CHAPTER 3

FINGERPRINT FEATURE EXTRACTION

3.1 FEATURE EXTRACTION METHODS

Extracting proper features from the biometrics traits for the study is vital for the satisfactory design of any image analysis system. Varieties of approaches are used in the literature for the feature extraction. Generally, these approaches are categorized as spatial domain and the frequency domain. In this work, fingerprint features have been identified by three approaches. By the first approach, FFT, 2D-DCT and PSD features have been extracted. Secondly, using DWT and SVD, a feature vector was generated. By the third approach, six ridge parameters along with the fingertip size of the fingerprint have been determined. However, only the ridge count (RC), ridge width (RW) and the fingertip size (FS) are selected for gender classification. Section 3.2 gives details of internal database and how the ten fingers are numbered and used in this study. In section 3.3, feature extractions through image transforms have been described. The generation of feature vector using DWT and SVD are demonstrated in section 3.4. Section 3.5 explains the way of finding the spatial features.

3.2 FINGERPRINT DATA ACQUISITION

The fingerprint samples were collected from subjects residing at various parts of Tamil Nadu, namely Chennai (all age groups), Trichy (ages 26-55 years), Hosur (ages 8-12 years) and Dharmapuri (all age groups). The
fingerprint images were collected using Fingkey Hamster II scanner manufactured by Nitgen biometric solution, Korea. The fingerprint image is of 8 bit grey level with a size of 300 x260 and resolution of 500 dpi.

### 3.2.1 Database

The population samples of internal database consist of 403 males and 410 females. All 10 fingers of each subject were scanned and thus in total, 8130 fingerprint have been used. The fingerprints were categorized into four age groups, viz., 8 to 12, 13 to 18, 19 to 25 and above 25. The number of samples in each age group is shown in Table 3.1.

<table>
<thead>
<tr>
<th>Age group</th>
<th>Male</th>
<th>Female</th>
<th>Total samples (No. of subjects X ten fingers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8-12</td>
<td>44</td>
<td>48</td>
<td>920</td>
</tr>
<tr>
<td>13-18</td>
<td>55</td>
<td>77</td>
<td>1320</td>
</tr>
<tr>
<td>19-25</td>
<td>198</td>
<td>193</td>
<td>3910</td>
</tr>
<tr>
<td>Above 25</td>
<td>106</td>
<td>92</td>
<td>1980</td>
</tr>
<tr>
<td><strong>Total Samples</strong></td>
<td><strong>403</strong></td>
<td><strong>410</strong></td>
<td><strong>8130</strong></td>
</tr>
</tbody>
</table>

The samples of publicly available databases mentioned in the Table 1.1 are also used for the feature extraction. However, as discussed in the section 1.6, their results are not independently verified and compared.

### 3.2.2 Finger Numbering

The fingers are numbered as follows. Starting from left little finger to right little finger, the fingers are numbered 1 to 10, as shown in Figure 3.1.
3.3 FREQUENCY DOMAIN FEATURES

Frequency domain features using image transform rely on the fact that the pixels in an image exhibit certain level of correlation with their neighboring pixels (Jayaraman et al. 2011). Increased recognition rate has been achieved using FFT and DCT coefficients (Saleh et al. 2009). Image transform transforms an image data in time domain to frequency domain. In addition, by transforming the data into frequency domain, the spatial redundancies in time domain can be minimized. The energy of the transformed data is represented by transform coefficients. It is evident that the significant information about the image is concentrated in a few coefficients. These coefficients are regarded as the weights. The constant valued basis function at the upper left (the first element (1, 1) of the transform matrix) is often called the DC coefficient (for the entire array). The DC coefficient represents average intensity of a block and carries most of the energy and the perceptual information. In this work, the DC coefficient has been considered as the fundamental coefficient (FC). All the remaining coefficients containing the frequency information are called AC coefficients. These AC coefficients describe their variations around the DC value. To extract the frequency domain features, the image transform of the fingerprints using FFT, DCT, and PSD is computed. PSD is defined as the Fourier
The following sub-sections describe the image transforms and extraction of FC.

3.3.1 Fingerprint Region Cropping

The fingerprint region cropping is performed as a pre-processing step to reduce the template size. As the cropping around the center have good ridge information, the fingerprint center point is selected first and then the image is cropped. In Figure 3.2, the input image and the cropped image is shown.

(a) Input image        (b) Cropped image

Figure 3.2 Fingerprint image and cropped image

3.3.2 Fast Fourier Transform (FFT) Coefficients

Fast Fourier Transform is computationally efficient form of Discrete Fourier Transform (DFT). FFT is used in numerical analysis to transform an image between spatial and frequency domain. It decomposes an image into sine and cosine of varying amplitude and phase (Park and Park 2005). The pair of Equations (3.1) and (3.2) is used for 2D-FFT.

\[
F[k,l] = \frac{1}{\sqrt{MN}} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} f[m,n]e^{-j2\pi\left(\frac{mk}{M} + \frac{nl}{N}\right)}
\]

(3.1)
where, $0 \leq m, k \leq M - 1, 0 \leq n, l \leq N - 1$

Two dimensional Fourier transforms simply involve a number of one dimensional Fourier transforms. More precisely, a two dimensional transform is achieved by first transforming each row, replacing each row with its transform and then transforming each column, replacing each column with its transform. Thus, a 2D transform of a 1K by 1K image requires 2K 1D transforms. This follows directly from the definition of the Fourier transform of a continuous variable or the discrete Fourier transform of a discrete system.

The transform pairs that are commonly derived in one dimension can also be derived for the 2 dimensional situations. The two dimensional pairs can often be derived simply by considering the procedure of applying transforms to the rows and then the columns of the two dimensional arrays as expressed below in Equations (3.3) to (3.6).

We define,

$$w_M[m,k] = \frac{1}{\sqrt{M}} e^{\frac{j2\pi mk}{M}}$$  \hspace{1cm} (3.3)$$

$$w_N[n,l] = \frac{1}{\sqrt{N}} e^{\frac{j2\pi nl}{N}}$$  \hspace{1cm} (3.4)$$

We further define

$$X'[k,n] = \sum_{m=0}^{M-1} w_M[m,k] \cdot x[m,n] \quad (k = 0,1,\ldots,M - 1)$$  \hspace{1cm} (3.5)$$
And rewrite the 2D transform as

\[ X[k,l] = \sum_{n=0}^{N-1} w_N[n,l] X'[k,n] \quad (l = 0, 1, \cdots, N - 1) \quad (3.6) \]

**Column Transform:** First consider the expression for \( X'[k, n] \). As the summation is with respect to the row index \( m \) of \( x[m, n] \), the column index \( n \) can be treated as a parameter, and the expression is the 1D Fourier transform of the \( n \)th column vector of \( x \), which can be written in column vector (vertical) form for the \( n \)th column as given below.

\[
\begin{bmatrix}
X'[0,n] \\
\vdots \\
X'[M-1,n]
\end{bmatrix}_{M \times 1} =
\begin{bmatrix}
\cdots & \cdots & \cdots \\
\cdots & \omega_M[m,k] & \cdots \\
\cdots & \cdots & \cdots
\end{bmatrix}_{M \times M} 
\begin{bmatrix}
x[0,n] \\
\vdots \\
x[M-1,n]
\end{bmatrix}_{M \times 1}
\]

or more concisely

\[ X'_n = W_M X_n, \quad (n = 0, \cdots, N - 1) \quad (3.7) \]

i.e., the \( n \)th column of \( X' \) is the 1D FT of the \( n \)th column of \( X \). Putting all \( N \) columns together, we have

\[ [X'_0, \cdots, X'_{N-1}] = W_M [x_0, \cdots, x_{N-1}] \]

or more concisely

\[ X' = W_M X \quad (3.8) \]

Where \( W_M \) is a \( M \) by \( M \) Fourier transform matrix.
Row Transform

Now we reconsider the 2D DFT expression above

\[ X[k,l] = \sum_{n=0}^{N-1} w_n[n,l] X'[k,n] \]  \hspace{1cm} (3.9)

As the summation is with respective to the column index \( n \) of \( X'[k,n] \), the row index \( k \) can be treated as a parameter, and the expression is the 1D Fourier transform of the \( k^{th} \) row vector of \( X' \), which can be written in row vector (horizontal) form for the \( k^{th} \) row as given below.

\[
\begin{bmatrix}
X[k,0],...,X'[k,N-1]
\end{bmatrix} = \begin{bmatrix}
X'[k,0],...,X'[k,N-1]
\end{bmatrix} \begin{bmatrix}
\cdots & \cdots & \cdots \\
\cdots & w_n[n,l] & \cdots \\
\cdots & \cdots & \cdots
\end{bmatrix}
\]

or more concisely

\[ X_k^T = X_k'^T W_N, \quad (k = 0,...,M-1) \] \hspace{1cm} (3.10)

i.e., the \( k^{th} \) row of \( X \) is the 1D FT of the \( k^{th} \) row of \( X' \). Putting all \( M \) rows together, we have

\[
\begin{bmatrix}
X_0^T \\
\vdots \\
X_{M-1}^T
\end{bmatrix} = \begin{bmatrix}
X_0'^T \\
\vdots \\
X_{M-1}'^T
\end{bmatrix} W_N
\]

or more concisely

\[ X = X' W_N \] \hspace{1cm} (3.11)

But as \( X' = W_M x \), we finally have

\[ X = W_M x W_N \] \hspace{1cm} (3.12)
This expression indicates that 2D DFT can be carried out by 1D transforming all the rows of the 2D signal \( x \) and then 1D transforming all the columns of the resulting matrix. The order of the steps is not important. The transform can also be carried out by the column transform followed by the row transform. Similarly, the inverse 2D DFT can be written as

\[
X = W_M^* x W_N^* \tag{3.13}
\]

It is obvious that the complexity of 2D DFT is \( O(N^3) \) (assuming \( M=N \)), which can be reduced to \( O(N^2 \log_2 N) \) if FFT is used. Figure 3.3 shows, the frequency spectrum of FFT.

