CHAPTER 6

TWO-LAYERED CASSON MODEL OF BLOOD FLOW INSIDE ASYMMETRIC STENOSED ARTERY

6.1 Introduction

Theoretical modelings on different blood flow situations have drawn an increasing interest and keen attention of many researchers. There could be noticed a great many attempts which are motivated to the viscometric studies and theoretical modeling on blood flow as well as in the investigation of flow variables, such as velocity profile, flow rate, pressure drop, apparent viscosity, pressure-flow relationship and resistance to flow, in a vascular channel (Pedly1980, Fung1984, Puniyani and Nimi 1998, Biswas 2000). Many authors have examined blood flow through analytical models (Skalak 1982, Biswas and Chakraborty 2009) and experimental (Bugliarello and Hayden 1962, Bugliarello and Sevilla 1970, Young and Tsai 1973) works with different perspectives. Consequently, a good many theoretical, analytical and numerical models have been proposed in the field of Biomechanics. Further, a great interest, attention and enthusiasm among the investigators in this particular domain of Biomechanics, have gained a momentum due to the fact that many cardiovascular (cvs) diseases are closely associated with the flow phenomenon in blood vessels i.e. in cvs (Dintenfass 1981, Caro 1981, Chien 1982).

It is already said that at high shear rate and large diameter arteries, blood behaves like a Newtonian fluid where in shear stress versus rate of strain relation is linear (Fung, 1981; Taylor, 1959). Young (1968) has examined the effects of stenosis on flow behaviour of blood. It has been noticed that resistance to flow and wall shear stress increase with the increase of stenosis size. Forrester and Young (1970) extended the work of Young (1968), including the effects of flow separation in an artery with mild stenosis. Results of experimental work on models of arterial stenosis, have been presented by Young and Tsai (1973). Sanyal and Maiti (1998) have addressed the pulsatile blood flow through a stenosed artery. They have discussed numerical solutions of axial velocity profile and pressure gradient graphically. Jayaraman and Tewari (1995) have studied blood flow in a catheterized curved artery. They assumed blood as an incompressible Newtonian fluid and usual zero-slip
boundary condition is used to analyze the flow. Sarkar and Jayaraman (1997) have put forward a model of blood flow through a catheterized stenosed artery by considering blood as a Newtonian fluid. The importance of slip velocity at flow boundaries has reported by many authors both theoretically and experimentally (Vand, 1948; Bloch, 1962; Brunn, 1975; Nubar, 1967; Bugliarello and Hayden, 1962; Bennet, 1967). Chaturani and Biswas (1984) have examined the effect of slip velocity on blood flow through a stenosed artery. Pulsatile flow of blood in a stenosed artery by using an axial velocity slip at the stenosed wall and by considering blood as Newtonian fluid, has been dealt by Biswas and Chakraborty (2010). The annular flow of blood in a catheterized tapered artery, is investigated by Biswas and Chakraborty (2009). Biswas and Laskar (2012) have studied a Newtonian fluid model of blood flow, in a catheterized Multistenosed artery with an axial velocity slip condition at the stenotic wall. Recently, Mathematical modeling of blood (Newtonian fluid) flow through inclined non-uniform stenosed artery, has been proposed by Biswas and Paul (2013). Charm and Kurland (1965) have reported the utility of Casson’s equation which can be used to analyze blood flow over a wide range of hematocrit and shear rates. The flow of Casson fluid in tapered tubes is dealt by Oka (1973). Chaturani and Samy (1982) have taken a stenosed two-layered mode with peripheral layer of Newtonian fluid and the core region of Casson fluid. Dash et al. (1995) have studied the changed flow pattern and estimated the increase of flow resistance in a narrow catheterized artery, by using the Casson fluid model of blood. Casson fluid models of blood flow for different flow situations and geometries have investigated by many authors (Chaturani and Palanisamy, 1990; Biswas and Bhattacherjee, 2003; Nagarani and Sarojamma, 2008). Sankar (2009) has proposed a two-layered pulsatile flow of blood inside a catheterized artery with core region as Casson fluid and plasma in a constricted rigid artery, by considering blood as a Casson fluid, has been studied by Verma et al.(2011)

It is seen that the abnormal growth of stenosis at the arterial wall is mostly non-symmetrical. With the above motivation, an attempt has been made to study the effects of slip (at the asymmetric stenosis) and the influence of flow variables (wall shear stress, velocity, flow rate, pressure gradient, apparent viscosity) for 2-layered model of Casson fluid flow through asymmetric constricted vessel with velocity slip
at interface. The motion of flow is assumed to be steady and laminar.

