CHAPTER 3

EXPERIMENTAL INVESTIGATION

3.1 INTRODUCTION

Experimental investigations were undertaken to investigate the wear performance and machinability of Al 2219-SiC\textsubscript{p} composite. The experimental set-up and experimental procedures used for characterization and machinability of the composite in this study are presented in this Chapter. An overview of the set-up includes a brief description of preparation of the composite, characterization for physical and mechanical properties, studies on wear behavior and machinability of the composite. The methodology followed for the present study is highlighted in the final section of this chapter.

The experimental investigation has been carried out in three stages. The first stage involves the fabrication of the composite and testing the composite for physical properties such as mass density and porosity, and mechanical properties such as hardness and compressive strength.

The second stage involves the study of wear performance and machinability of the composite. Wear performance of the composite has been studied in terms of wear, coefficient of friction, and temperature of the composite. Machinability of the composite relates to performing the drilling operation on the composite. The performance indicators considered are thrust force, torque, and surface roughness.
The third stage involves the optimization of multiple performance characteristics using Taguchi based Grey relational analysis method. A mathematical modeling is also developed using Response Surface Methodology (RSM) technique.

3.2 FABRICATION OF THE COMPOSITE

3.2.1 Workpiece Materials

Aluminium alloy Al 2219 powder of average particle size 44 µm is used as matrix material for the composite. Silicon carbide particulates of average particle size 23, 37 and 67 µm are used as reinforcements for the composite. The chemical composition of the matrix aluminium alloy and the properties of the matrix and reinforcement material are listed in Tables 3.1 to 3.3 respectively.

Table 3.1 Composition of Al2219

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Element</th>
<th>Composition by weight (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Silicon</td>
<td>0.20</td>
</tr>
<tr>
<td>2</td>
<td>Copper</td>
<td>6.00</td>
</tr>
<tr>
<td>3</td>
<td>Manganese</td>
<td>0.30</td>
</tr>
<tr>
<td>4</td>
<td>Zinc</td>
<td>0.10</td>
</tr>
<tr>
<td>5</td>
<td>Titanium</td>
<td>0.10</td>
</tr>
<tr>
<td>6</td>
<td>Magnesium</td>
<td>0.02</td>
</tr>
<tr>
<td>7</td>
<td>Vanadium</td>
<td>0.05</td>
</tr>
<tr>
<td>8</td>
<td>Zirconium</td>
<td>0.10</td>
</tr>
<tr>
<td>9</td>
<td>Aluminium</td>
<td>Balance</td>
</tr>
</tbody>
</table>
### Table 3.2 Properties of Al2219

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Properties</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Hardness, Brinell</td>
<td>115</td>
<td>500 kg load with 10 mm ball</td>
</tr>
<tr>
<td>2</td>
<td>Hardness, Vickers</td>
<td>130</td>
<td>Converted from Brinell Hardness Value</td>
</tr>
<tr>
<td>3</td>
<td>Ultimate Tensile Strength</td>
<td>414 MPa</td>
<td>AA; Typical</td>
</tr>
<tr>
<td>4</td>
<td>Tensile Yield Strength</td>
<td>290 MPa</td>
<td>AA; Typical</td>
</tr>
<tr>
<td>5</td>
<td>Elongation at Break</td>
<td>10%</td>
<td>AA; Typical; 1/16 in. (1.6 mm) Thickness</td>
</tr>
<tr>
<td>6</td>
<td>Modulus of Elasticity</td>
<td>73.1GPa</td>
<td>AA; Typical; Average of tension and compression.</td>
</tr>
<tr>
<td>7</td>
<td>Poisson's Ratio</td>
<td>0.33</td>
<td>Estimated from trends in similar Al alloys.</td>
</tr>
<tr>
<td>8</td>
<td>Fatigue Strength</td>
<td>103 MPa</td>
<td>AA; 500,000,000 cycles completely reversed stress</td>
</tr>
<tr>
<td>9</td>
<td>Shear Modulus</td>
<td>27GPa</td>
<td>Estimated from similar Al alloys.</td>
</tr>
<tr>
<td>10</td>
<td>Aging Temperature</td>
<td>163 – 191°C</td>
<td>From 10-36 hours at temperature</td>
</tr>
<tr>
<td>11</td>
<td>Density</td>
<td>2840 kg/m³</td>
<td>AA; Typical</td>
</tr>
</tbody>
</table>