![Figure 3.3 Frequency spectrum of FFT](image)

FFT transform applied to all the samples (all ten fingers) of internal database and its finger wise average FC for all age groups is presented in Figure 3.4. For each finger there is a difference in coefficient values for male and female. In all age groups, the coefficients are well below 50000 and more for female than male.
The average FC values calculated for male and female of each age group is presented in Table 3.2. All the coefficients are closer to each other and a common threshold is sufficient for all the age groups. The threshold assigned for the FFT is discussed in chapter 5.

**Figure 3.4 Comparison of FFT coefficients of male and female fingerprints**
Table 3.2 Average FFT fundamental coefficients of fingerprint

<table>
<thead>
<tr>
<th>Age group</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>8-12</td>
<td>61880</td>
<td>62582</td>
</tr>
<tr>
<td>13-18</td>
<td>60418</td>
<td>61433</td>
</tr>
<tr>
<td>19-25</td>
<td>59064</td>
<td>61183</td>
</tr>
<tr>
<td>&gt;25</td>
<td>61005</td>
<td>61810</td>
</tr>
<tr>
<td>Average</td>
<td>60592</td>
<td>61752</td>
</tr>
</tbody>
</table>

From the Table 3.2, it is visualized that the FC values are greater for female than male in all age groups. All the values of the coefficients are around 60000.

3.3.3 Discrete Cosine Transform (DCT) Coefficients

The equation for the two-dimensional DCT is

\[
F(m, n) = \frac{2}{\sqrt{MN}} C(m) C(n) \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) \cos \left(\frac{2\pi + 1}{2M} \right) \cos \left(\frac{2\pi + 1}{2N} \right)
\]

(3.14)

where \( C(m), C(n) = \frac{1}{\sqrt{2}} \) for \( m = 0 \) and \( C(m), C(n) = 1 \) otherwise.

The number of rows and columns of the input signal must be powers of two. The 2D DCT is computed by applying 1D DCT (vertically) to columns and the resulting vertical DCT is applied with 1D DCT (horizontally). The step by step process is given in Figure 3.5.
Figure 3.5 Step by step process for computing 2D DCT

Figure 3.6 shows the frequency spectrum of DCT.

Figure 3.6 Frequency spectrum of DCT

DCT transform is applied to all the samples (all ten fingers) and its average FC calculated for all age group is presented in Figure 3.7.
Figure 3.7 Comparison of DCT coefficients of male and female fingerprints

It is visualized that the FC values are greater for male than female in all age groups. The average FC values of male and female identified for male and female of each age group is presented in Table 3.3.
Table 3.3 Average DCT fundamental coefficients of fingerprint

<table>
<thead>
<tr>
<th>Age group</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>8-12</td>
<td>46872</td>
<td>45300</td>
</tr>
<tr>
<td>13-18</td>
<td>44764</td>
<td>42769</td>
</tr>
<tr>
<td>19-25</td>
<td>44855</td>
<td>42480</td>
</tr>
<tr>
<td>&gt;25</td>
<td>51020</td>
<td>49447</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>46878</strong></td>
<td><strong>44999</strong></td>
</tr>
</tbody>
</table>

3.3.4 Power Spectral Density (PSD) Coefficients

Spectral method divides a fingerprint image into blocks, converts each block of fingerprint image from spatial field to frequency field using Discrete Fourier Transform (DFT), and estimates ridge distance of a block image according to the distribution of harmonic coefficients. Let $F$ (signal) is Fourier transform of the signal and the PSD is found by Equations (3.15) and (3.16).

$$PSD = |\text{abs} (F (\text{signal}))|^2/N$$  \hspace{1cm} (3.15)

OR

$$PSD = F (\text{signal}) F^* (\text{signal})/N$$  \hspace{1cm} (3.16)

where $N$ is Normalization factor. The frequency spectrum of PSD is shown in Figure 3.8.
Figure 3.8 Frequency spectrum of PSD

The average FC values of male and female identified for in each age group is presented in Table 3.4.

Table 3.4 Average PSD fundamental coefficients of fingerprint

<table>
<thead>
<tr>
<th>Age group</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>8-12</td>
<td>15298332</td>
<td>15633534</td>
</tr>
<tr>
<td>13-18</td>
<td>14642274</td>
<td>15096644</td>
</tr>
<tr>
<td>19-25</td>
<td>14018949</td>
<td>14989595</td>
</tr>
<tr>
<td>&gt;25</td>
<td>14881773</td>
<td>15257641</td>
</tr>
<tr>
<td>Average</td>
<td><strong>14710332</strong></td>
<td><strong>15244353</strong></td>
</tr>
</tbody>
</table>

The finger wise average FC for all age groups is presented in Figure 3.9. It is visualized that the FC value is greater for female than male.
Figure 3.9 Comparison of PSD coefficients of male and female fingerprints

3.4 FINGERPRINT FEATURE EXTRACTION THROUGH DWT AND SVD

Wavelet transform is a popular tool in image processing and computer vision because of its complete theoretical framework, flexibility for choosing bases and low computational complexity (Zhang et al. 2004). As wavelet features have been popularized by the research community for a wide range of applications, including fingerprint recognition (Pokhriyal and Lehri 2010, Verma and Goel 2011) face recognition (Feng et al. 2001 and Mazloom
and Ayat 2008,) and gender identification using face (Yaprakkaya et al. 2010). They have confirmed the efficiency of the DWT approach for gender identification using fingerprint.

The SVD approach is selected for the gender discrimination because of its good information packing characteristics and potential strengths in demonstrating results. It is considered as an information-oriented technique since it uses Principal Components Analysis (PCA) and a form of factor analysis to concentrate information (Golub and Kahane 1965). DWT and SVD-based fingerprint feature extractions are described below.

3.4.1 Feature Vector of Discrete Wavelet Transform (DWT)

Wavelets have been used frequently in image processing for feature extraction, de-noising, compression, face recognition (Fazli and Heidarloo 2012) and image super-resolution. Two dimensional DWT decomposes an image into sub-bands that are localized in frequency and orientation. The decomposition of images into different frequency ranges permits the isolation of the frequency components introduced by “intrinsic deformations” or “extrinsic factors” into certain sub-bands. This process results in isolating small changes in an image mainly in high frequency sub-band images. Hence, DWT is a suitable tool for designing a classification system.

The 2-D wavelet decomposition of an image results in four decomposed sub-band images referred to as Low–Low (LL), Low–High (LH), High–Low (HL) and High–High (HH). Each of these sub-bands represents different image properties. Typically, most of the energy in images is in the low frequencies and hence decomposition is generally repeated on the LL sub-band only (dyadic decomposition). For k level DWT, there are \((3^k + 1)\) sub-bands available. The energy of all the sub-band coefficients is used as feature vectors individually which is called sub-band energy vector (E). The energy of each sub-band is calculated by using Equation (3.17).
where $x_k(i,j)$ is the pixel value of the $k^{th}$ sub-band and $R$, $C$ are width and height of the sub-band, respectively.

Figure 3.10 shows the block diagram of the feature extraction by using DWT. The input fingerprint image is first cropped and then decomposed by using DWT. For level 1, the number of sub-bands is 4 and 3 sub-bands are added for each next level. Thus, the increase in levels of DWT increases the features.