6.2 Mathematical Formulation

We consider the steady, laminar, and fully developed flow of blood (assumed to be incompressible) in the axial ($z$) direction through constricted artery with an axially non-symmetrical but radially symmetrical stenosis with constriction. The constriction in the artery is formed in the lumen of the artery and is considered as mild and gradual developments. In this study, we consider the shape of the stenosis as asymmetric. The artery length is assumed to be large enough as compared to its radius so that the entrance, exit and special wall effects can be neglected.

The model basically consists of- a core of red blood cell suspension in the middle layer and the peripheral plasma layer in the outer layer (as shown in Fig.6.1). It is assumed that the rheology of blood in the core region has been characterized as a non-Newtonian fluid obeying the law of Casson fluid model and fluid with the PPL as a Newtonian fluid with different viscosities $k_c$ and $\mu_1$ respectively.

In Fig.(6.1), $R(z)$ is the radius of the tube with stenosis, $R_0$ is the constant radius of the tube, $R_1(z)$ is radius of the artery in the core region such that $\alpha = \frac{R_1(z)}{R_0}$, $L_0$ is the length of the stenosis, $L$ the length of the tube, $d$ the stenosis location, $\delta_k$ and $\delta_1$ are the maximum height of the stenosis in the PPL and that in the core region respectively at $z = d + \frac{L_0}{2}$ such that the ratio of the stenotic height to the radius of the artery is much less than unity i.e. $\frac{\delta}{R_0} << 1$.

For one dimensional flow, equation of motion can be rewritten in the form

$$c + \frac{\mu_l}{r} \frac{d}{dr} \left[ r \frac{d}{dr} u(r) \right] = 0$$

(6.2.1)
The constitutive equation for Casson fluid is furnished in the form (Fung, 1981)

\[
\sqrt{\tau} = \sqrt{\tau_o} + \sqrt{\eta\dot{\gamma}}
\]

\[
\Rightarrow \dot{\gamma} = \frac{1}{k_c} \left( \sqrt{\tau} - \sqrt{\tau_o} \right)^2, \tau \geq \tau_o, \quad (6.2.2)
\]

\[
= 0, \quad \tau \leq \tau_o, \quad (6.2.3)
\]

where \( \tau \) is shear stress,

\( \tau_o \) is the yield stress,

\( \dot{\gamma} \) is the rate of shear strain,

\( k_c \) is the Casson’s viscosity.

To find strain rate (\( \dot{\gamma} \)), we integrate equation (6.2.1) twice and use the boundary condition of usual no-slip at wall (Schlichting, 1968)

at \( r=R(z) \), tube radius, \( u=0 \),

and symmetry condition: at \( r = 0 \) (tube axis), \( u \) is finite,

there yields the solution

\[
u(r) = \frac{c R^2(z)}{4 \mu} \left\{ 1 - \left( \frac{r}{R(z)} \right)^2 \right\}, \quad 0 \leq r \leq R(z) \quad (6.2.4)
\]

Shear stress at any distance \( r \) from the tube axis is given (Schlichting, 1968)

\[
\tau_{rz} = \mu \frac{du}{dr}
\]
\[ \frac{cR^2(z)}{4\mu} \left\{ 0 - \frac{2r}{R^2(z)} \right\} \]

\[ = -\frac{cr}{2} \]

\[ = \frac{r}{2} \frac{dp}{dz} \]  \hspace{1cm} (6.2.5)

Wall shear stress

\[ \tau_w = \tau_{rz} \bigg|_{r=R(z)} \]

\[ = \frac{R(z)}{2} \frac{dp}{dz} \]  \hspace{1cm} (6.2.6)

At the stenotic wall, shear stress is \( \tau_w \) and at a certain distance from the tube axis it is \( \tau_0 \) the yield stress (Fung, 1981) whose correspondence on the horizontal axis is \( r_c \), a critical radius (Fig. 6.1).

