ABSTRACT

In many cases, information about the behavior of a dynamical system is uncertain. In order to obtain a more realistic model, all uncertainties should be taken into account. Also, in several cases the uncertainties are not of statistical type, for having some linguistic information and when a measurement cannot be repeated are such cases. Fuzzy differential equations (FDEs) are a natural way to model dynamical systems under possibilistic uncertainty. Also, in modeling real world phenomena, fuzzy initial value problems (FIVPs) appear naturally. There will be times when solving, the exact solution may be unavailable or the means to solve it will be unavailable and therefore it is indispensable to make studies on numerical solutions of FDEs. The still-standing problem in the theory of FDEs is to find implementable numerical methods. Much more effort has been made in this direction as well.

Initially the numerical solutions of FIVPs were studied under Hukuhara or Seikkala differentiability concept. But solutions under this model have disadvantage of increasing length as time increases. To overcome this difficulty, the concept of generalized differentiability was introduced. This research work attempts to show that a good approximation to the numerical solutions of FIVPs can be obtained by second, third and fourth order Runge-Kutta methods with higher order derivative approximations. Further, the results show that smaller h, step size, values results in reduction in error. Further, the experimental results show that the approximations by the
proposed Runge-Kutta methods are better than that of respective classical Runge-Kutta methods.

A second order Runge-Kutta method with harmonic mean of \( k, s \) is used to find the numerical solution of FIVPs. The efficiency of the method is illustrated by solving linear and non-linear FIVPs. The proposed formula gives better approximation than the approximation by standard Euler method. And the method is close to the existing classical Runge-Kutta method of order two. Further, Rational Extrapolation method is used to improve the accuracy of numerical solutions of FDEs where the numerical solutions are obtained first through standard Euler method and then by Modified midpoint rule. The experimental result shows the efficiency of the Rational Extrapolation method over the Polynomial Extrapolation method. Also, Hybrid predictor-corrector methods are used to find the numerical solutions of FIVPs. In both predictor and corrector a non-step point is used for approximating the results. An implicit fourth order two-step hybrid predictor-corrector, and an explicit fifth order predictor corrector formulae are used to obtain the numerical solution of FIVPs.

In real life, information available is sometimes vague, inexact or insufficient and so the parameters of the problem are usually defined by the decision maker in an uncertain way and it is desirable to consider the knowledge of experts about the parameters as fuzzy data. There are situations where due to insufficiency in the information available, the evaluation of membership values is not possible upto our satisfaction. At the same time, the
evaluation of non-membership is also not always possible and consequently there remains a part in deterministic on which hesitation survives. Intuitionistic fuzzy set (IFS) theory is most suitable to handle such situations. Intuitionistic fuzzy differential equations (IFDEs) with initial conditions are taken for study and the concept of generalized differentiability of FDEs is extended to IFDEs. Standard Euler’s method is proposed for finding the numerical solutions of IFDEs. And the method is illustrated with linear and non-linear IFDs.