![Figure 3.10 DWT-based fingerprint feature extraction](image)

### 3.4.2 Feature Vector of Singular Value Decomposition (SVD)

Singular Value Decomposition (SVD) is an algebraic technique for factoring any rectangular matrix into the product of three other matrices. It is a linear transform used often in pattern recognition as it possesses good information packing characteristics (Theodoridis and Koutroumbas 2003). Mathematically and historically, it is closely related to PCA. In addition, it provides insight into the geometric interpretation of PCA. Also, SVD has long been considered fundamental to the understanding of PCA.

SVD is the factorization of any $k \times p$ matrix into three matrices each of which has important properties. That is, any rectangular matrix $A$ of $k$ rows by $p$ columns can be factored into $U$, $S$ and $V$ by using Equation (3.18).
\[ A = U S V^T \]  
(3.18)

where

\[ U = A A^T \]  
(3.19)

\[ V = A^T A \]  
(3.20)

S is a \( k \times p \) diagonal matrix with \( r \) non-zero singular values on the diagonal, where \( r \) is the rank of A. Each singular value is the square root of one of the Eigen values of both \( A A^T \) and \( A^T A \). The singular values are ordered so that the largest singular values are at the top left and the smallest singular values are at the bottom right, i.e., \( S_{1,1} \geq S_{2,2} \geq S_{3,3} \) and so on.

Among the three rectangular matrices, S is a diagonal matrix which contains the square root Eigen values from U or V in descending order. These diagonal elements are selected and stored as a vector called Eigen vector (V). As the internal database contains images of size 300 rows and 260 columns, the feature vector of SVD is of the size 1x260. The spatial feature extraction of the proposed system is shown in Figure 3.11.

![SVD-based fingerprint feature extraction](image)

**Figure 3.11 SVD-based fingerprint feature extraction**

### 3.4.3 DWT Level 6 and SVD Feature Vector

As an example, the feature vector V of size 1x260 obtained by SVD and the sub-band energy vector \( E_k \) of size 1x19 obtained for DWT level 6 have been concatenated to form the feature vector. The resulting feature vector is of the size 1x279 (1x260 +1x19). An example excerpt of the extracted data vector of a fingerprint sample 521_F_7 is shown in Table 3.5.
The values should be viewed from top left to bottom right viewing row wise from left right. The shaded values are the features of DWT.

### Table 3.5 Typical feature vector of DWT level 6 and SVD

| DWT level 6 and SVD feature vector | 54095 8234 3177 2936 2843 2521 2247 2190 2043 1965 1933 1690 1669 1573 1459 1450 1395 1362 1249 1197 1170 1098 1057 1036 | 1002 994 944 904 887 867 848 843 784 773 753 717 | 706 696 684 665 641 605 599 581 562 539 536 515 | 511 496 490 468 462 441 438 422 405 399 385 374 | 364 360 357 349 337 324 316 313 304 301 283 | 273 268 259 256 249 246 239 233 228 224 218 213 | 204 197 196 189 183 175 174 168 165 158 153 151 | 149 144 141 138 134 131 129 128 125 120 116 112 | 110 108 106 105 103 100 98 97 94 92 88 87 | 87 85 83 82 81 79 77 77 74 73 72 70 | 69 68 68 65 65 64 63 61 60 59 59 58 | 57 57 56 55 54 53 52 50 50 49 48 47 | 47 47 45 45 45 44 43 42 42 41 41 40 | 39 38 38 37 36 35 34 34 33 32 32 | 31 30 30 30 29 29 29 28 28 27 26 25 | 25 24 23 23 22 22 21 21 20 20 20 19 | 18 18 17 17 17 16 15 15 14 13 13 13 | 12 11 11 11 11 10 10 10 8 8 8 7 | 7 6 6 6 5 4 4 4 3 3 3 2 | 2 2 2 1 1 1 1 0 0 0 0 0 | 0 0 0 0 0 0 0 0 0 11696 649 696 115 | 253 323 78 83 108 56 79 68 83 45 43 26 | 15 15 5 |

### 3.5 SPATIAL FEATURES OF FINGERPRINT

In spatial approach, there are two types of features exists namely local and global features (Alonso-Fernandez et al. 2007). Global features represent overall attribute of a finger such as fingerprint patterns and singular points. Local features represent ridge characteristics and minutiae details.
More generally, fingerprint details are defined in three levels, i.e., Level 1, Level 2 and Level 3. Level 1 feature includes friction ridge flow, pattern type and singular points. These features are used to categorize fingerprints. Minutiae details are categorized in level 2 and these details are used to establish a fingerprint’s individuality or uniqueness. Some of the popular minutiae are ridge end (termination), ridge bifurcation, lake, independent ridge, point or island, spur and crossover. Level 3 features include all dimensional attributes of the ridge such as ridge width, ridge shape, pores, edge contour and other details such as incipient ridges, creases and scars. These features are barely used by automated fingerprint verification system.

In this work, initially, singular points are determined to find ridge parameters. Minutiae details of ridge termination, bifurcation and lake are determined to evaluate the fingerprint vertical orientation. For the gender classification, ridge count, ridge width and fingertip size are used.

In the following section, the pre-processing of the fingerprint is initially carried out. A novel line-based minutiae extraction is demonstrated. Vertical orientation of fingerprint is addressed. Seven spatial parameters of fingerprint are determined. The parameters of ridge count, ridge width and fingertip size, which are used for gender classification, are discussed in detail.

3.5.1 Pre-processing

The acquired fingerprint images from scanners may be degraded and/or corrupted due to many reasons. In practice, due to variations in impression conditions, skin conditions (aberrant formations of epidermal ridges of fingerprints, postnatal marks, occupational marks), ridge configuration, acquisition devices, and no cooperative attitude of subjects, etc., a significant percentage of acquired fingerprint images (approximately
10 %) is of poor quality (Hong et al. 1998). Thus, for a reliable extraction of singular points, minutiae details and other parameters, pre-processing techniques are essential.

### 3.5.1.1 Enhancement and binarization

Image enhancement process greatly restores the missing information and removes spurious minutiae. Image enhancement process consists of four main stages (O’Gorman et al. 2008), as given below;

- Normalization
- Image Orientation
- Image Estimation and
- Gabor filtering

Figure 3.12 shows the flow chart to obtain the enhanced binary image.

![Figure 3.12 Fingerprint pre-processing steps](image)

**Figure 3.12 Fingerprint pre-processing steps**
Normalization: In the process of normalization, the evaluation of moments represents a systematic method of shape analysis (Jahne 2002). The evaluation of central moments normalized central moments and moment invariants convey shape attributes and thus are useful for object recognition. Normalization does not change the clarity of the ridge and valley structures. Let $I(i,j)$ denote the gray-level value at pixel $(i, j)$ of the input image. The normalized grey level value at pixel $(i, j)$ is defined in Equations (3.21) and (3.22) (Hong et al. 1998).

\begin{align*}
N(i, j) &= M_o + \sqrt{\frac{V_o (I(i, j) - M)^2}{V}} \quad \text{If } I(i, j) > M, \quad (3.21) \\
&= M_o - \sqrt{\frac{V_o (I(i, j) - M)^2}{V}} \quad \text{Otherwise}, \quad (3.22)
\end{align*}

where $M$ and $V$ denote the estimated mean and variance of $I$, respectively, and $M_o$ and $V_0$ are the desired mean and variance values, respectively.

Image Orientation: Image orientation is essential in enhancement process as the Gabor filtering stage relies on the local orientation to effectively enhance the fingerprint image. A number of methods have been proposed to estimate the orientation field of fingerprint images (Kawagoe and Tojo 1984, Kulkarni et al. 2006, Rao 1990). The least mean square orientation estimation algorithm proposed by (Hong et al. 1998) is used to obtain the orientation.

Image Estimation: The first step in the frequency estimation stage is to divide the image into blocks of size $w \times w$. The next step is to project the grey-level values of all the pixels located inside each block along a direction orthogonal to the local ridge orientation. This projection forms an almost
sinusoidal-shape wave with the local minimum points corresponding to the ridges in the fingerprint.

**Gabor Filtering:** Before performing Gabor filtering, it is reasonable to classify pixels into recoverable and unrecoverable region. This is achieved by mask generation. As the Gabor filter has frequency selective and orientation selective properties, it effectively preserves the ridge structures while reducing the noise. A two-dimensional Gabor filter consists of a sinusoidal plane wave of a particular orientation and frequency, modulated by a Gaussian envelope (Daugman 1985). The application of Gabor filter \( G \) to obtain the enhanced image \( E \) is performed by Equation (3.23).

\[
E(i, j) = \sum_{u=-\frac{w_x}{2}}^{\frac{w_x}{2}} \sum_{v=-\frac{w_y}{2}}^{\frac{w_y}{2}} G(u, v; O(i, j), F(i, j)) N(i-u, j-v),
\]

The convolution of a pixel \((i, j)\) in the image requires the corresponding orientation value \( O(i, j) \) and ridge frequency value \( F(i, j) \) of that pixel. \( N \) is the normalized fingerprint image and \( w_x \) and \( w_y \) are the width and height of the Gabor filter mask, respectively. The filter bandwidth is determined by the standard deviation parameters \( \sigma_x \) and \( \sigma_y \) and defined by the Equation (3.24) and (3.25).

\[
\sigma_x = K_x F(i, j) \quad (3.24)
\]

\[
\sigma_y = K_y F(i, j) \quad (3.25)
\]

**Binarization:** For binarization, the global threshold technique is effective in separating the ridges from the valleys. This improves the contrast between the ridges and valleys in a fingerprint image and facilitates the extraction of
minutiae The binarization process involves examining the grey level value of each pixel in the enhanced image and if the value is greater than the global threshold, then the pixel value is set to a binary value one; otherwise, it is set to zero.