Taking \( \tau_{rz} = \tau_0 \), the expression for yield stress leads to the form:

\[ \tau_0 = \frac{r_c}{2} \frac{dp}{dz} = -\frac{cr}{2} \]  \hspace{1cm} (6.2.7)

For the two stresses \( \tau_0 \) and \( \tau_w \), there may arise two cases:

(i) if the shear stress \( \tau_{rz} \) is less than the yield stress i.e. \( r < r_c \) then blood will not flow and for that \( u(r) = 0 \), when \( \frac{dp}{dz} < \frac{2}{R(z)} \tau_0 \)

(ii) If \( \tau > \tau_0 \) i.e. \( r > r_c \) then blood will flow and for that \( u = u(r) \) when

\[ \frac{dp}{dz} > \frac{2}{R(z)} \tau_0 \]
Thus the Casson’s equation (6.2.2-6.2.3) can be written as

\[
\dot{\gamma} = \frac{1}{k_c} \left( \sqrt{\tau_{rz}} - \sqrt{\tau_0} \right)^2, \quad \tau_{rz} \geq \tau_0
\]  \tag{6.2.8}

\[
= 0, \quad \tau_{rz} \leq \tau_0
\]  \tag{6.2.9}

where the vanishing of the strain rate i.e. \(\dot{\gamma} = 0\)

\[
\Rightarrow \frac{d}{dr} u(r) = 0
\]

\[
\Rightarrow u(r) = \text{constant} = u_c \quad \text{where} \quad \tau_{rz} = \tau_0
\]

where \(u_c\) is the core velocity at \(r = r_c\) (core radius)

Thus for blood flow when \(r_c < R_1(z)\), there will be two regions viz. \(0 \leq r \leq r_c\) and \(r_c \leq r \leq R_1(z)\)

For the region \(0 \leq r \leq r_c\),

\[
\Rightarrow \frac{d}{dr} u(r) = 0,
\]

\[
\Rightarrow u(r) = u_c \quad \text{for} \quad 0 \leq r \leq r_c
\]

Which indicate that velocity profile will be flat and for the region \(r_c \leq r \leq R_1(z)\), \(u(r)\) will exhibit deviations from flat profile and Casson’s equation (6.2.8) has to be applied for this domain of blood flow (Young, 1984). From the above considerations and equations (6.2.5), (6.2.7) and (6.2.8), the governing equations of motion for one dimensional steady, laminar flow in an asymmetric stenosed artery of a Casson fluid the equation (6.2.8) is transformed to (Biswas and Nath, 2003)

\[
\frac{d}{dr} u_1(r) = \frac{1}{k_c} \left( \sqrt{\frac{-cr}{2}} - \sqrt{\frac{-c.r_c}{2}} \right)^2, \quad r_c \leq r \leq R_1(z)
\]  \tag{6.2.10}
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\[ = 0, \quad 0 \leq r \leq r_c. \]

And for the peripheral region

\[
c + \frac{1}{r} \frac{d}{dr} \left( r \mu_1 \frac{du_2}{dr} \right) = 0, \quad R_1(z) \leq r \leq R(z),
\]

(6.2.11)

where \( u_1 \) and \( u_2 \) are the velocities in the core and PPL, \( c = -\frac{dp}{dz} \) is the pressure gradient, \( r \) is the radial co-ordinate, \( r_c \) is a critical radius corresponding to \( \tau_y \), the yield stress.

### 6.3 Boundary Conditions

For solving the above equations (6.2.10-6.2.11) we use the following boundary conditions:

(i) \( u_2 = 0 \) at \( r = R(z) \), (no slip at the stenotic wall) \hspace{1cm} (6.3.a)

(ii) \( u_1 - u_2 = u_s \) at \( r = R_1(z) \), (slip at interface) \hspace{1cm} (6.3.b)

(iii) \( \frac{\partial u_2}{\partial r} = 0 \), at \( r = 0 \) \hspace{1cm} (6.3.c)

(iv) \( u_3(r) = u_1(r) \) at \( r = r_c \) \hspace{1cm} (6.3.d)

where \( u_s \) is the slip velocity at interface

### 6.4 Solution Of The Problem

The solutions of eqs. (6.2.10-6.2.11) with boundary conditions (eqs.6.3.a-d) are obtained as

\[
u_2 = \frac{c}{4 \mu_1} \left( R^2(z) - r^2 \right), \quad R_1(z) \leq r \leq R(z)
\]

(6.4.1)
\[ u_i = u_s + \frac{c}{4k_c} \left[ \left( R_i^2(z) - r^2 \right) + 2r_c \left( R_i(z) - r \right) - \frac{8}{3} \sqrt{r_c \left( \sqrt{R_i^3(z)} - R_i(z) \right)} \right] \]

\[ + \frac{c}{4\mu_i} \left( R^2(z) - R_i^2(z) \right), \quad r_c \leq r \leq R_i(z) \quad (6.4.2) \]

