Chapter 2

Radiation: ‘What’s and ‘How’s

In the previous chapter we gave a general introduction on the physical considerations we must have before studying the QGP Physics. Once the QGP fluid is created, we will try to know the properties of it; and we have discussed that we need internal probes for looking into such a short lived (life-time ∼10 fm) medium. The internal probes being used in the present discussion are the high energy particles produced very early, just after the collision takes place. They lose energy inside the medium and for a very high energy particle the radiative energy loss dominates. Radiative energy loss of high energy internal probes being the subject matter of the present dissertation, in this chapter we will emphasize on the physical considerations which ensure the radiation. Later, the phenomenon of radiation will be discussed from a very general perspective, mostly adapted from the Classical Electrodynamics\(^1\) textbooks, to understand the basic assumptions and analogies one use in calculating the radiation loss.

\(^{1}\)Why do we expect that this adaptation from electrodynamics will work in chromodynamics? We will discuss it in the long run.
2.1 Radiation: What do we mean?

When can we tell that radiation is given off by a particle? First of all, radiation means photons/gluons oozed out from a parent parton. That means, the emitted particle is ‘well separated’ from its parent particle. As long as the radiation is in coherence with the parent particle, it is not radiated. The time needed for the radiation to be well-separated (by, say, one Compton length) from its parent parton is called the formation time. Similarly, for pair-production the required distance of separation between the pairs is two Compton wavelength for the pair to be ‘seen’ as a collection of two distinct particles. Quantitative estimate [1] of the formation time, $l_0$ shows (in natural unit) that

\[ l_0 = \frac{2E(E - k)}{m^2k} \]  

where $E$ and $k$ are the energies of the radiating and the radiated particle respectively and $m$ is the mass of the emitting particle. Formation length actually comes due to uncertainty in momentum transfer [2] which blurs the information at which point of the trajectory of the radiating particle the radiation has taken place. Hence, formation length is also an extended region in space anywhere within which the radiation might have taken place. Apparently, there are different ways one can explain the formation length as far as different physical contexts are concerned [1].

The concept of formation length plays a very important role in case of determining the radiation distribution off a particle undergoing multiple scattering inside a medium. Fig. 2.1 may help
understand the effect it may have on the spectrum. Formation length is the maximum distance within which two radiations, if emitted, will not be resolved by the detector. So, according to Fig. 2.1, $ab$ is the formation length. Now, emissions are results of scattering events (as free particles cannot radiate). As a result, the above scenario translates into the fact that, two scatterings, if take place within the formation time, can reduce the counting of number of radiation quanta emitted. This phenomenon of suppression is, clearly, an interplay of two time-scales (or equivalently, lengths), the formation time ($\tau_f$) and the mean free time ($\tau_m$) and is the famous Landau-Pomeranchuk-Migdal suppression\[3\].

2.2 Radiation: What do we mean and How?

Radiation means dissipation of power even at infinite distance. It is calculated by integrating the Poynting’s vector $\vec{S}$ over a large surface. We know,

$$\vec{S} \sim \vec{E} \times \vec{B} \quad (2.2)$$

where $\vec{E}(\vec{B})$ is the electric (magnetic) field. The power radiated (over area $a$) is given by:

$$\mathcal{P}(r) = \oint \vec{S}.d\vec{a} \quad (2.3)$$

If $\lim_{r\to\infty}\mathcal{P}(r) \neq 0$, then we get radiation. This is the way we calculate the radiated power of an electric dipole or a point charge [4].

But we can examine the idea of radiation in a more intuitive picture with the help of Ref. [5]. We assume a particle travelling right and bouncing off a wall at the point ‘e’ (see Fig. 2.2). Its present position is ‘g’; and ‘f’ would be its position at present had there been no wall. Since, the message that the particle has bounced off travels with a finite speed $c$ (the velocity of light in
vacuum), the lines of force within a circle of radius $cT_0$, where $T_0$ is the time difference between the points $e$ and $g$, will reorient themselves according to the new position of the particle (Fig. 2.2). The field lines outside radius $cT_0$ will point towards the ‘would-have-been’ present position of the particle. Now, if we consider the surface encompassed by the lines $abceda$, we see that there is an imbalance in the density \(^2\) of lines of forces (LOFs) penetrating the surface $ad$ and surface $bc$. The density ratio corresponding to the surfaces is 2:3. According to the Gauss’s law, which tells that the net number of lines of forces penetrating a surface must be zero, we must have transverse field entering across $ad$ or $bc$. Now, if the acceleration (bounce) of the particle occurs for a time $t_0 << T_0$, we have a ring of width $ct_0$ (see Fig. 2.3) which connects the field lines oriented according to the present position of the particle and those oriented according to the position ‘$f$’ of the particle.