3.5.1.2 Thinning

Thinning is a morphological operation that successively erodes away the foreground pixels until they are one pixel wide. It is a skeleton image of the given fingerprint image. Many standard thinning algorithms (Guo and Hall 1989, Lam et al. 1992, Zhang and Suen 1984) are available to perform the thinning operation. In this work, a standard thinning algorithm (Guo and Hall 1989) has been used. This algorithm performs the thinning process in two sub iterations.

In the first iteration, the image is divided into two distinct sub-fields in a checkerboard pattern, and each sub-iteration deletes pixel p in a sub-field when p is a contour pixel, b(p) > 1, and XH(p) = .1. The second algorithm is a modification of thin to 8 connected skeletons and retains diagonal lines and 2 x 2 squares. Equations (3.26) to (3.28) represent the process of thinning algorithm.

G1: \( X_{H}(p) = 1 \)

G2: \( 2 \leq \min \{ n_1(p), n_2(p) \} \leq 3 \), where

\[
n_1(p) = \sum_{i=1}^{4} x_{1i}V_s2k
\]

and

\[
n_2(p) = \sum_{i=1}^{4} x_{2i}V_s2k + 1
\]
represent the number of 4 adjacent pairs of pixels in $N(p)$ containing one or two black pixels in the first iteration and $180^\circ$ in the second iteration.

$$G3: (x_2 \lor x_3 \lor \neg x_1) \land x_1 = 0$$

This algorithm is accessible in MATLAB. Figure 3.13 shows the generation of thinned image from the enhanced binary image.

![Enhanced binary image](image1.png) ![Thinned image](image2.png)

**Figure 3.13 Generation of thinned image from the binary image**

### 3.5.2 Minutiae Extraction

Minutiae extraction is one of the most essential processes required and the minutiae based techniques take less memory and consume less time for processing. They are widely used in analysis and security. Although about 150 different possible minutiae details have been identified, the most common types are termination, bifurcation, lake, independent ridge, point or island, spur and crossover. A good quality fingerprint typically contains about 40-100 minutiae (Hong et al. 1998). In fingerprint analysis, minutiae are more abstract than fingerprint pixels and a fingerprint can be represented by minutiae constrained with their properties and relations (Yuliang et al. 2006).
The two most prominent ridge characteristics called minutiae are ridge termination and ridge bifurcation (Maltoni et al. 2003). A novel method of locating the minutiae details of bifurcation, termination and lake has been experimented using connected component analysis. A connected component is the definition of a local neighbourhood describing the connections between adjacent pixels. Any set of pixels which is not separated by a boundary is called connected. A region or an object is called connected when we can reach any pixel in the region by walking from one neighbouring pixel to the next (Jahne 2002). There are different neighbourhood systems. In square grid, we have 4 and 8 neighbourhood systems; in rectangular grid we can define 3 and 12 neighbourhoods where the neighbours have either a common edge or common corner. In hexagonal grid, we can only define a 6 neighbourhood because pixels which have a joint corner, but no joint edges.

By using connected component analysis and the ridge tracing approach, all connected line segments with respect to all the pixels are determined. The minutiae extraction process is accomplished by block processing. In order to validate the minutiae, post processing is performed on the minutiae extracted image.

**Line Based Approach**

Region boundaries are traced in a thinned binary image T. These traced objects are processed with run length encoding. Through these runs, scanning, preliminary labels are assigned and recorded as label equivalences in a local equivalence table. The reliable regions (representing image) are assigned unique numbers after resolving the equivalence classes. Some of the regions (lines) for the given input image are shown in Figure 3.14.
The steps involved in the proposed algorithm for line-based minutiae extraction are as follows.

1. Start loop i for numbers of lines identified
   a. Initialize an image B of the same size as the fingerprint image being processed with 0’s.
   b. Set the pixel values corresponding to the \( i^{th} \) region as 1’s.
      Find indices \((r, c)\) of non-zero elements.
   c. Start another loop j for number of 1’s present in image B.
      i. Let \( x \) be equal to \( r(j) \) and \( y \) be equal to \( c(j) \)
      ii. Create a 3x3 square mask \( S \) around the \((x, y)\) pixel
      iii. If center pixel is equal to 1, calculate the sum of all the elements present in \( S \) excluding the centre element.
      iv. If the sum corresponds to 1, then mark the centre pixel co-ordinate as termination point and if the sum
corresponds to 3, then mark the centre pixel co-
ordinate as bifurcation point.

d.  End loop j.

2.  Also the minutiae called enclosure (or lake) for each line are identified by tracing the region boundaries as shown in Figure 3.15.

![Diagram](image1.png)

**Figure 3.15**  Connected objects with the identified lake (a minutiae)

If an enclosed area is identified, then, the corresponding boundary pixels are marked as enclosure.

3.  End loop i.

4.  In Post-processing stage the false and non-vital minutiae can be eliminated. Create morphological disk-shaped structuring element SE of radius R = 3.

5.  Erode the mask of the fingerprint area located during the image enhancement process (Tico and kuosmanen 2000).

6.  Now check whether the identified minutiae points are present within the new mask created in the previous step.
7. If the points lie beyond the mask, then those minutiae points can be discarded because those will not mostly be the actual termination or bifurcation but formed due to the end of the fingerprint acquired. An input image and its image with minutiae details are shown in Figure 3.16. The minutiae details shown are the termination (yellow colour), bifurcation (red colour) and lake (purple colour).

![Input fingerprint image and its minutiae](a) Input image (b) Minutiae details)

**Figure 3.16 Input fingerprint image and its minutiae**

### 3.5.3 Fingerprint Image Rotation

User behaviour on placement of the finger on the scanner is an important consideration while analysing any fingerprint. In reality, the fingerprints are not exactly vertically oriented; the fingerprints may be oriented up to ±45° away from the assumed vertical orientation (Jain et al. 2000).
A vertically orientated fingerprint can provide a comprehensive description of the subject that enables discovery of the embedded features. In this proposed orientation model, a fingerprint enrolled in different orientation is rotated clockwise or anticlockwise. The orientation of the image is estimated as the angle between the x axis and the major axis of the ellipse.

In this approach, bilinear interpolation is used initially to resize image to a standard or constant size. This can allow the interoperability of the scanned image. Normalization and segmentation are used to obtain the mask of the given input image followed by morphological closing on the normalized and segmented grayscale image. From the mask, the orientation or the angle between the x-axis and the major axis of the ellipse bounding the mask is calculated. Based on the orientation, the angle by which the fingerprint is to be rotated clockwise or anticlockwise is determined. When the angle is determined, whole fingerprint image is rotated. The process of fingerprint rotation and enhancement is explained below.

**Normalization and Segmentation:** This process identifies the ridge region of the input image. The given input image is segmented into blocks of size n x n and the standard deviation in each region is evaluated. If the Standard Deviation (SD) is above the threshold, it is marked as part of the fingerprint. The image is normalized to have zero mean and unit standard deviation so that the threshold specified is relative to a unit standard deviation. Mask image \(I_{M1}\) is obtained using Equations (3.29) to (3.31).

\[
I(x, y) = I(x, y) - \text{mean}(I) \quad (3.29)
\]

\[
I(x, y) = \frac{I(x, y)}{\text{STD}(I)} \quad (3.30)
\]

\[
I_1(X, Y) = \text{RM} + I(X, Y) \times \text{SQRT}(RV) \quad (3.31)
\]
where, RM is the required mean and RV is the required variance.

**Morphological Filtering:** Morphological Filtering is based mainly on some mathematical morphology transformations (Serra 1988, Soille 1999). Here we perform morphological closing on the normalized and segmented grayscale image \( I_{M1} \) (Equation 3.31) to obtain the closed image, \( I_{M2} \). The Structuring Element (SE) must be a single structuring element object, as opposed to an array of objects. The morphological close operation is a dilation followed by erosion, using the same structuring element for both operations. This fills the image regions and holes.