Also \( u_i = u_c \) for \( 0 \leq r \leq r_c \) where \( u_c \) is the core velocity, which can be defined as

\[ u_c = u_s + \frac{c}{4k_c} \left[ \left( \sqrt{R_i(z)} - \sqrt{r_c} \right)^3 \left( \sqrt{R_i(z)} + \frac{\sqrt{r_c}}{3} \right) \right] + \frac{c}{4\mu_i} \left( R^2(z) - R_i^2(z) \right) \quad (6.4.3) \]

The volumetric flow rate \( Q \), defined by

\[ Q = 2\pi \int_0^{R(z)} ru(r) dr \]

is obtained as (using eqs. 6.4.1-6.4.3)

\[ = \pi R_i^2(z) u_s + \frac{\pi c R_i^4(z)}{8\mu_i} \left\{ 1 - \left( \frac{R_i(z)}{R(z)} \right)^4 \right\} - \frac{\pi R_i^4(z)}{8k_c} \left( \frac{dp}{dz} \right) \psi(\beta) \quad (6.4.5) \]

where \( \psi(\beta) = \left\{ 1 - \frac{16\sqrt{\beta}}{7} + \frac{4\beta}{3} - \frac{1}{21} \beta^4 \right\} \)

\[ l = \frac{r_c}{R_i(z)} \quad \beta = l^2 \]

From equation (6.4.5), the pressure gradient term can express as

\[ \frac{dp}{dz} = \frac{8\mu_i}{1 - \mu_i \psi(\beta) R_i^4(z) - R^4(z)} \left( \frac{Q}{\pi} - u_i R_i^2(z) \right) \quad (6.4.6) \]
where $\mu'_i = \frac{H_i}{k_c}$

Integrating between the limits $P = P_i$ at $z = 0$ and $P = P_0$ at $z = L$, we get the pressure drop as

$$p_i - p_0 = 8\mu_1 \int_{z=0}^{L} \left[ R^4(z) - \left(1 - \mu'_i \psi(\beta)\right)R_1^4(z) \right]^{-1} \left(\frac{Q}{\pi} - u_s R_i^2(z)\right) dz \quad (6.4.7)$$

The resistance to flow ($\lambda$) is defined by

$$\lambda = \frac{p_i - p_0}{Q}$$

$$= 8\mu_2 \left[ R_0^4 \left(1 - \left(1 - \mu'_i \psi \left(\frac{r_c}{\alpha R_0}\right)\right)\alpha^4 \right) \right]^{-1} \left[ \frac{1}{\pi} - Q^{-1} \left(\alpha R_0\right)^2 u_s \right] \left(\frac{L-L_0}{L}\right)$$

$$+ \int_{z=d}^{d+L_0} \left[ R^4(z) - \left(1 - \mu'_i \psi(\beta)\right)R_1^4(z) \right]^{-1} \left(\frac{1}{\pi} - Q^{-1} u_s R_i^2(z)\right) dz \quad (6.4.8)$$

where $Q_1 = Q$, at $R(z) = R_0$

The average pressure gradient in the axial direction can be defined as

$$\left(\frac{dp}{dz}\right)_{av} = \frac{\int_{r=0}^{R(z)} r \left(\frac{dp}{dz}\right) dr}{\int_{r=0}^{R(z)} r dr}$$

$$= \left(\frac{dp}{dz}\right)$$

(6.4.9)
Apparent viscosity can be expressed from the formula\[ \mu_a = \frac{\pi c R^4(z)}{8Q} \] as

\[
\mu_a = \left[ \frac{8\mu_s}{cR^2(z)} \left( \frac{R_i(z)}{R(z)} \right)^2 + \mu_1^{-1} \right]^{-1} \left( 1 - \left( 1 - \mu_1(\beta) \right) \left( \frac{R_i(z)}{R(z)} \right)^4 \right) \tag{6.4.10}
\]