### Table 3.3 Properties of SiC

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Mechanical</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Density</td>
<td>3.21 gm/cc</td>
</tr>
<tr>
<td>2</td>
<td>Porosity</td>
<td>0 %</td>
</tr>
<tr>
<td>3</td>
<td>Color</td>
<td>black</td>
</tr>
<tr>
<td>4</td>
<td>Flexural Strength</td>
<td>550 MPa</td>
</tr>
<tr>
<td>5</td>
<td>Elastic Modulus</td>
<td>410 GPa</td>
</tr>
<tr>
<td>6</td>
<td>Poisson’s Ratio</td>
<td>0.14</td>
</tr>
<tr>
<td>7</td>
<td>Compressive Strength</td>
<td>3900 MPa</td>
</tr>
<tr>
<td>8</td>
<td>Hardness</td>
<td>2800 kg/mm²</td>
</tr>
<tr>
<td>9</td>
<td>Maximum Use Temperature</td>
<td>1650 °C</td>
</tr>
</tbody>
</table>
3.2.2 Blending and Consolidation of Powders

The matrix and reinforcement powders are blended together through mechanical alloying process using planetary type ball mill apparatus. The photograph of the apparatus is shown in Figure 3.1. The mechanical alloying process is performed in a ball mill, making possible the introduction of hard dispersion particles into a relatively soft metal matrix.

The composite powders, so produced, are then pressed and consolidated by cold compaction process followed by sintering. The cold compaction process is carried out in a compression moulding machine. The powders are compacted into cylindrical specimens of diameter 10 mm and length 25 mm. The photograph of the compression molding machine and the muffle furnace used for aging are shown in Figure 3.2 and 3.3. The die used for compaction and the consolidation process are shown in Figure 3.4a-b. Samples of the composite specimens fabricated are also shown in Figure 3.5. The consolidation is done in room temperature at a pressure of 2 MPa. The composite specimens fabricated are then taken for sintering in a muffle furnace at atmospheric conditions under three different sintering temperatures viz., 500, 550 and 600°C for 4 hours each. This is followed by cooling with normal water and artificial aging is then carried out for all the cooled specimens in the same muffle furnace at 170°C for 10 hours.

![Figure 3.1 Photograph of planetary ball mill used for blending](image)
Figure 3.2 Photograph of compression molding machine used for consolidation

Figure 3.3 Photograph of muffle furnace used for sintering and aging
Figure 3.4  Photographs of (a) Die used for consolidation (b) consolidation process

Figure 3.5 Photograph of sample composite specimen
3.3 TESTING THE COMPOSITE FOR MICROSTRUCTURE, PHYSICAL AND MECHANICAL PROPERTIES

3.3.1 Microstructure

The uniform distribution of the dispersion reinforcement particles in the matrix is confirmed through SEM (Scanning Electron Microscope) micrographs.

3.3.2 Density

The mass density of the composite specimens is calculated using the equation given below. The density for the specimens is measured after heat treatment.

\[
\rho = \left( \frac{\text{Mass}}{\text{Volume}} \right) \text{ (kg/m}^3\text{)}
\]  

(3.1)

The theoretical density of the composite specimens is calculated using rule of mixtures as given below:

\[
\rho_c = \rho_r (1 - V_m) + \rho_m V_m
\]  

(3.2)

Where,  
\( \rho_c \) = Density of the composite (kg/m\(^3\))  
\( \rho_r \) = Density of the reinforcement (kg/m\(^3\))  
\( V_r \) = Volume fraction of the reinforcement  
\( V_m \) = Volume fraction of the matrix

3.3.3 Porosity

The porosity of the composite specimens is determined based on the theoretical and experimental values of density of composites. Porosity is calculated as shown in Equation (3.3)
The porosity of the composite specimens may arise on account of the compaction of the mixture containing two different materials, one being very hard reinforcement particles and the other being a very soft matrix material. Theoretical density is calculated using rule of mixtures and the calculated density using Equation 3.1.

\[
Porosity = \left( \frac{\text{TheoreticalDensity} - \text{CalculatedDensity}}{\text{TheoreticalDensity}} \right) \times 100 \text{ (\%)} \quad (3.3)
\]

The hardness of the heat treated (sintering followed by aging) composites is measured using Vickers hardness tester under a load of 1 kg. The load is applied for 30 seconds. In order to eliminate possible segregation effect a minimum of three hardness readings are taken for each specimen at different locations of the test samples.