Dilation of a MASK image \( I_{M1}(x, y) \) by a structuring element SE \((m, n)\) is denoted by \( I_{M2}(x, y) \)

\[
I_{M2}(x, y) = D(I_{M1}, SE)(x, y) = \max\{I_{M1}(x - m, y - n) + SE(m, n)\} \tag{3.32}
\]

Erosion of \( I_{M2}(x, y) \) by a structuring element SE \((m, n)\) is denoted by \( I_{M3}(x, y) \)

\[
I_{M3}(x, y) = E(I_{M2}, SE)(x, y) = \min\{I_{M2}(x + m, y + n) - SE(m, n)\} \tag{3.33}
\]

**Fingerprint Orientation Estimation:** Taking the MASK image as input, the orientation or the angle between the x-axis and the major axis of the ellipse bounding the MASK \( I_{M3} \) image that has the same second-moments as the region is calculated. The normalized second central moment of a pixel with unit length is 1/12. Equations (3.34) to (3.36) define the angle with respect to x axis to major axis of the ellipse, y axis to major axis of the ellipse, and between them.

\[
G_{XX} = \sum x^2 N + (1/12) \tag{3.34}
\]
Equations (3.37) to (3.40) describe the algorithm used to find the orientation.

If \( G_{YY} = G_X \)

\[
\text{Num} = G_{YY} - G_{XX} + \sqrt{(G_{YY} - G_{XX})^2 + 4 \times G^2_{XY}}
\]

\[
\text{Den} = 2 \times G_{XY}
\]

Else

\[
\text{Num} = 2 \times G_{XY}
\]

\[
\text{Den} = G_{XX} - G_{YY} + \sqrt{(G_{XX} - G_{YY})^2 + 4 \times G^2_{XY}}
\]

End

If \( \text{Num} == 0 \) \& \( \text{Den} == 0 \)

Orientation = 0

Else

\[
\text{Orientation} = \frac{180}{\pi} \times \tan^{-1}\left(\frac{\text{Num}}{\text{Den}}\right)
\]

End

**Orientation Angle Identification**: The orientation obtained in the Equation (3.40) may be negative or positive. If the orientation angle is negative, Equation (3.41) is used to estimate the angle of rotation. Similarly, if the orientation angle is positive, Equation (3.42) is used to estimate the
angle of rotation. This is done to ensure whether the rotation should be clockwise (negative) or anticlockwise (positive).

If Orientation < 0

\[ A = -(90 + \text{Orientation}) \]  \hspace{1cm} (3.41)

Else

\[ A = (90 - \text{Orientation}) \]  \hspace{1cm} (3.42)

End.

**Vertical Orientation:** Using the calculated angle of rotation (A) the whole range is rotated. Nearest neighbour interpolation is used to set the values of pixels in the rotated image that are outside the rotated image to 0. Obtain the mask for the rotated image by slightly shrinking the mask using morphological erosion with an SE 'disk' of radius 3. Finally, by converting the values of the region outside the mask to max value (255), we will be able to obtain the final rotated image. Figure 3.17 shows the results of the rotation process.

![Fingerprint image rotation process](image)

(a) Input image  \hspace{1cm} (b) Masked image  \hspace{1cm} (c) Mask after filtering \hspace{1cm} (d) Rotated image

**Figure 3.17 Fingerprint image rotation process**

In addition to the internal database, the FVC fingerprint data base is also used to evaluate this work. Table 3.6 presents a few FVC samples and the angle by which they need to be rotated for vertical orientation.
### Table 3.6 FVC samples and their angle of rotation

<table>
<thead>
<tr>
<th>FVC database</th>
<th>Finger ID</th>
<th>Angle of rotation</th>
<th>vertical alignment direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>FVC2004 DB1_A</td>
<td>10_6</td>
<td>-8.37</td>
<td>Clockwise</td>
</tr>
<tr>
<td></td>
<td>26_5</td>
<td>9.29</td>
<td>anticlockwise</td>
</tr>
<tr>
<td></td>
<td>27_6</td>
<td>-10.32</td>
<td>Clockwise</td>
</tr>
<tr>
<td></td>
<td>99_5</td>
<td>-8.69</td>
<td>Clockwise</td>
</tr>
<tr>
<td></td>
<td>102_3</td>
<td>-16.59</td>
<td>Clockwise</td>
</tr>
<tr>
<td>FVC2002 DB1_B</td>
<td>103_3</td>
<td>15.15</td>
<td>anticlockwise</td>
</tr>
<tr>
<td></td>
<td>106_3</td>
<td>-21.96</td>
<td>Clockwise</td>
</tr>
<tr>
<td></td>
<td>110_5</td>
<td>26.51</td>
<td>anticlockwise</td>
</tr>
<tr>
<td></td>
<td>105_7</td>
<td>-10.70</td>
<td>Clockwise</td>
</tr>
<tr>
<td>FVC2000 DB2_B</td>
<td>107_2</td>
<td>1.85</td>
<td>anticlockwise</td>
</tr>
<tr>
<td></td>
<td>108_5</td>
<td>-0.52</td>
<td>Clockwise</td>
</tr>
<tr>
<td></td>
<td>109_6</td>
<td>3.99</td>
<td>anticlockwise</td>
</tr>
</tbody>
</table>

Figure 3.18 shows the results of two FVC database images and two internal database images.

(a) Examples of actual and rotated FVC database images

(b) Examples of actual and rotated internal database images

**Figure 3.18 Actual and vertically oriented fingerprint images**
FVC2004_DB1_7_6 image and its rotated image with the angle of rotation -10.3171 degrees is shown left in Figure 3.18 (a). Similarly, FVC2004_DB1_8_6 image and its rotated image with the angle of rotation +8.7009 degrees are shown right in Figure 3.18 (a). Internal DB image 233_9 (shown left) and internal database143_10 (shown right) with its rotated image are shown in Figure 3.18 (b). The angle of rotation is -11.2726 and -5.0534 degrees, respectively.

**Performance evaluation of rotation algorithm:** The performance of the fingerprint rotation algorithm is evaluated using the line based minutiae extraction method discussed in section 3.4.2. Five samples of FVC2004 DB1_A and five samples of internal database have been selected randomly. Feature extraction is performed in the input image and the minutiae of bifurcation (B) and termination (T) has been noted. Again the minutiae extraction is carried out in the rotated image. Both minutiae obtained for input and rotated image are compared in Table 3.7.

**Table 3.7 Comparison of minutiae between input and rotated image**

<table>
<thead>
<tr>
<th>Fingerprint Image ID</th>
<th>Minutiae</th>
<th>Input Image</th>
<th>Rotated image</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Bifurcation</td>
<td>Termination</td>
</tr>
<tr>
<td>FVC2004 DB1_A_7.6</td>
<td></td>
<td>25</td>
<td>10</td>
</tr>
<tr>
<td>FVC2004 DB1_A_8.5</td>
<td></td>
<td>14</td>
<td>19</td>
</tr>
<tr>
<td>FVC2004 DB1_A_10.6</td>
<td></td>
<td>27</td>
<td>15</td>
</tr>
<tr>
<td>FVC2004 DB1_A_26.5</td>
<td></td>
<td>26</td>
<td>15</td>
</tr>
<tr>
<td>FVC2004 DB1_A_27.6</td>
<td></td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>Internal DB_116_4</td>
<td></td>
<td>26</td>
<td>7</td>
</tr>
<tr>
<td>Internal DB_143_10</td>
<td></td>
<td>14</td>
<td>18</td>
</tr>
<tr>
<td>Internal DB_214_8</td>
<td></td>
<td>12</td>
<td>5</td>
</tr>
<tr>
<td>Internal DB_224_9</td>
<td></td>
<td>14</td>
<td>4</td>
</tr>
<tr>
<td>Internal DB_233_9</td>
<td></td>
<td>18</td>
<td>9</td>
</tr>
</tbody>
</table>
The number of bifurcations and terminations observed in the input image and the vertically oriented images are not differing or differing by one or two. In the process of gender classification, the main requirement is the delta and core points. In this work, fundamental coefficients identified from the image transforms are taken, and it is found that the vertical orientation did not affect the values. Similarly, the DWT and SVD features were also not affected. In addition, the ridge count, ridge width and the fingertip size calculated were not showing difference with the input image and vertically oriented image.

The minutiae of the input image FVC2004_DB1_A_75_5 and its rotated image are shown in Figure 3.19. It could be observed that, the minutiae are present in the same place as located in the input image.