Stress components at stenotic wall (\( \tau_w \)), interface (\( \tau_{R(z)} \)) and yield stress(\( \tau_y \)) can be obtained from the formula \( \mu = \left( \frac{cr}{2} \right) \frac{dp}{dz} \) in the following forms

\[
\tau_w = -\mu \frac{\partial u_z}{\partial r} \text{ at } r = R(z)
\]

\[
= -\frac{c}{2} R(z)
\]

\[
= \frac{R(z)}{2} \frac{dp}{dz} \tag{6.4.11}
\]

\[
\tau_{R(z)} = \left( -\frac{c}{2} \right) R_i(z)
\]

\[
= \frac{R_i(z)}{2} \frac{dp}{dz} \tag{6.4.12}
\]

\[
\tau_y = \left( -\frac{c}{2} \right) r_c
\]

\[
= \frac{r_c}{2} \frac{dp}{dz} \tag{6.4.13}
\]

The non dimensional form of the flow variables can be expressed, by using the following non-dimensional variables:
\[
\bar{z} = \frac{z}{R_0}, \quad \bar{d} = \frac{d}{R_0}, \quad \bar{R} = \frac{R}{R_0}, \quad \bar{R}_1 = \frac{R_1}{R_0},
\]

\[
\bar{\delta} = \frac{\delta}{R_0}, \quad \mu'_1 = \frac{\mu_1}{k_c}, \quad \left( \frac{dp}{dz} \right) = \frac{dp}{dz},
\]

\[
\bar{A} = \frac{A}{R_0^{n-1}}, \quad (\bar{L}_c, \bar{L}_0) = \left( \frac{L_c, L_0}{R_0} \right), \quad \bar{u} = \frac{u}{u_0}
\]

Where \( \bar{\lambda} = \frac{\lambda}{\lambda_0}, \quad \bar{Q} = \frac{Q}{Q_0} \),

\[
u_0 = \frac{c R_0^2}{4 \mu_1}, \quad F = \frac{\rho g}{C}, \quad \tau_{R(z)0} = -\left( \frac{dp}{dz} \right) \frac{R_0}{2},
\]

\[
Q_0 = \frac{\pi c R_0^4}{8 \mu_2}, \quad \left( \frac{dp}{dz} \right)_0 = -\frac{8 \mu_1 Q_0}{\pi R_0^4}, \quad \lambda_0 = \frac{8 \mu_1 L}{\pi R_0^4},
\]

Velocity functions:

\[
\bar{u}_s = \bar{R}^2 \left[ 1 - \left( \frac{r}{R(z)} \right)^2 \right], \quad \frac{R_1(z)}{R(z)} \leq \frac{r}{R(z)} \leq 1 \tag{6.4.14}
\]

\[
\bar{u}_l = \bar{u}_s + \mu'_1 \left[ \bar{R}_l^2 - \bar{R}^2 \left( \frac{r}{R(z)} \right)^2 + 2 \tau \left( \bar{R}_l - \bar{R} \left( \frac{r}{R(z)} \right) \right) - \frac{8}{3} \sqrt{\tau_c} \left( \sqrt{\bar{R}_l^3} - \sqrt{\left( \frac{R(z)}{R(z)} \right)^3} \right) \right]
\]

\[
\left( \bar{R}_l^2 - \bar{R}_1^2 \right), \quad \frac{R_c}{R} \leq \frac{r}{R} \leq \frac{R_1}{R} \tag{6.4.15}
\]

\[
\bar{u}_c = \bar{u}_s + \mu'_1 \left( \sqrt{\bar{R}_1} - \sqrt{\tau_c} \right)^3 \left( \sqrt{\bar{R}_1} + \frac{1}{3} \sqrt{\tau_c} \right) + \left( \bar{R}_1^2 - \bar{R}_1 \right), \quad 0 \leq \frac{r}{R} \leq \frac{r_c}{R} \tag{6.4.16}
\]
Flow rate
\[ \bar{Q} = 2\bar{u}_s\bar{R}_1^2 + \left[ \bar{R}_1^4 - \bar{R}_1^2 \left( 1 - \mu'_t\psi\left( \bar{r}^2 \right) \right) \right] \]  
\[ (6.4.17) \]

where \( \psi\left( \bar{r}^2 \right) = \psi\left( \frac{\bar{r}}{\bar{R}_1} \right) \), \( \mu'_t = \frac{\mu_t}{k_c} \)

Pressure gradient:
\[ \frac{dp}{dz} = \left[ \bar{R}_1^4 - \left\{ 1 - \mu'_t\psi\left( \bar{\beta} \right) \right\} \bar{R}_1^4 \right]^{-1} \left( \bar{Q} - 2\bar{u}_s\bar{R}_1^3 \right) \]
\[ = \left( \frac{dp}{dz} \right)_{av} \]  
\[ (6.4.18) \]