3.3.5 Compressive Strength

The compressive strength of the composites is estimated using Universal testing machine of 100 tonne capacity. The compressive load is applied on the composite specimens with a traverse speed of 0.25 m/min. The compression test is carried out at room temperature.
3.4 WEAR PERFORMANCE OF THE COMPOSITE

Dry sliding wear tests are conducted using a pin-on-disc wear testing apparatus (model: TR 201 Wear and friction monitor, Ducom make, Bengaluru, India) under varying applied normal loads on the composite specimen, and at varying rotational speeds of the disc. All the wear tests are carried out as per ASTM G-99 standard under unlubricated condition in a normal laboratory atmosphere at 50-60% relative humidity and at a temperature range of 28-32°C. The counter disc used in the apparatus is EN32 steel disc of hardness 65 HRC. The composite specimens are used as pins for the tests. They are 25 mm in length and 10 mm in diameter. The surfaces of the pin sample and the steel disc are ground using emery paper prior to each test. In order to ensure effective contact of fresh surface with the steel disc, the fresh samples are subjected to sliding on emery paper of 240 grit size fixed on the steel disc. During sliding, the load is applied on the specimen through cantilever mechanism and the specimens brought in intimate contact with the rotating disc at a track diameter of 52 mm. The samples are cleaned with acetone before every test.

The testing parameters considered during wear test and their levels, performance indicators are shown in Table 3.3 and 3.4. The performance indicators during the test are recorded from the digital display interfaced with the wear testing machine through a data acquisition system. Wear and coefficient of friction are recorded from the data acquisition system and the temperature of the pin is recorded using a non-contact IR type temperature detector, fixed at a distance of 20 mm from the contacting surface. A set of three samples is tested in every experimental condition, and the average for each set of three tests is measured. Each set of test is carried out for a period of 30 minutes. The schematic diagram and the photograph of the experimental set up are shown in Figure 3.6 and 3.7 respectively.
Table 3.4 Input Parameters and their levels for wear test

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Parameter</th>
<th>Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Weight fraction of reinforcement</td>
<td>10%, 20% &amp; 30%</td>
</tr>
<tr>
<td>2</td>
<td>Sintering temperature</td>
<td>500°C, 550°C &amp; 600°C</td>
</tr>
<tr>
<td>3</td>
<td>Applied normal load</td>
<td>10N, 20N &amp; 30N</td>
</tr>
<tr>
<td>4</td>
<td>Rotational speed of the Disc</td>
<td>400rpm, 500rpm &amp; 600rpm</td>
</tr>
</tbody>
</table>

Table 3.5 Performance indicators and their units for wear test

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Performance indicator</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Wear (Linear)</td>
<td>µm</td>
</tr>
<tr>
<td>2</td>
<td>Coefficient of Friction</td>
<td>No Unit</td>
</tr>
<tr>
<td>4</td>
<td>Temperature of the Pin</td>
<td>°C</td>
</tr>
</tbody>
</table>

Figure 3.6 Schematic diagram of wear and friction monitor equipment
3.5 MACHINABILITY OF THE COMPOSITE

Machinability of the composite is studied by conducting drilling experiments on the specimen. The experiments are conducted in ARIX VMC 100 CNC drilling machine under room temperature and in dry condition. The photograph of the machine with dynamometer attachment is shown in the Figure 3.8. The summary of the experimental conditions is listed in Table 3.5. Experiments are conducted for full factorial design. Each experiment is repeated three times so as to get consistent values and in order to remove the influence of tool wear each experimental condition is performed with a new drill. In the system, the Kistler piezoelectric dynamometer is used to measure the thrust force and torque. The surface roughness of the hole is evaluated (Ra according to ISO 4287/1) with a Mitutoyo surf test (make - Japan: model – SJ 301) measuring instrument with cut off length 2.5 mm, as shown in the Figure 3.9. Surface roughness readings are taken at three different positions around the circumference of the hole and approximately mid-way down the depth of
the hole. The surface roughness of each hole is taken as the mean of the three readings. The testing parameters considered during wear test and their levels, performance indicators are shown in Table 3.6 and 3.7 respectively.

