CHAPTER 4

MMSE LINEAR PRECODER DESIGN WITH PREDICTION ERROR CORRECTION

4.1 INTRODUCTION

In this chapter, MMSE based linear precoder is designed to overcome the effect of channel estimation error, feedback delay, spatial correlation and prediction error. The future CSI is predicted using 2-D Wiener prediction filter. The prediction error coefficient is obtained from the normalized mean square error of the prediction filter. The actual CSI is modelled to correct the prediction error. Then, the MMSE precoder is designed using CSI model which corrects the prediction error. The outage analysis is derived for the predicted CSI MMSE precoder and the prediction error corrected CSI MMSE precoder.

4.2 LITERATURE REVIEW

The performance of the MIMO wireless communication system depends on the degree of knowledge of the CSI. For a given wireless communication system, the best performance is achieved when the perfect CSI is available at the base station or at both links. However, in practical communication systems, the imperfect CSI may arise from various sources such as channel estimation errors, antenna correlation and the outdated channel estimates with respect to the current channel. Hence, current CSI is not sufficient and future channel conditions need to be known to adapt
transmission parameters. The accuracy of the transmit channel estimation depends on the channel characteristics. In wireless communication systems, mobility can make the available channel information out of date. The Doppler spread induced by the motion of the subscribers and scatterers has a strong influence on time processing algorithm. The Doppler spread is large in macro cells which serve high mobility subscribers and it increases with high operating frequencies. Hence, in the case of a larger Doppler spread, the reciprocity cannot be applied i.e. the uplink channel estimates cannot be used for the downlink. A timely update of the CSI is an obvious solution to improve the system performance in a time varying channel. Hence, it is necessary to estimate the downlink channel in accordance with the Doppler rate. Larger Doppler rate implies faster channel variation.

In the literature, the researchers have focused on preprocessing concepts based on channel prediction. To compensate the effect of feedback delay, the prediction algorithm is proposed by Kobayashi et al (2006) and the results are compared with the zero forcing (ZF) and MMSE methods. The performance of MIMO- SVD system in Rician channels are analyzed in the presence of channel estimation error and feedback delay in the literature (Au et al. 2008). The channel prediction for equalization in MIMO wireless communication systems by considering the decision delay is proposed by Komninakis et al. (2002). A Pre-RAKE transmitter with long range prediction is employed over rapidly varying multipath fading channel in the literature (Guncavadi & Duel-Hallen 2001). The channel prediction based on Wiener filtering is used in adaptive beam forming Multiple Input Single Output (MISO) systems in the literature (Ramya & Bhashyam 2007). The effect of channel prediction error on the BER performance of adaptive modulation based on transmit beamforming is analyzed by Zhou & Giannakis (2004). Dong et al. (2005) discussed the pre-RAKE beamforming for predicted channel. The outage probability has been analyzed by Ramya & Bhashyam
(2009) for antenna selection in MIMO systems using delayed feedback. All these above mentioned work have considered channel prediction using Kalman filter or Wiener filter, to design the precoder.

Since the practical scenario is considered in this thesis, the CSI available at the BS is imperfect. The 2-D prediction applied in the section 3.5 is used here to predict the future CSI. However, the channel prediction error (i.e.) the difference between the actual channel taps and the predicted channel is unavoidable. From normalized MSE, the correlation coefficient between the actual channel and the predicted channel is derived. Then, using the relationship among the actual channel, predicted channel and the correlation coefficient, the actual channel is obtained. Subsequently, the linear MMSE precoder is derived for the actual channel model and BER performance and outage performance are analyzed for the predicted channel and prediction error corrected channel.

The structure of the chapter is organized as follows. The MMSE precoder design with prediction error correction is discussed in section 4.3. The outage analysis is derived in section 4.4. Section 4.5 presents the simulation results of the proposed algorithm and section 4.6 concludes the paper.