Resistance to Flow:
\[ \bar{z} = \left[ 1 - \left\{ 1 - \mu'_t\psi\sqrt{\alpha^{-1}\bar{r}} \right\} \alpha^4 \right]^{-1} \left( 1 - 2\alpha^2 \frac{\bar{u}_s}{\bar{Q}_1} \right) \left( \frac{L - L_0}{L} \right) \]
\[ + \frac{1}{L} \int_{z-d}^{d} \left[ \bar{R}_1^4 - \left\{ 1 - \mu'_t\psi\left( \bar{\beta} \right) \right\} \bar{R}_1^4 \right]^{-1} \left( 1 - 2\bar{u}_s\bar{R}_1^3 \right) \]
\[ (6.4.19) \]

Apparent viscosity:
\[ \bar{\mu}_a = \left[ 1 + 2\frac{\bar{u}_s}{\bar{R}^2} \left( \frac{\bar{R}_1}{\bar{R}} \right)^2 \right]^{-1} \left\{ 1 - \mu'_t\psi\left( \bar{\beta} \right) \right\} \left( \frac{\bar{R}_1}{\bar{R}} \right)^4 \]  
\[ (6.4.20) \]

Flow Geometry

For PPL
\[ \bar{R}(\bar{z}) = \begin{cases} 1 - \bar{A} \left[ \bar{L}_0^{n-1} (\bar{z} - \bar{d}) - (\bar{z} - \bar{d})^n \right], & \bar{d} \leq \bar{z} \leq \bar{d} + \bar{L}_0 \\ 1, & \text{Otherwise} \end{cases} \]  
\[ (6.4.21) \]
For the Core region

\[
\bar{R}_1(\bar{z}) = \alpha - A \left[ L_0^{-1} \left( \bar{z} - \bar{d} \right)^n - \left( \bar{z} - \bar{d} \right)^n \right], \quad \bar{d} \leq z \leq \bar{d} + L_0
\]

\[
= \alpha, \quad \text{otherwise}
\]

(6.4.22)

### 6.5 Results and Discussions

In the present analysis three successive growths at lumen of an artery, slip and no-slip cases at interface for investigating two-layered in uniform stenosed tube are employed in Fig 6.1. In the model body fluid blood is assumed to behave as a non-Newtonian fluid, inhabiting an yield property known as Casson fluid. In such a visco-inelastic fluid, shear stress versus vs rate of strain relationship is non-linear and as the Casson fluid possess a finite yield stress \( \tau_y \), there may arise two flow situations mainly (a) if the shear stress at a radial distance \( r \) is not higher than the yield stress \( \tau_y \) \((\tau_{r_1} \leq \tau_y\)), blood will not flow and if otherwise i.e. if the shear stress is lower its yield stress \( \tau_y \) value \((\tau_{r_1} \geq \tau_y\)), blood will be possible. Analytical expression for flow variables are obtained and the variations of axial velocity of both regions, rate of flow, pressure gradient, shear stress and apparent viscosity at the interface of fluids in the constricted region etc. have been presented graphically (Figs 6.2-6.22)

In this two-layered blood flow there are three regions namely \( 0 \leq r \leq r_c \) (critical radius), \( r_c \leq r \leq R_1 \) and \( R_1 \leq r \leq R \), where \( r_c \) is a critical radius, when \( r = r_c \), then \( u(r) = u_c \) (a constant velocity) and \( \tau_{r_1} = \tau_y \). (b) when \( R(z) = R_0 = R_1(z) \), \( \tau_y = 0 \) (or \( r_c = 0 \)) it leads to Poiseuille flow of blood (behaving as a Newtonian fluid) with slip or zero-slip at the vessel wall.

#### 6.5.1 Variation of velocity

The variations of velocity is shown in Figs.(6.2-6.8). As expected, velocity attains a constant magnitude in the yield stress zone and thereafter shows parabolic trends in core region and PPL. As slip velocity increases, velocity increases in core and PPL.
regions. Velocity attains the highest magnitude in all three layers for mild formation of stenosis and the lowest one in severe case of constriction. Although velocity shows a non-parabolic trend in core region but in PPL region, it exhibits a parabolic profile. It increases with shape parameter $n = 2$ (symmetric case) to $n > 2$ (asymmetric stenosis).