![Minutiae of FVC2004_DB1_A_75_5 and its rotated image](image)

**Figure 3.19  Minutiae of FVC2004_DB1_A_75_5 and its rotated image**

It is evident from figure 3.19, that the minutiae are not affected by the image rotation
3.5.4 Singular Points

Fingerprints are categorized based on their global pattern of ridges and valleys. Basically, the fingerprints are classified into three basic classes as arch, whorl and loop. These types are further categorized as plain arch, tented arch, right loop, left loop, whorl, central pocket, twin loops and accidents. Fingerprint types are shown in Figure 3.20. Fingerprint types (a)-(e) are most common and (f)-(h) are rare occurrences.

![Fingerprint types](image)

(a) Left loop  (b) Right loop  (c) Whorl  (d) Plain arch
(e) Tented arch  (f) Central pocket  (g) Twin loop  (h) Accidental

*Figure 3.20 Fingerprint types*

The distribution of the fingerprint classes illustrated in Figure 3.20 are in nature and not uniform (Wilson 1994). General distribution of the most common fingerprint types are shown in a pie chart in Figure 3.21.
Figure 3.21 Percentage distributions of most common fingerprint types

It is observed that the left loops, right loops and whorl types together making up 93.4% of all fingerprints.

Each type has one or more core and delta points referred to as singular points. The singular point area is defined as a region where the ridge curvature is higher than normal and where the direction of the ridge changes rapidly (Boer et al. 2001, Srinivasan and Murthy 1992). Except the plain arch which does not have singular points, remaining types are identified easily as these points are well defined in literature (Li et al. 2008). These singular points are useful for fingerprint indexing i.e., for classification of fingerprint types (Zhang and Yan 2004) for fingerprint alignment and orientation field modelling (Gu and Zhou 2003, Sherlock and Monro 1993) and for identification/verification.

A core point is the turning point of an innermost ridge. In biometrics and fingerprint scanning, core point refers to the center area of a fingerprint. The following rules of Federal Bureau of Investigation (FBI) govern the selection of core of a loop (Hoover 2006).
• When the innermost sufficient recurve contains no ending ridge (Figure 3.22 (a)) or rod rising as high as the shoulders of the loop, the core is placed on the shoulder of the loop farther from the delta.

• When the innermost sufficient recurve contains an **odd number of rods** (Figure 3.22 (b)) rising as high as the shoulders, the core is placed upon the end of the center rod.

• When the innermost sufficient recurve contains an **even number of rods** (Figure 3.22 (c), the core is placed upon the end of the farther one of the two center rods.

![Figure 3.22 Core point identification](http://www.gutenberg.org/files/19022/19022-h/19022-h.htm)
A delta point is a place where a ridge is bifurcated (or) a delta point is a place where two ridges run side by side and diverge. Figure 3.23 demonstrates these two possible delta points.

![Delta points](image)

(a) Delta point location  
(b) Examples of delta point locations

**Figure 3.23 Delta point identification**

When there is a choice between two or more possible deltas, the following rules govern (Hoover 2006).

- The delta may not be located at a bifurcation which does not open toward the core. In this case (Figure 3.24 (a)), E is not a delta point, but D is a delta point.

- When there is a choice between a bifurcation and another type of delta, the bifurcation is selected. In Figure 3.24 (b), A is not a delta point, but D is a delta point.
When there are two or more possible deltas which conform to the definition, the one nearest the core is chosen, as shown in Figure 3.24(c). It should be noted that X is not a delta point, but D is a delta point.

The delta may not be located in the middle of a ridge running between the type lines toward the core, but at the nearer end only as shown in Figure 3.24 (d). It is evident that A is not a delta point, but D is a delta point.

The multi-scale detection algorithm based on both the continuous orientation field and the modified Poincare Index (Bo et al. 2008) has been
used for the automatic detection of core and delta. However, manual selection of singular points was also carried out for the images deteriorated.

In a loop pattern, the ridges enter from right side (for right loop) and left side (for left loop), re-curve and pass out or tend to pass out the same side they entered. In loop pattern, there is only one delta and core point. In the whorl pattern, the ridges are usually circular and contain two core point and two delta points. In an arch pattern (two types viz., plain arch and tented arch), the ridges enter from one side, make a rise in the center and exit generally on the opposite side. For plain arch, the ridge rises slightly and for the tented arch the ridge projection will be steeper. The plain arch type does not have delta and core points. Thus, for the processing, these points need to be chosen manually. However, the tented arch has a core and a delta point. The central pocket pattern consists of one or more free re-curving ridges. Delta like pattern is observed, one nearest and another furthest to the core point. In general, the delta point furthest to the core point will be considered for further processing. In the twinned loop pattern, the re-curving ridges present two loop formations separate and apart. There are two possible core and delta points in twin loop pattern.

The core and delta points of these generally available six patterns are identified automatically. The selection of core and delta points is elaborated in Table 3.8.
### Table 3.8 Fingerprint patterns and singular points

<table>
<thead>
<tr>
<th>Types</th>
<th>Number of cores</th>
<th>Number of deltas</th>
<th>Choosing core</th>
<th>Delta position with respect to core</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tented arch</td>
<td>1</td>
<td>1</td>
<td>A point at which a ridge has steep rise</td>
<td>Beneath</td>
</tr>
<tr>
<td>Left loop</td>
<td>1</td>
<td>1</td>
<td>Point at which inner most ridge re-curve</td>
<td>Right</td>
</tr>
<tr>
<td>Right loop</td>
<td>1</td>
<td>1</td>
<td>Point at which inner most ridge re-curve</td>
<td>Left</td>
</tr>
<tr>
<td>Whorl</td>
<td>2</td>
<td>2</td>
<td>Top core point of the circular pattern</td>
<td>Left or right delta point which is nearest to the core selected.</td>
</tr>
<tr>
<td>Plain arch</td>
<td>No core</td>
<td>No delta</td>
<td>A point in a ridge which have a high peak in its middle among all ridges</td>
<td>A point in a bottom most ridge which is almost straight</td>
</tr>
<tr>
<td>Central pocket</td>
<td>1</td>
<td>1</td>
<td>The point where re-curve occurs.</td>
<td>Opposite side of the core identified</td>
</tr>
<tr>
<td>Twin loops</td>
<td>2</td>
<td>2</td>
<td>Rising re-curve of the inner ridge point</td>
<td>Delta available beneath the core selected</td>
</tr>
</tbody>
</table>

Seven types of fingerprints for which the core and delta point locations identified are shown in Figure 3.25. The probabilities of the other classes such as accidental, lateral pocket loop and the composite are rare patterns. If gender classification is required for these rare patterns, the core and delta points are located manually and then the spatial parameters are calculated.
3.5.5 Ridge Parameters

The ridge parameters exhibit a number of properties which shall be used for gender determination and age categorization. In general, following fingerprint parameters are used for varieties of applications right from the authentication to gender classification.

- Core to delta Euclidean distance
- Core to Delta angle
- Ridge Orientation
- Ridge Frequency Estimation
- Ridge Count
- Ridge Width
As an initial step of pre-processing, fingerprint region has been extracted. The normalized input image is broken into 16X16 blocks. Standard deviation of each block is calculated. If the standard deviation is above the threshold (0.1), that block contains fingerprint region. As a next step thin image is created. In the thinning process, the morphological thinning operation is performed on the enhanced image till the ridge becomes single pixel wide. The single pixel image is then negated to represent by a white pixel.

A fingerprint image 180_F_1 has been considered to test the algorithm. The core and delta point have been identified manually. The selected core and delta points are marked and these points are connected by a green line as shown in Figure 3.26.

![Figure 3.26 Fingerprint sample with manual selection of core and delta point](image)

With respect to these singular points all the spatial parameters and the fingertip size of the fingerprint have been determined by the following algorithm.

1. Core to delta Euclidean distance is calculated from the coordinates of the core and delta points
The Euclidean distance is calculated by Equation (3.43)

\[
\text{Euclidean distance} = \sqrt{(C_x - D_x)^2 + (C_y - D_y)^2}
\]  \hspace{1cm} (3.43)

where, \(C_x\) and \(C_y\) are the coordinates of the core pixel and \(D_x\) and \(D_y\) are the coordinates of the delta pixel. Let the values identified be \(C_x=118\), \(C_y=201\), \(D_x=194\) and \(D_y=250\) for a fingerprint image. From Equation (3.43), the distance is calculated as 90.4268. The distance measured in millimetre has been determined as given below.

\[
\text{Euclidean distance} = 0.0833 \times 90.4268 = 7.55 \text{mm}
\]

All internal database images are with the size of 260 pixels wide and 300 pixels height. The value 0.0833 is calculated by dividing 25 mm by 300 which is the height of the image. The 25 mm is proportional to the resolutions per inch of the scanner used.