**Table 3.6 Summary of experimental conditions**

<table>
<thead>
<tr>
<th>Workpiece</th>
<th>Cold compacted cylindrical specimens of Al 2219 with average reinforcement particle sizes 23, 37 and 67 µm SiC and weight fractions – 10%, 15% and 20%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutting tool and machine</td>
<td>PCD drill – $\varphi$ 5 mm (twist) and 118° point angle, ARIX VMC 100 CNC drilling machine</td>
</tr>
<tr>
<td>Cutting conditions</td>
<td>Spindle speed – 500, 1000, 1500 rpm Feed rate – 10, 15, 20 mm/min Environment – Dry</td>
</tr>
</tbody>
</table>

**Figure 3.8** Photograph of vertical machining centre used for drilling with dynamometer attachment
Table 3.7 Input Parameters and their levels for drilling test

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Parameter</th>
<th>Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Weight fraction of reinforcement</td>
<td>10%, 20% &amp; 30%</td>
</tr>
<tr>
<td>2</td>
<td>Sintering temperature</td>
<td>500°C, 550°C &amp; 600°C</td>
</tr>
<tr>
<td>3</td>
<td>Spindle speed</td>
<td>500, 1000, 1500 rpm</td>
</tr>
<tr>
<td>4</td>
<td>Feed rate</td>
<td>10, 15, 20 mm/min</td>
</tr>
</tbody>
</table>

Table 3.8 Performance indicators and their units for drilling test

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Performance indicator</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Thrust force</td>
<td>N</td>
</tr>
<tr>
<td>2</td>
<td>Torque</td>
<td>Nm</td>
</tr>
<tr>
<td>4</td>
<td>Surface roughness. Ra</td>
<td>µm</td>
</tr>
</tbody>
</table>
3.6 OPTIMIZATION OF THE PROCESS PARAMETERS

The process parameters considered for the wear test and machinability tests are optimized using Taguchi based Grey relational method.

3.6.1 On Taguchi’s Quality Engineering

Dr. Genichi Taguchi, a mechanical engineer, devised a new concept known as quality engineering. He defined quality as “the loss imparted to the society from the time a product is shipped”. Dr. Taguchi stresses on the concept that organizations must try to achieve the target criteria instead of trying to manufacture within limits. According to him a loss is imparted even if the value diverts from the target value within the specified limit. Dr. Taguchi devised the concept of loss function to quantify the loss imparted from a given product. Although he devised 68 loss functions, it can be grouped under 3 major classifications. The loss functions are shown in Figure 3.10.

- Nominal the better
- Higher the better
- Lower the better

![Figure 3.10](image)

Figure 3.10 Taguchi’s loss function (a) Nominal the better (b) smaller the better (c) Higher the better
(a) **Nominal the better**

The Figure 1.3a shows the performance characteristics for the notion “Nominal the better”. When the value for the performance characteristics “y” is within the specifications the loss is 0 and when it is outside the specifications the loss is A. The quadratic equation shown below describes the loss function. In this situation loss occurs as soon as the performance characteristics “y” departs from the target “τ”. The quadratic loss function is

\[ L = k(y - \tau)^2 \quad (3.4) \]

Where,  
\[ L = \text{Loss function} = \text{cost incurred as quality deviates from the target} \]
\[ y = \text{Performance characteristics} \]
\[ \tau = \text{Target} \]
\[ k = \text{Quality loss coefficient} = \frac{A}{(y - \tau)^2} \]