6.5.2 Variation of flow rate

Flow rate Figs. (6.9-6.11) attains the greatest magnitude at the two ends of stenosis for all values of shape parameter $n$ but the lowest magnitude is attained at the throat for $n=2$ (symmetric form) and near the termination position for $n > 2$ (asymmetric case). It is the highest for mild form of stenosis and the lowest for severe stenosis, although minimum magnitude is attained at the stenotic throat. However, in all cases of stenosis, flow rate is higher with slip velocity than that obtained with zero-slip velocity.

6.5.3 Variation of pressure gradient

The variation of pressure gradient, shown in Figs. 96.12-6.15, reveals that pressure gradient decreases as shape parameter $n$ increases (from $n=2$ to $n > 2$). It is lowest with slip and increasing slip velocity for symmetric and asymmetric forms of stenosis.

6.5.4 Apparent viscosity

The variation of apparent viscosity, exhibited in Figs. (6.16-6.17), shows that as yield stress increases, it increases in all three stenosis formations. It decreases with velocity slip. The apparent viscosity attains the minimum value for mild stenosis case and the maximum magnitude for severe growth. It is found that $\mu_a$ (mild case) < $\mu_a$ (moderate forms) < $\mu_a$ (severe growth).

6.5.5 Wall shear stress

Wall shear stress Figs. (6.18-6.22) exhibits that it decreases with slip and it further decreases as slip velocity increases. It attains the highest magnitude in severe form of stenosis and the lowest value at the mild form of stenosis. As $Q$ increases, it increases for all three types of growth.
6.5 Conclusions

The present chapter deals with steady, laminar and one-dimensional (1-D) flow of in the two-layered region of a uniform asymmetric stenosed artery in Fig 6.1. In order to study the effect of velocity slip at the fluid interface, an analysis has been developed here by employing a slip condition at the interface of fluids in case of a mild, moderate and severe arterial stenosis. Analytical expression for flow variables are obtained and the variations of axial velocity of both regions, rate of flow, pressure gradient, shear stress and apparent viscosity at the interface of fluids in the constricted region etc. have been presented graphically (Figs 6.2-6.22)

A second form of the flow variables are obtained and their variations are shown graphically. It is found that velocity in a function of R, z, k_c, L_0 etc. The following observation can be recorded from the present analysis

(a) In the two-layered Casson fluid flow, there arises two region for flow (for flow situation) viz. \( \tau_x \leq \tau_y \) and \( \tau_x \geq \tau_y \) and in the former case, no flow will occur whence as in this later blood flow will be possible.

(b) For two-layered blood flow within a uniform stenosed arterial region, there are regions viz. \( 0 \leq r \leq r_c, r_c \leq r \leq R_1(z) \) and \( R_1(z) \leq r \leq R(z) \)

(c) It includes (i) Poiseuille flow of blood (acting as non-Newtonian fluid) with wall-slip or at the vessel boundary. (ii) one-dimensional stenosed flow of both Newtonian fluid and Casson fluid with slip or zero-slip at uniform arterial wall

(d) The behaviour of Velocity is presented in the Fig (6.2-6.8). It attains a fixed magnitude in yield stress zone and alters parabolically in core and ppl regions. It increases with shape parameter \( n = 2 \) (symmetric case) to \( n > 2 \) (asymmetric stenosis). It attains the highest magnitude for mild growth and lowest value for severe stenosis. However, in all forms of stenosis development, velocity increases with a velocity slip as well as with an increasing magnitude of slip.

(e) The variations of flow rate are shown in Figs (6.9-6.11), it shows that the magnitude attained with \( n = 2 \) is lower than that with \( n > 2 \). Flow rate reaches the maximum for mild stenosis and the minimum for severe stenosis.
(f) The behaviour of pressure gradient, as shown in Figs 6.12-6.15. However, pressure gradient \( C = \frac{dp}{dz} \), indicates the highest value for mild growth and the lowest magnitude for severe stenosis. It increases with velocity slip.

(g) Apparent viscosity in Fig (6.16-6.17), is lowered with velocity slip. It attains the greatest value for severe growth and the lowest magnitude for mild formation. It is found that \( \bar{\mu}_a \) (mild case) < \( \bar{\mu}_a \) (moderate forms) < \( \bar{\mu}_a \) (severe growth).

Thus in view of the above analysis, for flow variables it could be concluded that with an employment of axial velocity slip at fluids interface in the constricted uniform channel. Damages to a diseased or occluded artery could be reduced. In this analysis, slip has been employed to accelerate the flow in the constricted two-layered region in one hand and to reduce the impedance to this unidirectional flow, on the other. However, in the analysis, some arbitrary measures of slip \( u_s = 0, 0.5, 1 \), \( \tau_y, n = 2, 6, 9 \); \( k_c = 2cp \), have been used and stenotic wall at rigid, non-porous and visco-elastic as well as flow is steady and uni-directional.