4.3 PRECODER DESIGN WITH PREDICTION ERROR CORRECTION

The system model described in chapter 3 (section 3.3) and the two dimensional prediction discussed in chapter 3 (section 3.5) are considered in this chapter. The predicted channel vector \( \hat{h}(n + \Delta) \) of size \( KN_t \times 1 \) is given as \( \hat{h}(n + \Delta) = W_{2D} \tilde{h} \) and it is rearranged to form the \( K \times N_t \) predicted channel matrix \( H_{pred} \). The actual channel is denoted as \( H_{act} \). Let the prediction error \( e \) is given as
\[ \mathbf{e} = \mathbf{H}_{\text{act}} - \mathbf{H}_{\text{pred}} \]  

(4.1)

The normalized MSE of the prediction is given as

\[ c = \frac{E[|\mathbf{e}|]}{E[|\mathbf{H}_{\text{act}}|^2]} \]  

(4.2)

The normalized MSE of the 2D prediction is given as

\[ c = \frac{\text{Tr}(\mathbf{R}_s - (\mathbf{r}_t^T \otimes \mathbf{R}_s)(\mathbf{R}_{tx}^T \otimes \mathbf{R}_s + \sigma_e^2 \mathbf{I})^{-1}(\mathbf{r}_t \otimes \mathbf{R}_s)^T)}{\text{Tr}(\mathbf{R}_s)} \]  

(4.3)

The correlation between the actual channel and the predicted channel is given as

\[ \rho_c = \frac{\sqrt{E[|\mathbf{H}_{\text{act}}|^2] E[|\mathbf{H}_{\text{pred}}|^2]} - (E[\mathbf{H}_{\text{act}}^* \mathbf{H}_{\text{pred}}])}{E[|\mathbf{H}_{\text{act}}|^2] E[|\mathbf{H}_{\text{pred}}|^2]} \]  

(4.4)

where \( 0 < \rho_c < 1 \). If \( \rho_c = 1 \), the predicted channel is equal to the actual channel and if \( \rho_c = 0 \), the predicted channel is independent of the actual channel. The normalized MSE of the channel prediction can be related with the correlation coefficient \( \rho_c \) by \( \mathbf{e} = 2(1 - \rho_c) \), (Choi & Murch 2003).

The correlation coefficient has also been calculated in the literature (Ramya & Bhashyam 2007) and the authors analyzed the requirement of the length of filters in accordance with the correlation coefficient. Using the fact that channel gains form a Gaussian process, the relationship between the actual channel and the predicted channel can be modelled as in the literature (Ramya & Bhashyam 2007),

\[ \mathbf{H}_{\text{act}} = \rho_c \mathbf{H}_{\text{pred}} + \frac{2}{\sqrt{1 - \rho_c^2}} \mathbf{v} \]  

(4.5)
where $\mathbf{v}$ is the zero mean unit variance Gaussian random variable independent of the channel. From this model, it is evident that given the predicted channel vector $\mathbf{H}_{\text{pred}}$, the actual channel can be modelled to be Gaussian with mean $\mathbf{H}_{\text{pred}}$ and variance $(1 - \rho_c^2)$ i.e., $\mathbf{H}_{\text{act}} = \mathcal{N}(\mathbf{H}_{\text{pred}}; (1 - \rho_c^2))$. This actual channel vector $\mathbf{H}_{\text{act}}$ can be used to design precoding matrix elements.

The optimal precoding matrix for the prediction error corrected CSI can be obtained by replacing $\mathbf{H}_{\text{act}}$ in Equation (3.11) with Equation

$$
\mathbf{G} = \arg\min_{\mathbf{a} \in \mathbb{F}_+} \mathbb{E} \left[ \left\| \mathbf{a} \left( \rho_c \mathbf{H}_{\text{pred}} + \sqrt{1 - \rho_c^2} \mathbf{v} \right) \mathbf{G}x + \mathbf{a}n - \mathbf{x} \right\|^2 \right] \quad (4.6)
$$

$$
\mathbf{G} = \arg\min_{\mathbf{a} \in \mathbb{F}_+} \mathbb{E} \left[ \left\| \mathbf{a} \left( \rho_c \mathbf{H}_{\text{pred}} + \sqrt{1 - \rho_c^2} \mathbf{v} \right) \mathbf{G}x \right\|^2 \right] + \frac{\mathbb{E}(\|n\|^2)}{\mathbb{a}^2} \quad (4.7)
$$

