1.1 Overview

One of the spectacular developments in string theory has been the conjectured duality between a conformal field theory (CFT) and a theory of gravity in anti-de Sitter (AdS) space-time, which goes by the name of the “Anti de-Sitter space/Conformal Field Theory correspondence” in string theory parlance, or the “AdS/CFT correspondence” [1–3]* in short. The correspondence offers an innovative approach to access the strong coupling regime of a wide class of quantum field theories, that are otherwise inaccessible by standard field-theoretic techniques. The essence of the correspondence is to establish a mapping in a well-defined sense between a gauge theory respecting conformal invariance and a string theory in AdS space-time. It can be shown that under certain conditions a strongly coupled gauge theory can be mapped to a weakly coupled string theory, which reduces to classical gravity thereby, permitting a perturbative analysis. This is what makes the correspondence

*A comprehensive review of the correspondence can be found in [4].
so appealing - one just solves the problem perturbatively in the weakly coupled string dual and reverts back to the strongly coupled gauge theory of interest! In the process one invokes the so-called “AdS/CFT dictionary” that relates gauge-theoretic quantities to their string theory counterparts. In recent years, a considerable effort has been directed towards exploiting this remarkable correspondence to unravel the salient features of the plasma phase of gauge theories that admit a dual string description. Apart from generic theoretical interests, the phenomenology of ultra-relativistic heavy ion collisions, i.e., collisions of atomic nuclei where the center-of-mass energy per nucleon far exceeds the nucleon rest mass, has also acted as a catalyst for undertaking such studies. In fact, inspiration has acted the other way too, and precious insights gained from theoretical studies using the correspondence have paved way for exploring uncharted frontiers in the collider experiments. It is this thriving symbiotic relationship between heavy ion phenomenology and the AdS/CFT correspondence that will be the underlying theme of the thesis.

Experiments currently underway at the Relativistic Heavy Ion Collider (RHIC) at the Brookhaven National Laboratory and the Large Hadron Collider (LHC) at the European Organization for Nuclear Research (CERN) have provided fascinating insights into properties of Quantum Chromodynamics (QCD) matter at extreme high temperature and/or energy density. Experimental signatures suggest that in the energy scale accessed at the colliders, QCD matter appears in the guise of a new phase - “Quark-Gluon Plasma” (QGP) which emerges after a phase transition from the hadronic state to a deconfined state of quarks and gluons. Experimental evidence further indicates that the QGP formed does not behave as a weakly coupled gas of quarks and gluons but resembles a strongly coupled fluid [5–7]. For instance, while computing the ratio of the shear viscosity $\eta$ to the entropy density $s$, weak and strong coupling results [8,9] are found to differ not only quantitatively but also parametrically, and experimental data [10] supports the strong coupling result.
Later this ratio was shown to be universal for all strongly coupled gauge theories in the limit of large number of colors and permitting a dual gravitational description [11]. In such a scenario one of the major questions confronting us is whether the perturbative framework suffices to explain the relevant physics issues at a temperature of few hundred MeV’s (as attained in the colliders) or we should take recourse to a formalism that is robust at strong coupling. Although in a non-Abelian plasma weak coupling effects are distinctly different from strong coupling ones, a priori it is not apparent which observations and features owe their origin to weak coupling and which of them can be attributed to the strong coupling behavior. Hence, one of the exigent task at hand is to systematically disentangle the effects of strong coupling and weak coupling. Thus, in our endeavor to explain the wealth of experimental data accumulated from heavy ion collisions, a pressing requirement is a cross-fertilization of perturbative and non-perturbative ideas. Conventional field theory, which is essentially based on a perturbative framework, is well-suited to explain the weak coupling features. On the other hand, it is very crucial that we also have at our disposal a suitable machinery to explain the effects stemming from a large value of the coupling. A theoretical explanation of any strongly coupled phenomenon is always a challenging assignment since strong coupling imposes severe restrictions upon the applicability of the time-tested tools of traditional perturbative field theory. Lattice field theory has emerged as a viable alternative to investigate systems bearing the stamp of strong coupling effects, in a non-perturbative framework, but not without its own baggage of shortcomings. Indeed, it has successfully explained a multitude of thermodynamic properties of hot and dense QCD matter like critical temperature, nature of phase transition, equation of state, etc., but the very premises upon which it is formulated, make it incapable of handling real-time dynamics that are of relevance to QGP physics. It is thus highly desirable that one seeks alternative avenues to investigate gauge theories characterized by high values of the coupling
parameter. This makes us turn our attention to the AdS/CFT correspondence which has been immensely successful in explaining a plethora of strongly coupled phenomena across a diverse range of fields and energy scales, be it QCD, QGP, condensed matter physics, or even fluid dynamics.