This theoretical model could be improved if an appropriate magnitude of a velocity slip, yield stress of blood, PPL thickness, two-dimensional flow etc. are considered in the modeling. In order to propose such an improved model of blood flow through a two-layered uniform stenosed artery, the present model could be used as base. In blood flow modeling, it has been reported that apart from possessing a finite yield stress \( (\tau_y) \), blood exhibit a Newtonian behaviour. In order to include both the characteristic of blood, Bingham plastic fluid, in a two-layered constricted artery region has been dealt with in the next chapter 7.
Fig. 6.1: Schematic diagram of Two-Layered Casson Model of Blood Flow inside Asymmetric Stenosed Artery

Fig. 6.2: Variation of axial velocities against radial distance for different values of $n$ and $\bar{u}_s = 0.00, k_c = 2cp$
Fig. 6.3  Variation of axial velocities against radial distance
for different values of n and $u_\ast = .05, k_c = 2cp$

$k'_c = 2cp, \bar{\delta}_i = .14$
$\bar{\alpha}_i = 1, u_\ast = .05$

$z = d + \frac{L_0}{4}$  
n = 6

$z = d + \frac{3L_0}{4}$  
n = 9

$z = d + \frac{L_0}{2}$  
n = 2

Fig. 6.4: Variation of axial velocities against radial distance
for different values of n and $u_\ast = .1, k'_c = 2cp$

$k'_c = 2cp, \bar{\delta}_i = .14$
$\bar{\alpha}_i = 1, u_\ast = .1$
Fig. 6.5: Variation of axial velocity against radial distance for $u_s = 0, k_c = 2cp$ and $a=2$

Fig. 6.6: Variation of axial velocity against radial distance for $u_s = .1, k_c = 2cp$ and $a=2$
Fig. 6.7: Variation of axial velocity against radial distance for $u_s = 0, k_c = 2cp$ and $n=6$

Fig. 6.8: Variation of axial velocity against radial distance for $u_s = 1, k_c = 2cp$ and $n=6$
Fig. 6.9: Variation of flow rate against axial distance
for $u_s = 0, 0.05, 0.1, k_c = 2cp$ and $n = 2, 6, 9$

Fig. 6.10: Variation of flow rate against axial distance
for $kR_0 = 0.1, u_s = 0, 0.1, n = 2$
Fig. 6.11: Variation of flow rate against axial distance for $u_2 = 0, 1, k_c = 2c_1$ and $n=6$.

Fig. 6.12: Variation of pressure gradient against axial distance and $u_z = 0$.

$\mu_r^* = 0.62, \alpha = 0.8, \bar{u}_z = 0$

$\bar{\sigma}_r = 0.15, \bar{\sigma}_t = 0.12, k_c = 2c_1$
Fig. 6.13: Variation of pressure gradient against axial distance and $u_s = .05, k_c = 2c_p$

Fig. 6.14: Variation of pressure gradient against axial distance and $u_s = .1, k_c = 2c_p$
Fig. 6.15: Variation of pressure gradient against axial distance and $u_s = 0, .05, .1, k_c = 2cp, n = 2, 6, 9$

Fig. 6.16: Variation of apparent viscosity against axial distance for $\tau_y = .1, \tau_y = .04, k_c = 2cp$ and $n = 2$
Fig. 6.17: Variation of apparent viscosity against axial distance for $\tau_y = 0, u_z = 0, u_s = .1, k_c = 2cp$ and $n=2$

Fig. 6.18: Variation of wall shear stress against axial distance for $u_s = 0, .05, .1, \tau_y = 0$ and $k_c = 2cp$
Fig. 6.19: Variation of wall shear stress against axial distance for $\bar{u}_s = 0, .05, .1, n = 9$

Fig. 6.20: Variation of wall shear stress against axial distance for $\bar{u}_s = 0, .05, .1$
Fig. 6.21: Variation of wall shear stress against axial distance for $n=6$, $\tau_y = 0$, $\bar{u}_s = 0$, $Q = 0.5, 1.5, k_c = 2cp$

Fig. 6.22: Variation of wall shear stress against axial distance for $\bar{u}_s = 0, 1, n = 2$