(b) **Smaller the better**

The Figure 1.3b shows the performance characteristics for the notion “Smaller the better”. The target value for this notion is “0” and there are no negative values for the performance characteristics. Examples of performance characteristics include radiation leakage from a microwave appliance, response time of a computer, pollution from an automobile etc.

\[ L = ky^2 \quad (3.5) \]

Where \( L, y, \tau \) and \( k \) have similar definitions

Here, \( k = A/y^2. \)
(c) Higher the better

The Figure 1.3c shows the performance characteristics for the notion “Higher the better”. The target value for this notion is “∞” which gives a zero loss. There are no negative values and the worst case is for y = 0. Actually, “larger the better” is the reciprocal of “smaller the better” and because of the difficulty of working with “∞”, some practitioners prefer to work with the reciprocal. Thus a larger the better performance characteristics of m/s becomes a smaller the better characteristics of s/m. examples of performance characteristics are bond strength of adhesives, welding strength etc.

\[ L = \frac{k}{y^2} \]  

(3.6)

Where \( L \), \( y \), \( \tau \) and \( k \) have similar definitions.

Here, \( k = Ay^2 \).

3.6.2 Orthogonal Arrays (OA)

Orthogonal Arrays (OAs) are a simplified method of putting together an experiment. To determine the proper orthogonal array usually the following steps are used.

- Determine the number of factors and their levels.
- Determine the degrees of freedom.
- Select an orthogonal array.
- Consider any interactions.
3.6.3 Degrees of Freedom

It determines the minimum number of treatment combination. It is equal to the sum of

- (Number of levels – 1) for each factor.
- (Number of levels – 1) (Number of levels – 1) interaction.
- One for the average.

Once the number of factors, their levels and their degrees of freedom are known, then the OA can be selected using the array selector table shown in Figure 3.11. The array selector table is a table designed on the rule that “The number of treatment conditions is equal to the number of rows in the OA and must be equal to or greater than the degrees of freedom.” In this study for both wear test and machinability test, the process parameters are optimized using L9 orthogonal array shown in Table 3.8.

![Array selector table](image)

Figure 3.11 Array selector table
Table 3.9 L₉ Orthogonal Array

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Factor 1</th>
<th>Factor 2</th>
<th>Factor 3</th>
<th>Factor 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
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<tr>
<td>3</td>
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</tr>
<tr>
<td>9</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

3.6.4 Signal to Noise Ratio (S/N Ratio)

The S/N Ratio is another important contribution of Dr. Genichi Taguchi. It was developed as a proactive equivalent to the loss function. In layman’s terms, S/N ratio can be defined as the ratio of amount of energy for the intended function to the amount of energy wasted.

\[
\text{S/N ratio} = \frac{\text{Signal}}{\text{Noise}}
\]  

Signal factors are set by the designer or the operator to obtain the intended value of the response variable. Noise factors are not controlled or are very difficult or expensive to control. The higher the S/N ratio, the better is the performance of the parameter, since higher S/N ratio means, amount to signal compared to noise is greater.
The formulas for the S/N ratios are given as follows.

- **Nominal the better**, 
  \[
  \frac{S_N}{N \text{ ratio}} = 10 \log_{10} \left( \frac{\left( \frac{\gamma_i}{s^2} \right)}{\frac{1}{n}} \right) \tag{3.8}
  \]

- **Smaller the better**, 
  \[
  \frac{S_N}{N \text{ ratio}} = -10 \log_{10} \frac{1}{n} \sum_{i=1}^{n} y_i^2 \tag{3.9}
  \]

- **Higher the better**, 
  \[
  \frac{S_N}{N \text{ ratio}} = -10 \log_{10} \frac{1}{n} \sum_{i=1}^{n} \frac{1}{y_i^2} \tag{3.10}
  \]

Where  
- \( n \) = Number of trials 
- \( y_i \) = Signal Factor 
- \( s^2 \) = Noise Factor

### 3.6.5 Optimization Procedure

First the parameters corresponding to the design of the experiment i.e. the orthogonal array are noted. Next a response table for the S/N ratios are generated to find the optimal level of each experiment. The parameters are ranked based on their order of importance. The confirmation tests are conducted for the optimal condition and the experimental and predicted values are compared. The formula to determine the predicted value is 

\[
\gamma = \gamma_m + \sum_{i=1}^{p} \bar{\gamma}_i - \gamma_m \tag{3.11}
\]

Where  
- \( \gamma_m \) = Total mean S/N Ratio. 
- \( \bar{\gamma}_i \) = Mean S/N ratio at the optimal level.