2. Core to Delta angle estimated using Two Point Slope Formula

The angle between core and delta is calculated by Equations (3.44) and (3.45).

\[
\text{Slope} = \frac{(D_y-C_y)}{(D_x-C_x)}
\]  \hspace{1cm} (3.44)

\[
\angle CTD = \tan^{-1} (\text{slope})
\]  \hspace{1cm} (3.45)

For the computed coordinates, the angle between core and delta is 32.8114 degrees.
3. Ridge orientation
   a. Using gradient of Gaussian filter, the gradient of fingerprint image in the X and Y direction is generated.
   b. The local ridge orientation at each point is estimated by finding the principal axis of variation in the image gradients.

4. Ridge frequency estimation
   a. Using the estimated ridge orientation, mean orientation within the block has been calculated.
   b. By rotating the image block, the ridges become vertical.
   c. The image is cropped, so that the rotated image does not contain any invalid regions
   d. Columns are summed down to get a projection of the grey values down the ridges.
   e. Peaks in projected grey values are determined by performing grayscale dilation and then finding where the dilation equals the original values.
   f. The spatial frequency of the ridges is determined by dividing the distance between the first and last peaks by the (No of peaks - 1).

The values of the above parameters are determined and listed in Table 3.9. The images are randomly selected from the internal data base of age group of 19-25. As the delta point selection depends on the pattern of the fingerprint, the angle from core to delta and the ridge orientations of the images may be negative or positive. If the delta point is located left with respect to core point, the core to delta angle will be negative. However, the orientation is positive or negative depends on the flow of ridges which varies from pattern to pattern.
Popular and frequently occurring fingerprint patterns and the appropriate identification of core and delta points are elaborated in section 3.4.4 (vide Table 3.8 and Figure 3.25).

Table 3.9 Spatial parameter values of selected fingerprints

<table>
<thead>
<tr>
<th>Fingerprint Image Details</th>
<th>Core to Delta Euclidean distance (mm)</th>
<th>Core to delta angle (degrees)</th>
<th>Ridge orientation core to delta</th>
<th>Overall</th>
<th>ridge spatial frequency (core to delta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>180_F_1</td>
<td>7.5581</td>
<td>32.8114</td>
<td>17.155</td>
<td>3.2375</td>
<td>0.011499</td>
</tr>
<tr>
<td>289_F_2</td>
<td>13.6974</td>
<td>86.1627</td>
<td>-55.9107</td>
<td>-12.9713</td>
<td>0.0092343</td>
</tr>
<tr>
<td>357_F_3</td>
<td>7.6071</td>
<td>42.7803</td>
<td>12.4455</td>
<td>5.6034</td>
<td>0.011584</td>
</tr>
<tr>
<td>407_F_4</td>
<td>10.937</td>
<td>35.3854</td>
<td>17.4171</td>
<td>-24.8627</td>
<td>0.0097645</td>
</tr>
<tr>
<td>471_F_5</td>
<td>9.7082</td>
<td>11.8887</td>
<td>25.217</td>
<td>4.2801</td>
<td>0.01167</td>
</tr>
<tr>
<td>590_F_6</td>
<td>13.8067</td>
<td>84.8056</td>
<td>-44.0624</td>
<td>-13.7716</td>
<td>0.0078952</td>
</tr>
<tr>
<td>600_F_7</td>
<td>10.2649</td>
<td>82.0672</td>
<td>-45.7449</td>
<td>17.7953</td>
<td>0.0087251</td>
</tr>
<tr>
<td>685_F_8</td>
<td>9.6455</td>
<td>-50.2578</td>
<td>-15.7961</td>
<td>-3.2646</td>
<td>0.010944</td>
</tr>
<tr>
<td>700_F_9</td>
<td>11.9655</td>
<td>-43.5891</td>
<td>-26.9385</td>
<td>-14.9622</td>
<td>0.011001</td>
</tr>
<tr>
<td>883_F_10</td>
<td>6.5933</td>
<td>-73.8557</td>
<td>-15.7784</td>
<td>-7.7343</td>
<td>0.01177</td>
</tr>
<tr>
<td>170_M_1</td>
<td>13.1286</td>
<td>28.8427</td>
<td>10.9671</td>
<td>0.80211</td>
<td>0.011844</td>
</tr>
<tr>
<td>205_M_2</td>
<td>7.6167</td>
<td>36.9953</td>
<td>-0.15053</td>
<td>50.508</td>
<td>0.011105</td>
</tr>
<tr>
<td>250_M_3</td>
<td>12.3435</td>
<td>55.4516</td>
<td>7.6327</td>
<td>-14.7424</td>
<td>0.009797</td>
</tr>
<tr>
<td>301_M_4</td>
<td>10.7458</td>
<td>66.218</td>
<td>34.8739</td>
<td>-3.2646</td>
<td>0.0096423</td>
</tr>
<tr>
<td>409_M_5</td>
<td>9.2665</td>
<td>41.7191</td>
<td>28.8053</td>
<td>-0.8228</td>
<td>0.0097547</td>
</tr>
<tr>
<td>545_M_6</td>
<td>10.152</td>
<td>-80.0738</td>
<td>-36.9346</td>
<td>-8.1269</td>
<td>0.0078704</td>
</tr>
<tr>
<td>580_M_7</td>
<td>12.1335</td>
<td>-37.1847</td>
<td>-5.1389</td>
<td>2.5821</td>
<td>0.0107</td>
</tr>
<tr>
<td>660_M_8</td>
<td>12.2046</td>
<td>-43.0632</td>
<td>-16.7488</td>
<td>4.9221</td>
<td>0.010072</td>
</tr>
<tr>
<td>701_M_9</td>
<td>11.2577</td>
<td>55.2512</td>
<td>13.9642</td>
<td>-21.2772</td>
<td>0.011717</td>
</tr>
<tr>
<td>913_M_10</td>
<td>12.0231</td>
<td>-46.1233</td>
<td>-14.3849</td>
<td>-2.091</td>
<td>0.0099934</td>
</tr>
</tbody>
</table>

5. Ridge Count

The ridge count is defined as the number of ridges intervening between the delta and core (Hoover 2006). In normal practice, the technical employees count each ridge which crosses an imaginary line drawn from the delta to core. In this proposed method, instead of considering counting only between the core and delta, an effort is taken to count the ridges of the entire
fingertip. To enable this, an imaginary line is drawn between core to delta and an imaginary line is drawn at 135 (referred as principal diagonal) and 45 degree (referred as other diagonal) from the core point upwards. Figure 3.27 illustrates a fingerprint with the imaginary lines.

![Image of fingerprint with imaginary lines]

**Figure 3.27 A fingerprint with imaginary lines to find ridge count**

In the thinned image, along the required line of interest, the number of white pixels gives the ridge count. Ridge count along core to delta, along principal diagonal and along other diagonal are measured. Let $a$ be the ridge count between core to delta, $b$ be the ridge count in the principal diagonal and $c$ be the ridge count in other diagonal. The total ridge count is calculated by Equation (3.46).

$$RC = a + \frac{1}{2}(b + c)$$

An excerpt of the ridge counts calculated for male and female is presented in Table 3.10. All the fingerprints are taken from the age group of 13-18. The fingerprint image detail is given as fingerprint id; gender and the
finger number (refer Figure 3.1). For example, 272_F_1 indicates the fingerprint id in the internal database as 272, the gender as female and the ‘1’ represents the little finger. Average ridge count is calculated as per Equation (3.46).

**Table 3.10 Average ridge count of selected fingerprints**

<table>
<thead>
<tr>
<th>Fingerprint</th>
<th>Image Details</th>
<th>Number of ridges</th>
<th>Average ridge count (RC = a+(b+c)/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Core to Delta (a)</td>
<td>Principal diagonal (b)</td>
</tr>
<tr>
<td>272_F_1</td>
<td>10</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>129_F_2</td>
<td>10</td>
<td>18</td>
<td>20</td>
</tr>
<tr>
<td>482_F_3</td>
<td>6</td>
<td>17</td>
<td>11</td>
</tr>
<tr>
<td>564_F_4</td>
<td>15</td>
<td>11</td>
<td>18</td>
</tr>
<tr>
<td>848_F_5</td>
<td>11</td>
<td>16</td>
<td>19</td>
</tr>
<tr>
<td>477_F_6</td>
<td>12</td>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td>563_F_7</td>
<td>12</td>
<td>12</td>
<td>16</td>
</tr>
<tr>
<td>873_F_8</td>
<td>12</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>869_F_9</td>
<td>13</td>
<td>22</td>
<td>25</td>
</tr>
<tr>
<td>868_F_10</td>
<td>10</td>
<td>18</td>
<td>26</td>
</tr>
<tr>
<td>137_M_1</td>
<td>12</td>
<td>15</td>
<td>21</td>
</tr>
<tr>
<td>166_M_2</td>
<td>10</td>
<td>23</td>
<td>14</td>
</tr>
<tr>
<td>261_M_3</td>
<td>15</td>
<td>23</td>
<td>19</td>
</tr>
<tr>
<td>342_M_4</td>
<td>17</td>
<td>22</td>
<td>20</td>
</tr>
<tr>
<td>503_M_5</td>
<td>18</td>
<td>14</td>
<td>18</td>
</tr>
<tr>
<td>556_M_6</td>
<td>22</td>
<td>20</td>
<td>18</td>
</tr>
<tr>
<td>843_M_7</td>
<td>16</td>
<td>13</td>
<td>19</td>
</tr>
<tr>
<td>895_M_8</td>
<td>12</td>
<td>21</td>
<td>19</td>
</tr>
<tr>
<td>903_M_9</td>
<td>15</td>
<td>29</td>
<td>11</td>
</tr>
<tr>
<td>912_M_10</td>
<td>17</td>
<td>23</td>
<td>17</td>
</tr>
</tbody>
</table>