where $\mathbf{H}_{\text{pred}}$ is the predicted CSI and $\rho_c$ is the prediction error coefficient. Since the error due to the estimation cannot be avoided, the transmitter or receiver will have only the estimate of the actual CSI. Hence, the predicted CSI at the receiver with channel estimation error is modelled as $\mathbf{H}_{\text{pred}} = \hat{\mathbf{H}}_{\text{pred}} + \mathbf{E}$, where $\mathbf{E}$ is the channel estimation error matrix with $K \times N_t$ independent elements with zero mean and estimation error variance $\sigma_e^2$. Also, it is assumed that the error matrix $\mathbf{E}$ is independent of the data vector $\mathbf{x}$ and the noise vector $\mathbf{n}$. Further, the optimal precoding matrix $\mathbf{G}$, when $\hat{\mathbf{H}}_{\text{pred}}$ is given, is obtained as,

$$
\mathbf{G} = \arg\min_{\mathbf{a} \in \mathbb{F}_+} \mathbb{E} \left[ \left\| \mathbf{a} \left( \rho_c \hat{\mathbf{H}}_{\text{pred}} + \mathbf{E} + \sqrt{1 - \rho_c^2} \mathbf{V} \right) \mathbf{G}x - \mathbf{x} \right\|^2 + \frac{\mathbb{E}(\|n\|^2)}{\mathbb{a}^2} \right] \quad (4.8)
$$
Since the equality power constraint is assumed in this thesis, \( \operatorname{Tr}(G^H G) = P \) and if we let \( T = a G \), then \( a^2 = \frac{\operatorname{Tr}(T^H T)}{P} \); and now the optimal precoding matrix \( G \) is obtained from

\[
T = \arg \min_{a \in \mathbb{R}^+} \left\{ E \left[ \| (\hat{\rho}_c \bar{A}_{\text{pred}} + E + \frac{2}{\sqrt{1 - \rho_c^2}} V) T x - x \|^2 \mid \hat{A}_{\text{pred}} \right] + \frac{K o^2 \operatorname{Tr}(T^H T)}{P} \right\} \tag{4.9}
\]

The first term of the Equation (4.9) is expanded as follows

\[
E \left[ \| (\hat{\rho}_c \bar{A}_{\text{pred}} + E + \frac{2}{\sqrt{1 - \rho_c^2}} V) T x - x \|^2 \right] = E \left[ (\hat{\rho}_c (\bar{A}_{\text{pred}} + E) + \frac{2}{\sqrt{1 - \rho_c^2}} V) T x - x \right]^H \left( (\bar{A}_{\text{pred}} + E) + \frac{2}{\sqrt{1 - \rho_c^2}} V \right) T x - x \left| \hat{A}_{\text{pred}} \right] \tag{4.10}
\]
\[
E \left[ \left( \rho_c x^H \hat{F}_{\text{pred}}^H \hat{F}_{\text{pred}} \right) + \left( \rho_c x^H \mathbf{E} \right) + \left( \sqrt{1 - \rho_c^2} x^H \hat{F} \mathbf{V} \hat{F}^H \right) - \right.
\]
\[
\left. x^H \left( \rho_c \hat{F}_{\text{pred}} \mathbf{T} x + \rho_c \mathbf{E} \mathbf{T} x + \left( \sqrt{1 - \rho_c^2} \mathbf{V} \mathbf{T} x \right) \right) \right|_{\hat{F}_{\text{pred}}}
\]

On evaluating the Equation (4.11),

\[
= \rho_c^2 \mathbf{Tr} \left( \mathbf{T}^H \hat{F}_{\text{pred}}^H \hat{F}_{\text{pred}}^\top \mathbf{T} \right) + \rho_c^2 \sigma_p^2 \mathbf{Tr} \left( \mathbf{T}^H \mathbf{T} \right) + 2 \rho_c \mathbf{Tr} \left( \hat{F}_{\text{pred}}^H \mathbf{T} \right) + (1 - \rho_c^2) \mathbf{Tr} \left( \mathbf{T}^H \mathbf{T} \right) \mathbf{I}
\]