P = Number of the main designed parameters that affect the quality characteristics

Further main effects plot and interaction plots of the data means are used to indicate the interactions between the various input and output parameters.

3.7 GREY RELATIONAL ANALYSIS

The Grey Relational Analysis (GRA) associated with the Taguchi method represents a rather new approach to optimization.

The grey theory is based on the random uncertainty of small samples which developed into an evaluation technique to solve certain problems of system that are complex and having incomplete information.

GRA is basically an extension of Taguchi’s technique, and hence it is usually called as Taguchi based GRA. However, Taguchi is a single objective optimization technique whereas, GRA is a multiple objective optimization technique.

GRA is applied by several researchers to optimize control parameters having multiple responses through grey relational grade. The flow chart in the Figure 3.12 depicts the steps in the GRA. Simultaneously it also depicts the correlation between GRA and Taguchi’s technique.

Each of the steps depicted have been clearly explained as follows:

Step 1: Experiment design and execution

The Design of Experiment (DOE) is similar to the Taguchi’s technique. Based on the number of input parameters and their levels, an
Orthogonal Array is selected from the array selector table. Next the experiments are conducted based on the design of the OA.

**Step 2: Signal – to – Noise ratio calculation**

Calculation of S/N ratio is same as explained above, where the formulas given under Taguchi’s technique are used based on the criteria that is specified.

**Step 3: Grey relational Pre – processing**

GRA involves pre – processing stages wherein the output parameters are normalized. Normalization is a very important step in GRA. The reason is that GRA is a multi – objective optimization technique and depends on more than one parameter for its result.

The output parameters being analyzed normally are of different units. Further the range of these values may be different. For example, one parameter might be in the range (1 – 10 m/s). On the other hand the other parameter might be in the range (100 – 10000N). When GRA is applied to these values as such then they will lead to erroneous results. These errors might be negligible in some cases, however as a standard the values are normalized prior to GRA.

During normalization, all the output parameters are converted into dimensionless values. Further, the range of the parameters is restricted to 0 – 1, where “0” indicates the lowest value while “1” indicates the highest value.

Comparing the various parameters based on this standard is very easy. Moreover, the chances of errors in these normalized values are very low.
Figure 3.12 Steps in Grey Relational Analysis

Normalization process of GRA involves three notions similar to Taguchi’s technique i.e. nominal the better, smaller the better and higher the better.

The formulas for the normalization process are as follows

- Nominal the better,

\[ Y(k) = 1 - \frac{|x_i^0(k) - x^0|}{\max x_i^0(k) - x_i^0(k)} \]  \hspace{1cm} (3.12)

- Smaller the better,

\[ Y(k) = \frac{\max x_i^0(k) - x_i^0(k)}{\max x_i^0(k) - \min x_i^0(k)} \]  \hspace{1cm} (3.13)
• Higher the better,

\[ Y(k) = \frac{x_i^0(k) - \min x_i^0(k)}{\max x_i^0(k) - \min x_i^0(k)} \quad (3.14) \]

Where, 
- \( Y(k) \) = The normalized value for the \( k \)th trial.
- \( x_i^0(k) \) = The value of the output parameter for the \( k \)th trial.
- \( \min x_i^0(k) \) = The smallest value of the output parameter \( "x" \) for the \( k \)th trial.
- \( \max x_i^0(k) \) = The largest value of the output parameter \( "x" \) for the \( k \)th trial.

**Step 4: Reference Sequence Definition**

Before proceeding any further, the designer must set a reference sequence for each of the output parameter. The reference sequence indicates the best that can be achieved.

The reference sequence thus indicates the targets which are required to be achieved, i.e. it indicates the ideal solution.

