wall shear stress decreases. Also as shape parameter values $n$ increases, $\bar{\tau}_{R(z)} \rvert_{n=2} < \bar{\tau}_{R(z)} \rvert_{n=6} < \bar{\tau}_{R(z)} \rvert_{n=9}$. As slip velocity ($u_s$) increases, $\bar{\tau}_{R(z)}$ decreases. As flow rate increases, shear stress increases. As stenosis height increases from mild to severe constriction, it increases from a lower magnitude to a higher value. In all stenosis cases, wall shear stress computed with slip is expected to be lower than that with no-slip. In case of moderate and severe stages of stenosis, wall shear stress increases with slip. Wall shear stress increases as $\bar{Q}$ increases but it decreases with the employment of velocity slip. It attains lower magnitude for mild stenosis and higher magnitude for moderate and severe cases.

3.6.5 Apparent viscosity

Apparent viscosity $\bar{\mu}_a$ is obtained from eq. (3.6.21) and its variation with heights of stenosis and slip or no-slip cases, is plotted against axial distance $z$ in Figs.(3.36-3.37). The following are noticed in the profiles:

(i) In case of no-slip, profiles drawn for $\bar{\mu}_a$ is linear and the trend is non-linear, for slip at interface.

(ii) It decreases with an introduction of slip at interface and as slip increases, it decreases.

(iii) For zero-slip at interface, $\bar{\mu}_a$ attains a constant magnitude, in all three stages of stenosis considered.

(iv) As expected, $\bar{\mu}_a$ computed with slip condition, is seen to be lower than that value, obtained with no-slip at interface.
(v) As axial coordinate ranges from either end to the throat of stenosis, \( \mu_L \) decreases from a higher value at the initiation or termination of stenosis to a lower one at the minimum constricted area.

(vi) Pressure gradient decreases with arise in slip velocity. It increases as \( \overline{Q} \). Also, \( \mu_L \) decreases as stenosis size increases from mild to severe growths and this trend is reflected for both type of shape parameter (n \( \geq \) 2).

### 3.7. Conclusion

In the present investigation, steady flow of blood (Newtonian fluid) through a uniform artery with the presence of an axially non-symmetrical but radially symmetric mild stenosis is considered (Fig.1). Analytic expressions of flow variables are obtained and variations of velocity, shear stress, flow rate, pressure gradient and apparent viscosity are shown graphically (Figs.3.2-3.37). The present analysis includes Poiseuille flow of blood, 1-dimentional blood flow models through uniform vessel, with or without constriction, with slip or zero-slip cases as its special cases.

In Figs. (3.26-3.29) it is revealed that wall shear stress in the stenotic region increases rapidly from its approached value at \( z=d \) (at initiation) and attains its maximum at the throat of stenosis (at \( z=d+L_0 \)) for symmetric stenosis at n=2 and for asymmetric constriction (at n=6, 9), the maximum value of \( \overline{\tau}_{R(z)} \) is reached near the end (\( z=d+L_0 \)) of stenosis. There from, it decreases rapidly in the downstream, attaining the minimum value at the termination (\( z=d+L_0 \)) of constriction. As expected, it will reduce due to an employment of velocity slip at interface.

In this investigation, it is easily noticed that consideration of a 2-layered blood flow with asymmetric stenosis in uniform blood vessel, would provide better results than 1-layered axi-symmetric stenosed uniform tubes. Further it is noticed that employment of velocity slip at interface, could reduce wall shear stresses. It is seen that speed and rate of flow can be accelerated and wall shear stress can be reduced due to an employment of velocity slip. Apart from this, damage to the vessel wall could be lowered at one hand and normal blood flow can be restored on the other. Further, as
slip might play a significant role in blood flow modeling, its accurate and exact measure in accordance with physiological situations, is quite important. The present mathematical model may be used as a device or tool in such cases.

In literature, it has been reported that blood is a highly viscous fluid and under certain flow situations, it exhibits a Newtonian behaviour. In order to take account of this Newtonian nature of blood, the next chapter 4 deals with two-layered model of blood flow inside inclined asymmetric artery with an employment of velocity slip condition at interface.

![Diagram](image)

Fig. 3.2: Variation of axial velocity against radial distance for n=2,6,9 and $u_s = 0$
Fig. 3.3: Variation of axial velocity against radial
distance for slip velocity $\bar{u}_s = 0.05$

$$\mu' = 0.62, \alpha = 0.8, \bar{u}_s = 0.05$$
$$\delta_s = 0.15, \delta_l = 0.12$$

Fig. 3.4: Variation of axial velocity against radial
distance for slip velocity $\bar{u}_s = 0.1$

$$\mu' = 0.62, \alpha = 0.8, \bar{u}_s = 0.1$$
$$\delta_s = 0.15, \delta_l = 0.12$$
Fig. 3.5: Variation of axial velocities against radial distance and slip velocity $\bar{u}_s = .00, n = 2, \mu'^2 = .62$

Fig. 3.6: Variation of axial velocities against radial distance and slip velocity $\bar{u}_s = .1$
Fig. 3.7: Variation of axial velocities against radial distance for mild, moderate, severe, n=2 and slip velocity $u_s = 0.05$.

Fig. 3.8: Variation of axial velocities against radial distance for mild, moderate, severe, n=6 and slip velocity $u_s = 0.00$. 