Now, we want to find out the field inside the ring (the shaded region in Fig. 2.3). The field has two components, the transverse component, $E_t$, and the radial component $E_r$. From Fig. 2.3,\(^2\)

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\(^2\)As there are infinite number of LOFs passing through any surface, we can talk of density and not the numbers of them.
\[
\frac{E_t}{E_r} = \frac{v_0}{c} \frac{T_0 \sin \theta}{c t_o} = \frac{f' T_0 \sin \theta}{c}
\] (2.4)

where \( f' = v_0/t_0 \) is the acceleration. We can find out the radial field from the Gauss’s law considering a Gaussian pillbox across the inner radius of the ring. Since the sides of the pillbox are of vanishing width, the radial component of field is almost same on the each side of the shell’s inner surface. If we remember \( E_r \sim 1/R^2 \), the transverse field is given by,

\[
E_t \propto f' \sin \theta R
\] (2.5)

and since \( E_t \sim 1/R \) (hence electric energy density \( \sim 1/R^2 \)), the integral over infinitely large surface will give constant value of power dissipation. Hence the transverse field will be responsible for radiation. We notice from Eq. 2.5 that \( E_t \) is dependent on \( \theta \). It is maximum at \( \theta = 90^0 \) and minimum at \( \theta = 0^0, 180^0 \). Hence density of ‘kinks’ will be greater along the perpendicular direction of motion, and it decreases as \( \theta \) approaches 0 or 180. Also, the transverse (or radiation) field depends on the magnitude of acceleration. Greater the acceleration (or deceleration, because the magnitude matters, anyway) greater the radiation.

### 2.3 Calculating radiation loss: Count or Find Poynting’s Vector

So, we learn that an accelerated charge radiates. The radiated power is calculated by integrating the Poynting’s vector over an infinitely large surface. But, in section 1.5 we demand that we have ‘counted’ the ‘number’ of emitted quanta given off by an accelerated charge. Are the two approaches equivalent? In next few paragraphs we will endeavour to bridge the gap between
Figure 2.4: Particle of charge $q$ moves with constant velocity $v$ and passes P at a distance $b$ them. For that, we need to study fields of a uniformly moving charge and the fate of it when acceleration occurs.

The fields of a uniformly moving charge are given in [6]. Our aim is to extract some ideas which will help us understand how we can find radiation loss by ‘counting’ the number of emitted quanta. If a particle of charge $q$ is moving along X-axis with velocity $v$ and passes an observation point P on Y-axis with the closest distance of approach $b$ (see Fig. 2.4), the fields in the observer’s frame (i.e. laboratory frame) are given by [6],

$$
E_x = -\frac{q\gamma vt}{(b^2 + \gamma^2 v^2 t^2)^{3/2}} \\
E_y = \frac{\gamma qb}{(b^2 + \gamma^2 v^2 t^2)^{3/2}} \\
B_z = vE_2
$$

(2.6)

where $\gamma = (1 - v^2)^{-1/2}$, $vt$ is the X distance at which the charge is (as observed by P) after a time $t$. It will be interesting if we study the variation of the fields $E_x$ and $E_y$ with $vt$. We see
\[ E_y = \frac{\gamma q b^2}{b^2} \left( 1 + \frac{\gamma^2 v^2 t^2}{b^2} \right)^{-3/2} \]  

Figure 2.5: Variation of \( E_y \) with \( vt \)

We observe that \( E_y \) is maximum at \( vt = 0 \) and decays along the \( \pm vt \) axes. The maximum value of \( E_y \) is \( E_{y,\text{max}} = \gamma q / b^2 \) and the time duration within which the field assumes an appreciable value is (see Fig. 2.5).

\[ \Delta t \sim b (\gamma v)^{-1} \]  

Similarly, the longitudinal field rapidly varies from \(+ve\) to \(-ve\) values. So the observer observes a pulse of plane polarized radiation. This observation leads us to picturize that a rapidly moving charge, when Lorentz boosted, acquires virtual quanta (up to frequency \( \omega_{\text{max}} \sim 1/\Delta t \) [7]), called Weizsäcker-Williams (WW) quanta [6]. A violent collision of particles results in acceleration imparted to the moving charges. While the fast Fourier components of the field (or equivalently, WW quanta), whose transverse momenta \( k \geq a_0 \), where \( a_0 \) is the inverse acceleration time, can manage to follow the charge, the softer part is ‘left behind’ [8]. The WW quanta which are now detached from the charge are regenerated along a new direction. So, there is radiation — the
radiation off accelerated charge particles in the form of real particles. So it makes no harm if we start counting the number of such quanta emitted (which can be done by the help of quantum field theory techniques) in stead of finding out the Poynting’s vector and integrating it over a large surface. But, we must remember one thing, we use the word ‘count’ in a sense that we want to find out the number of quanta emitted within certain range of variables. It should better be called the radiation distribution which means, probability of finding a particle with certain values of some physical quantities (like momentum or the angle of emission) is always zero. So what we will ‘count’ is the number of particles emitted within a range of momentum and angle the emission makes with the parent parton.

Before the curtain down of this chapter, let us be familiar with the jargons which are popularly used in stead of the terms used in the discussion so far. Hence, the high-energy (i.e at an energy where the mass difference between heavy and light particles becomes irrelevant), collimated beam of internal probes along with the WW gluons (a jargon which we have already discussed) is called a jet shower (loosely jet). The probe particle around which hovers the WW cloud is called the leading particle\(^3\); and the energy loss phenomenon of jets is called jet quenching.

\(^3\)with which the medium interacts. The WW gluons become noticeable only after they metamorphose as radiations.
Bibliography