**Step 5: Grey Relational Coefficient (GRC) calculation**

The next step is the calculation of the GRC. The GRC for any output parameter can be calculated using the formula.

\[ \eta(j) = \frac{\Delta_{\min} + \zeta \Delta_{\max}}{\Delta_{\min} + \zeta \Delta_{\max}} \quad (3.15) \]

Where
- \( \eta(j) \) = GRC for the \( j \)th output parameter.
- \( \Delta_{oi} \) = Deviation Sequence.
- \( \zeta \) = weighting coefficient.
However, if all the output parameters are of equal importance the value of the weighting coefficient is taken as “0.5”.

Thus the GRC is calculated for each of the output parameters. The closer the value is to “1”, the better is its performance, i.e. it satisfies the conditions of higher/lower to greater extent.

**Step 6: Grey Relation Grade**

Calculation of the Grey Relational Grade (GRG) is the final step in GRA. GRG is the step where the optimization technique converts from a single objective optimization to a multi–objective optimization. The GRG is calculated as an average of the GRCs of all the output parameters at a given level of the OA. The formula for GRG is

\[
\text{GRG} = \frac{1}{n} \sum_{i=1}^{n} \eta(j) \tag{3.16}
\]

Where, \(n\) = the number of output parameters.

**Step 7: Determination of optimal factor levels**

For a factor \(A\) that has three levels, 1, 2 and 3, for example, the main effect of factor \(A\) at level 1 \((mA1)\) is equal to the average GRG whose factor \(A\) in experimental scenarios is at level 1, and the main effect of factor \(A\) at level 2 \((mA2)\) is equal to the average GRG whose factor \(A\) in experimental scenarios is at level 2 and so on. The higher the main effect is, the better the factor level is. Therefore, the optimal levels for factor \(A\) will be the one whose main effect is the highest among all levels.
Step 8: Confirmation Test

Confirmation tests are carried out to predict and verify the enhancement of the quality characteristics using the optimal parametric combination. The estimated grey relational grade using optimal level of machining parameters can be calculated as

\[ \gamma = \gamma_m + \sum_{i=1}^{n} \gamma_i - \gamma_m \]  

(3.17)

Where  \( \gamma_m = \) Total mean grey relational grade.

\( \gamma_i = \) Mean grey relational grade at the optimal level.

\( P = \) Number of the main designed parameters that affect the quality characteristics

In this study, GRA, a normalization evaluation technique is used to solve the complicated multi-performance characteristics optimization effectively.

3.8 RESPONSE SURFACE METHODOLOGY

Response Surface Methodology (RSM) is a collection of mathematical and statistical techniques useful for the modeling and analysis of problems in which a response of interest is influenced by several variables and the objective is to optimize this response. When treatments are from a continuous range of values, then a Response Surface Methodology is useful for developing, improving, and optimizing the response variable. For
example, if the response variable “y” is a function of the variables “x1” and “x2” then it can be expressed as

\[ y = f(x_1, x_2) + \varepsilon \]  

(3.18)

The variables \(x_1\) and \(x_2\) are independent variables where the response \(y\) depends on them. The dependent variable \(y\) is a function of \(x_1, x_2,\) and the experimental error term, denoted as \(\varepsilon.\) The error term \(\varepsilon\) represents any measurement error on the response, as well as other type of variations not counted in \(f.\) It is a statistical error that is assumed to distribute normally with zero mean and variance \(\sigma^2.\) In most RSM problems, the true response function \(f\) is unknown. In order to develop a proper approximation for \(f,\) the experimenter usually starts with a low-order polynomial in some small region. If the response can be defined by a linear function of independent variables, then the approximating function is a first-order model. A first-order model with 2 independent variables can be expressed as

\[ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon \]  

(3.19)

In general all RSM problems use either one or the mixture of the both of these models. In each model, the levels of each factor are independent of the levels of other factors. In order to get the most efficient result in the approximation of polynomials the proper experimental design must be used to collect data. Once the data are collected, the Method of Least Square is used to estimate the parameters in the polynomials. The response surface analysis is performed by using the fitted surface. The response surface designs are types of designs for fitting response surface. Therefore, the objective of studying RSM can be accomplished by
- Understanding the topography of the response surface (local maximum, local minimum, ridge lines)

- Finding the region where the optimal response occurs.