The correspondence, in its primitive incarnation \([1]\), conjectured a bold duality between type IIB string theory living on \(AdS_5 \times S^5\) and \(\mathcal{N} = 4, SU(N_c)\) super Yang-Mills (SYM) theory, with \(N_c\) being the number of colors, living on the 4-dimensional boundary of \(AdS_5\). Since then, the duality has been the subject of intense theoretical investigations and generalized to encompass a wider variety of gauge theories under its ambit and is now more appropriately called the gauge/string duality or the holographic duality (since it relates a field theory to a theory of gravity in one higher dimension). The generalizations empower us to study less symmetric and hence, more realistic physical systems, making the duality even more potent.

Having said so, it must be admitted, that this approach is plagued by its own limitations. The duality does not provide precision tools for QCD physics and can at best be considered a complementary toolkit offering a semi-quantitative insight into the strong coupling regime of QCD. In spite of intense efforts the exact dual to QCD has remained elusive. But the duality does hold for a large class of solvable models that share many features with QCD. In the absence of any well-controlled machinery, the duality remains our best bet for deciphering the rich structure underlying QCD physics. Presently, various “toy” gauge theories admitting string duals are engaged to carry out the dual computations, and most intriguingly, the results extracted, in many instances, agree with those of QCD predictions and experimental observations (at least qualitatively). In fact, many of the results exhibit a kind of universality among the different theories hinting at the existence of a universality class. Further, in spite of the limitations, the results are all obtained from first principle
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calculations in non-Abelian field theories. This makes it worthwhile to pursue this complementary path further and try to obtain a better understanding of strongly coupled gauge theories.

By now there is a vast literature computing, holographically, different quantities of experimental interest in QCD-like gauge theories in the deconfined phase. However, most of the works concern QGP that is locally isotropic. The primary aim of the thesis is to use holographic ideas to elicit lessons about strongly coupled QGP when effects of anisotropy might be dominant. While a study of how anisotropy affects quantities of experimental relevance is interesting in its own right, what makes it more appealing is that anisotropy is one of the hallmarks of the plasma during its early stage right after its birth. Thus, a proper understanding of anisotropy-induced modifications is absolutely imperative in our endeavor to understand better the early-time dynamics of QGP.