6. Ridge width

Cummins (1941) presented extensive reports on human epidermal ridge breadth and demonstrated that the ridge breadth varies considerably between sexes. The growth in ridge breadth was reported by Hecht in 1924 (Cummins 1941) as the average ridge widths are 0.30 to 0.35 mm (up to 10
years of male or female), 0.40 to 0.50 mm (for adult women) and 0.50 mm and above (for adult men). The thickness of the ridge was computed by Verma and Agarwal (2008) by counting the number of pixels between consecutive maxima points of the projected image.

In the previous work by other researchers, the fingerprint image was divided into many overlapping blocks. Within each block, ridge orientation is determined and then the ridge width is calculated (Badawi et al. 2006). However, the ridge orientation varies according to different fingerprint patterns. Considering this varying orientation, a new approach is used to find the ridge width. In this approach, in addition to the colours used in ridge count process, additional two lines are drawn with respect to core and delta points as shown in Figure 3.28.

![Figure 3.28 A fingerprint with imaginary lines to find ridge width](image)

In the thinned image, along the required line of interest, the number of black pixels divided by the ridge count gives the average ridge width. Let A, B, C, D and E be the average ridge width along core to delta points, along principal diagonal, along other diagonal, along the horizontal line with respect to core and along the horizontal line with respect to delta. RW is calculated by using Equation (3.47). The data obtained are given in micrometre.
The average ridge widths are presented for male and female fingerprints in Table 3.11. All the fingerprints are taken from the age group of 19-25.

Table 3.11 Average ridge width of selected fingerprints

<table>
<thead>
<tr>
<th>Fingerprint Image Details</th>
<th>Core to Delta (A)</th>
<th>Principal diagonal (B)</th>
<th>Other diagonal (C)</th>
<th>Along horizontal with respect to core (D)</th>
<th>Along horizontal with respect to delta (E)</th>
<th>Average ridge width (RW = ( \frac{A+B+C+D+E}{5} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>180_F_1</td>
<td>325</td>
<td>560</td>
<td>350</td>
<td>340</td>
<td>550</td>
<td>425</td>
</tr>
<tr>
<td>289_F_2</td>
<td>150</td>
<td>340</td>
<td>465</td>
<td>375</td>
<td>560</td>
<td>378</td>
</tr>
<tr>
<td>357_F_3</td>
<td>245</td>
<td>330</td>
<td>360</td>
<td>270</td>
<td>360</td>
<td>313</td>
</tr>
<tr>
<td>407_F_4</td>
<td>325</td>
<td>425</td>
<td>435</td>
<td>455</td>
<td>485</td>
<td>425</td>
</tr>
<tr>
<td>471_F_5</td>
<td>395</td>
<td>370</td>
<td>360</td>
<td>395</td>
<td>485</td>
<td>401</td>
</tr>
<tr>
<td>590_F_6</td>
<td>210</td>
<td>590</td>
<td>495</td>
<td>610</td>
<td>485</td>
<td>478</td>
</tr>
<tr>
<td>600_F_7</td>
<td>100</td>
<td>510</td>
<td>500</td>
<td>570</td>
<td>455</td>
<td>427</td>
</tr>
<tr>
<td>685_F_8</td>
<td>250</td>
<td>415</td>
<td>350</td>
<td>305</td>
<td>425</td>
<td>349</td>
</tr>
<tr>
<td>700_F_9</td>
<td>275</td>
<td>310</td>
<td>360</td>
<td>340</td>
<td>455</td>
<td>348</td>
</tr>
<tr>
<td>883_F_10</td>
<td>235</td>
<td>460</td>
<td>305</td>
<td>310</td>
<td>395</td>
<td>341</td>
</tr>
<tr>
<td>170_M_1</td>
<td>300</td>
<td>555</td>
<td>395</td>
<td>305</td>
<td>360</td>
<td>383</td>
</tr>
<tr>
<td>205_M_2</td>
<td>325</td>
<td>440</td>
<td>440</td>
<td>325</td>
<td>375</td>
<td>381</td>
</tr>
<tr>
<td>250_M_3</td>
<td>300</td>
<td>380</td>
<td>480</td>
<td>360</td>
<td>420</td>
<td>388</td>
</tr>
<tr>
<td>301_M_4</td>
<td>215</td>
<td>470</td>
<td>490</td>
<td>390</td>
<td>675</td>
<td>448</td>
</tr>
<tr>
<td>409_M_5</td>
<td>330</td>
<td>415</td>
<td>735</td>
<td>490</td>
<td>710</td>
<td>536</td>
</tr>
<tr>
<td>545_M_6</td>
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<td>390</td>
<td>495</td>
<td>600</td>
<td>2065</td>
<td>727</td>
</tr>
<tr>
<td>580_M_7</td>
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<td>385</td>
<td>465</td>
<td>355</td>
<td>445</td>
<td>401</td>
</tr>
<tr>
<td>660_M_8</td>
<td>330</td>
<td>575</td>
<td>415</td>
<td>390</td>
<td>565</td>
<td>455</td>
</tr>
<tr>
<td>701_M_9</td>
<td>260</td>
<td>410</td>
<td>305</td>
<td>295</td>
<td>465</td>
<td>347</td>
</tr>
<tr>
<td>913_M_10</td>
<td>305</td>
<td>335</td>
<td>610</td>
<td>340</td>
<td>480</td>
<td>414</td>
</tr>
</tbody>
</table>

7. Fingertip size of the fingerprint

The fingertip size of the selected images of male and female is presented in Table 3.12. All the fingerprint images are in the age group of above 25 taken from the internal database.
Table 3.12 Fingertip size of selected fingerprints

<table>
<thead>
<tr>
<th>Fingerprint image details</th>
<th>Fingertip size (mm²)</th>
<th>Fingerprint image details</th>
<th>Fingertip size (mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>107_1</td>
<td>403</td>
<td>101_1</td>
<td>437</td>
</tr>
<tr>
<td>206_2</td>
<td>430</td>
<td>208_2</td>
<td>505</td>
</tr>
<tr>
<td>438_3</td>
<td>384</td>
<td>345_3</td>
<td>507</td>
</tr>
<tr>
<td>426_4</td>
<td>389</td>
<td>411_4</td>
<td>491</td>
</tr>
<tr>
<td>438_5</td>
<td>478</td>
<td>501_5</td>
<td>536</td>
</tr>
<tr>
<td>437_6</td>
<td>487</td>
<td>724_6</td>
<td>523</td>
</tr>
<tr>
<td>517_7</td>
<td>413</td>
<td>742_7</td>
<td>475</td>
</tr>
<tr>
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<td>448</td>
<td>750_8</td>
<td>503</td>
</tr>
<tr>
<td>760_9</td>
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<td>478</td>
</tr>
<tr>
<td>772_10</td>
<td>338</td>
<td>762_10</td>
<td>402</td>
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</tbody>
</table>

3.6 SUMMARY

This chapter has described fingerprint features and various parameters required to classify gender. Three methods of feature extraction have been discussed. In frequency domain analysis, FFT, DCT and PSD were used to find the fundamental coefficients. It has been identified that the coefficients are greater for female with FFT and PSD in this study. As far as the DCT coefficients are concerned, the male values are greater than female. All the coefficients were determined from the cropped fingerprint images. Because of the unique and good information packing characteristics, DWT and SVD were used to obtain the frequency and spatial features respectively. By combining both features by concatenation, feature vector of size 1x279 was obtained. Various spatial parameters of fingerprint were determined.
These parameters were computed with respect to core and delta points which together are referred to as singular points. Automatic identification of singular points for various fingerprint patterns was discussed. In addition to ridge count and ridge width, the fingertip size of the fingerprint was used as spatial parameters for the proposed work. As the fingerprint image may vary based on the character and situation, the fingerprint may be of different pressure and different orientation. Solution was provided for obtaining vertically oriented fingerprint and the algorithm was evaluated using line-based minutiae extraction algorithm developed in this research work.