The goal is to move rapidly and efficiently along a path to get to a maximum or a minimum response so that the response is optimized.

The relationship between a set of independent variables and the response $y$ is determined by a mathematical model called regression model. When there are more than two independent variables the regression model is called multiple – regression model. In general, a multiple-regression model with $q$ independent variable takes the form of

$$y_i = \beta_0 + \beta_1x_{i1} + \beta_2x_{i2} + \cdots + \beta_qx_{iq}^2 + \varepsilon_i (i = 1, 2, \ldots, N)$$  \hspace{1cm} (3.20)

Where, $n > q$. The parameter $\beta_j$ measures the expected change in response $y_i$ per unit increase in $x_i$ when the other independent variables are held constant. The $i$-th observation and $j$-th level of independent variable is denoted by $x_{ij}$.

The multiple-regression model can be written in a matrix form

$$y = X\beta + \varepsilon$$  \hspace{1cm} (3.21)

Where, $y = (n \times 1)$ vector of observations

$X = (n \times k)$ matrix of levels of independent variables

$\beta = (k \times 1)$ vector of regression coefficients

$\varepsilon = (n \times 1)$ vector of random errors
A good estimated regression model shall explain the variation of the dependent variable in the sample. How well the estimated model fits the data can be measured by the value of $R^2$. The $R^2$ lies in the interval $[0,1]$. When $R^2$ is closer to the 1, the better the estimation of regression equation fits the sample data. In general, the $R^2$ measures percentage of the variation of $y$ around $y$ that is explained by the regression equation. However, adding a variable to the model always increases the $R^2$, regardless of whether or not that variable is statistically significant. Thus, some experimenters would rather use adjusted $R^2$. When variables are added to the model, the adjusted $R^2$ will not necessarily increase. In fact, if unnecessary variables are added, the value of adjusted $-R^2$ will often decrease.

$$R^2 = \frac{SS_R}{SS_T} = 1 - \frac{SS_E}{SS_T} \quad (3.22)$$

$$\text{Adj.}R^2 = 1 - \frac{SS_E/(n-q-1)}{SS_T/(n-1)} \quad (3.23)$$

where the meanings of the terms will be discussed under the topic ANOVA. An adjusted $-R^2$ value of above 80% is usually preferred to indicate that the mathematical model so generated fits the given situation.

A statistical software package Minitab (version 16) was used for Response Surface Methodology (RSM) to develop model relating quantitative experimental factors and responses. Central Composite Design was used in RSM technique.
3.9 EXPERIMENTAL METHODOLOGY

The methodology of the proposed research is illustrated in the Figure 3.13.

**Studies on Characterization and machinability of Al 2219 – SiC\(_p\)**

**Work piece material**
Matrix: Al2219, Reinforcement: SiC\(_p\) of average particle size 23, 37 and 67 μm and weight fraction 10, 15 and 20%

**Fabrication**
Cold compaction at 20 bar followed by sintering at different temperatures 500, 550 and 600°C for 4 hours and aging at 170°C for 10 hours

**Testing for mechanical properties**
Mass density, porosity, hardness, compressive strength

**Machinability study**
Drilling using PCD drill of 5 mm diameter – feeds 10, 20 and 30 mm/min and cutting speeds 1000, 1500 and 2000 rpm
**Performance indicators**- thrust force, torque, and surface roughness of the hole
(Machine used: CNC vertical machining centre)

**Wear characterization**
Pin on disc dry sliding wear testing – loads 10, 20 and 30N, disc speeds 400, 500 and 600 rpm
**Performance indicators**- wear (linear), coefficient of friction, and temperature of the pin
(Machine used: Pin on disc tribometer-make:Ducom)

**Results and discussions**
Effect of reinforcement particle size, its weight fraction, sintering temperature and microstructure, process parameters on the performance of machining and wear

**Optimization**- Taguchi based Grey Relational analysis
**Modelling** - Response Surface Methodology (RSM)

**Conclusions & Scope for future work**

Figure 3.13 Experimental methodology