In collisions with a non-zero impact parameter, i.e., when the nuclei do not collide head-on, anisotropic pressure gradient develops in the overlapping region of two colliding nuclei, transforming the initial coordinate-space anisotropy into an observed momentum-space anisotropy, through interactions between the produced particles, leading to an anisotropic particle distribution. The early success of relativistic ideal hydrodynamics in explaining various results at RHIC provided empirical evidence in favor of fast thermalization and isotropization - at time scales $\tau_{\text{iso}} \sim 0.5$ fm. In an attempt to make better agreement with experimental results this was subsequently generalized to relativistic viscous hydrodynamics, which, however, predicted the presence of a sizable pressure anisotropy. It was found that the transverse pressure exceeds the longitudinal (along the beam direction) one with the difference being the largest for time $\leq 2$ fm. Thus viscous hydrodynamical simulations suggest that isotropization may occur as late as $\tau_{\text{iso}} \sim 2$ fm. Currently, the question of the degree of momentum-space anisotropy in QGP is open to intense theoretical debate and is
deemed worthy of an in-depth study. Recent studies [12,13] suggest that large momentum-space anisotropies may be present for most part of the time evolution both at weak and strong coupling. The plasma, just after its birth in relativistic heavy ion collisions, is locally anisotropic and far away from equilibrium for a time $t < \tau_{\text{out}}$. It settles down in an isotropic state only after time $\tau_{\text{iso}} > \tau_{\text{out}}$, so that the standard hydrodynamic description of the plasma makes sense only if we want to probe the plasma at time scale $t > \tau_{\text{iso}}$. One would, of course, like to make progress and study the plasma in the time scale $t < \tau_{\text{out}}$ when it is far away from equilibrium. However, in the present state of development, studying the far-from-equilibrium dynamics of the hot plasma and its temporal evolution to an equilibrium state, is a rather difficult task. Instead, we focus our attention upon an intermediate temporal window $\tau_{\text{out}} < t < \tau_{\text{iso}}$, where the plasma is in equilibrium but yet to attain isotropy, and which is much more accessible via our current theoretical tools. To probe the plasma in this time domain, it is essential that one takes into account the inherent anisotropy present in the system. It is suspected that the magnitude of this momentum-space anisotropy can be so high that it may even violate the central assumption of canonical viscous hydrodynamical treatments - which is to linearize around an isotropic background. It has been shown that large linear corrections result in unphysical results such as negative particle pressures, negative one-particle distribution functions, etc. [14]. Another closely related aspect of the same problem is that microscopic models of the early stages of relativistic heavy ion collisions indicate that the produced system is highly anisotropic [15]. In the theory of the Color Glass Condensate (CGC), at very early proper times, $\tau \ll 1/Q_s$, where $Q_s$ is the saturation scale, the classical gluon fields lead to an energy-momentum tensor of the form $T_{\mu\nu} = \text{diag}(\varepsilon, \varepsilon, \varepsilon, -\varepsilon)$ [16,17] implying a negative value of the longitudinal pressure with the transverse pressure equal to the energy density. At later proper times, $\tau \gg 1/Q_s$,
both analytical perturbative approaches [18] and full numerical simulations [19] furnish the form $T_{\mu\nu} = \text{diag}(\varepsilon, \varepsilon/2, \varepsilon/2, 0)$ showing that the longitudinal pressure is zero. Consequently, proper matching of the results of the microscopic models with the hydrodynamic description (where the energy-momentum tensor should be close to the isotropic form) is not easy. Another vital issue is that physically one expects entropy production to vanish in two limits: the ideal hydrodynamical limit (vanishing shear viscosity) and the free streaming limit (infinite shear viscosity). However, within the realm of viscous hydrodynamics, entropy production is a monotonically increasing function of the shear viscosity. In the large shear viscosity limit, viscous hydrodynamics becomes a poor approximation and one has to seek an alternative framework. The afore-mentioned difficulties spurred the development of a reorganization of viscous hydrodynamics in which one incorporates the possibility of large momentum-space anisotropies at the leading order. The inclusion of large anisotropies also allows for direct matching with theories such as CGC. This framework has been dubbed anisotropic hydrodynamics [20–26]. Motivated by these field-theoretic developments, there has also been a surge in interest in investigating anisotropic plasma in the spirit of the gauge/string duality.

In the thesis, we consider two specific holographic toy models of anisotropic plasma to compute different quantities related to heavy probe quarks that are of direct relevance to collider experiments. While there are many such quantities, we focus specifically on those quantities, where there is a promise of significant cross-fertilization between experimental data and insights obtained via the gauge/string duality. One of the quantities we compute is the bound state quark-antiquark ($Q-\bar{Q}$) potential $E(L)$ as a function of the quark-antiquark separation ($L$). This provides information regarding the suppression of quarkonium production (like $J/\Psi$ states) which, in turn, sheds light upon the temperature of the matter and