$\mu' = 0.62, \alpha = 0.8$
Fig. 3.9: Variation of axial velocities against radial distance for mild, moderate, severe, n=6 and slip velocity $\bar{u}_s = .1$

Fig. 3.10: Variation of axial velocities against radial distance for mild, moderate, severe, n=6 and slip velocity $\bar{u}_s = .05$
Fig. 3.11: Variation of axial velocities against radial distance for mild, moderate, severe, n=9 and slip velocity $u_s = 0.0$

Fig. 3.12: Variation of axial velocities against radial distance for mild, moderate, severe, n=9 and slip velocity $u_s = 0.1$
Fig. 3.13: Variation of axial velocities against radial distance for mild, moderate, severe, n=9 and slip velocity $\bar{u}_s = .05$

Fig. 3.14: Variation of flow rate against axial distance $n=2, 6, 9$ and $\bar{u}_s = 0, \alpha = .8$
Fig. 3.15: Variation of flow rate against axial distance for $n=2,6,9$ and $\overline{u_s} = .05, \alpha = .8$

$\mu'_2 = .62, \alpha = .8, \overline{u_s} = .05$

$z = d + \frac{L_0}{4}$

$n=6$

$z = d + \frac{L_0}{2}$

$\frac{n-2}{2}$

$n=9$

$z = d + \frac{3L_0}{4}$

Fig. 3.16: Variation of flow rate against axial distance for $n=2,6,9$ and $\overline{u_s} = .1, \alpha = .8$

$\mu'_2 = .62, \alpha = .8$

$\overline{u_s} = .1$
Fig. 3.17: Variation of flow rate against axial distance for slip velocities $u_s = 0, .05, .1$

Fig. 3.18: Variation of flow rate against axial distance for mild, moderate, severe $n = 2$, and $\overline{u}_s = 0$
Fig. 3.19: Variation of flow rate against axial distance for mild, moderate, severe and $n=2, \overline{u_s} = .05$

Fig. 3.20: Variation of flow rate against axial distance for mild, moderate, severe and $\overline{u_s} = .1$
Fig. 3.21: Variation of flow rate against axial distance for mild, moderate, severe and \( n = 6, \overline{u_s} = 0 \).

\[ \mu'_2 = 0.62, \alpha = 0.8, \overline{u_s} = 0, n = 6 \]

Fig. 3.22: Variation of flow rate against axial distance for mild, moderate, severe and \( n = 6, \overline{u_s} = 0.05 \).

\[ \mu'_2 = 0.62, \alpha = 0.8, n = 6, \overline{u_s} = 0.05 \]
Fig. 3.23: Variation of flow rate against axial distance for mild, moderate, severe and $n=6, \bar{u}_s = 0.1$

Fig. 3.24: Variation of flow rate against axial distance for mild, moderate, severe and $n=9, \bar{u}_s = 0$
Fig. 3.25: Variation of flow rate against axial distance for mild, moderate, severe and $n=9, \bar{u}_{s} = 0.1$.

Fig. 3.26: Variation of shear stress against axial distance for different slip velocities $\bar{u}_{s}$.
Fig. 3.27: Variation of shear stress against axial distance for different values of \( \bar{Q} \)

\[ \mu' = .62, \alpha = .8, \bar{u}_i = 0 \]
\[ \bar{\delta}_i = .15, \bar{\delta}_j = .12 \]

\( \bar{Q} = .5 \)
\( \bar{Q} = 1 \)
\( \bar{Q} = 1.5 \)

Fig. 3.28: Variation of wall shear stress against axial distance for mild, moderate, severe and \( n=2, \bar{u}_i = 0, \bar{u}_i = .05, \bar{u}_i = .1 \)
Fig. 3.29: Variation of wall shear stress against axial distance for mild, moderate, severe and $n=9$, $\bar{u}_z = 0, 0.05, 1$

Fig. 3.30: Variation of pressure gradient against axial distance for $u_z = 0, 0.05, 1$ and $\bar{u}_z = 0.15, 0.12$
Fig. 3.31: Variation of pressure gradient against axial distance for different flow rate $\tilde{Q} = .5, 1, 1.5$

$\mu' = .62, \alpha = .8, \bar{\delta} = .15, \bar{\delta} = .12$

$Q = 1.5, n = 6$

$Q = 1, n = 2$

$Q = .5, n = 9$

$\bar{u_s} = .1$

$\bar{u_s} = 0$

Fig. 3.32: Variation of pressure gradient against axial distance for $\bar{u_s} = 0, \tilde{n} = .1$
Fig. 3.33: Variation of pressure gradient against axial distance for $Q = 0.5, Q = 1, Q = 1.5$ and $n = 2$

$\mu'_s = 0.62, \alpha = 0.8, \bar{u}_s = 0, n = 2$

Fig. 3.34: Variation of pressure gradient against axial distance for $\bar{Q} = 0.5, \bar{Q} = 1, \bar{Q} = 1.5$ and $n = 6, \bar{u}_s = 0$
Fig. 3.35: Variation of pressure gradient against axial distance for \( Q = 0.5, Q = 1, Q = 1.5 \) and \( n = 9 \)

\[ \mu' = 0.62, \alpha = 0.8, \overline{u_i} = 0 \]

Fig. 3.36: Variation of apparent viscosity against axial distance for mild, moderate, severe and \( n = 2, \overline{u_i} = 0, 0.1 \)

\[ \mu' = 0.62, \alpha = 0.8, n = 2 \]
Fig. 3.37: Variation of apparent viscosity against axial distance for mild, moderate, severe and n=6, $\bar{u}_s = 0.1$