$^{\dagger}$We shall also denote the bound state potential by $V(L)$ interchangeably.
the degree of color screening. An alternative mode to explore this screening is simply to compute the screening length $L_{\text{max}}$, the distance beyond which the bound states melt into the plasma. Other quantities that we evaluate include the jet quenching parameter $\hat{q}$ that measures the radiative energy loss of an energetic parton plowing through the plasma and the drag coefficient $\gamma$, which encodes the amount of collisional energy loss undergone by a probe quark as it executes stochastic motion in the plasma. The drag coefficient is, in turn, related to the relaxation time, $t_{\text{relax}}$, which is a characteristic time scale beyond which the plasma thermalizes. We shall try to compute these quantities at various stages of the thesis and observe how they carry the imprint of anisotropy. In this context let us also issue the caveat that since all the computations will be performed using “toy” models, one should exercise utmost care in attempting to connect our results to realistic QCD plasma. Nevertheless, it is a fruitful undertaking since many of the toy models belong to the same universality class as QCD and hence, provide precious qualitative insights into the rich dynamics that underlies QCD. In fact, remarkably, we find that the stamp of anisotropy on many of the quantities that we compute are qualitatively very similar.

1.2 Plan of the Thesis

To help the reader navigate through the thesis, we provide here a short description of the ensuing chapters and their contents. Chapter 2 aims to provide a concise overview of aspects of QCD and QGP relevant to our purpose and to heavy ion phenomenology. This is followed by a lightning review of the AdS/CFT correspondence. We further establish the relevance of the gauge/string duality in the context of QGP. In chapter 3 we discuss holographic computation of Wilson loops in strongly coupled thermal $\mathcal{N} = 4$ SYM plasma following well-defined prescription in the literature and learn how to extract various heavy
quark observables from the expectation values of different Wilson loops. Thereby we set the stage for the computations to be carried out in the more general scenarios of anisotropic plasma in the subsequent chapters. Chapter 4 concerns the first of our anisotropic models - the finite temperature, strongly coupled, non-commutative Yang-Mills (NCYM) plasma. To start with we compute the jet quenching parameter $\hat{q}_{\text{NCYM}}$ from a light-like Wilson loop and working in light-cone coordinates right from the outset. Then we compute the dipole potential $E(L)$ as a function of the dipole length $L$ when the dipole is moving along the commutative direction. We show numerically how $E(L)$ varies with $L$ with the dipole velocity $v$ and the non-commutativity $\theta$ as parameters. We are also able to arrive at an analytical expression for the screening length in a restricted domain of the parameter space of $v$ and $\theta$. Finally, for the sake of completeness, by taking the limit $v \to 1$ we recompute the expectation value of a light-like Wilson loop to extract the expression for $\hat{q}_{\text{NCYM}}$ that matches with the expression found out earlier. We consider the effect of non-commutativity upon $\hat{q}_{\text{NCYM}}$ for both small and large values of $\theta$ and attempt to connect our results with recent collider data by giving some numerical estimates using benchmark values of $\theta$ available in the literature. Chapter 5 deals with different aspects of massive probe quarks in a topologically deformed SYM theory - the second of our anisotropic models, where the deformation parameter depends upon one of the space coordinates thereby injecting anisotropy into the theory. To have analytical handle over our computations, we confine ourselves only to small values of anisotropy whence the metric components and the other relevant fields can be written analytically (perturbatively). The interaction potential and the screening length of mesonic bound states are found out for different orientations of the dipole in the plasma and the effect of anisotropy on the dissociation of mesons is discussed. The findings are compared with those obtained in other anisotropic models. We further analyze the
Brownian motion of a non-relativistic heavy probe quark in the plasma. The concomitant Langevin equation supplies information regarding the drag force, the random force autocorrelator and the relaxation time. The validity of the fluctuation-dissipation theorem in an anisotropic medium is verified from a holographic perspective. Finally, we conclude in chapter 6 with a summary of the work done. We also outline potential future avenues along which the work done in the thesis can be advanced further to know more about anisotropy-induced modifications in hot and dense strongly coupled QGP.