Now, the optimal precoding matrix \( \mathbf{T}_{\text{opt}} \) is obtained as derived in the chapter 3 and is given as

\[
\mathbf{T}_{\text{opt}} = \left( \rho_c^2 \hat{F}_{\text{pred}}^H \hat{F}_{\text{pred}}^\top + K \rho_c^2 \sigma_p^2 + \mathbf{I} \right)^{-1} \rho_c \hat{F}_{\text{pred}}^H \left( 1 - \rho_c^2 + \frac{K \sigma_p^2}{\mathbf{p}} \mathbf{I} \right)^{-1}
\]

With the equality power constraint, the BS precoding matrix is given by

\[
\mathbf{G}_{\text{opt}} = \sqrt{\mathbf{P}} \mathbf{T}_{\text{opt}} \sqrt{\mathbf{I} - \mathbf{Tr} \left( \mathbf{T}_{\text{opt}}^H \mathbf{T}_{\text{opt}} \right)}
\]

\[4.4 \quad \text{OUTAGE ANALYSIS}\]

Mobility of the vehicle and the feedback delay together often cause the estimated channel outdated. In previous work on transmit beamforming and precoding, the adaptation at the transmitter was done using the predicted channel. Here, the prediction error corrected CSI is used for adaptation. In
this section, the outage analysis is done for predicted CSI precoder and the prediction error corrected CSI precoder.

4.4.1 Outage Analysis for the Predicted CSI Precoder

Yoo & Goldsmith (2006) derived a lower bound of the mutual information between the transmitter and receiver for a given $\hat{H}_{\text{pred}}$ and is given as

$$I(X; Y|\hat{H}_{\text{pred}}, H_{\text{act}}) \geq \log \left( 1 + p_t^2 H_{\text{act}}^H H_{\text{act}} Q / (R_c \sigma_e^2 + \sigma_n^2) \right)$$ (4.15)

where $H_{\text{act}}$ is the actual channel estimate, $Q$ is the input covariance matrix, $p_t$ is the transmit power, $\sigma_e^2$ is the noise variance and $\sigma_n^2$ is the channel estimation error variance. Since the channel is here assumed to be spatially correlated also, the Equation (4.15) is modified as mentioned in the thesis (Ding 2008),

$$I(X; Y|\hat{H}_{\text{pred}}, H_{\text{act}}) \geq \log \left( 1 + P_t H_{\text{act}}^H H_{\text{act}} Q / \text{Tr}(R_t Q)(R_t \sigma_e^2 + \sigma_n^2) \right)$$ (4.16)

The input covariance matrix $Q$ is calculated as $Q = E[(G \times \times G^*)] = \text{Tr}(GG^*)$, where $\text{Tr}(\cdot)$ is the trace of matrix. The mutual information now becomes,

$$I(X; Y|\hat{H}_{\text{pred}}, H_{\text{act}}) \geq$$

$$\log \left( 1 + P_t H_{\text{act}}^H H_{\text{act}} \text{Tr}(GG^*) / \text{Tr}(R_t Q)(R_t \sigma_e^2 + \sigma_n^2) \right)$$ (4.17)

Therefore, the outage probability is

$$P_{\text{out}} = P(\log \left( 1 + P_t H_{\text{act}}^H H_{\text{act}} \text{Tr}(GG^*) / \text{Tr}(R_t Q)(R_t \sigma_e^2 + \sigma_n^2) \right) < R$$ (4.18)
where \( R \) is the rate of transmission. Let 

\[
\alpha = \frac{P_t}{\text{Tr}(R_0Q)(P_0\sigma^2 + \sigma^2)}
\]

\[
P_{\text{cut}} = P\left(\|H_{\text{act}}^*H_{\text{act}}\text{Tr}(GG^*) < \frac{e^{R-1}}{\alpha}\right) = P\left(\|H_{\text{act}}\|^2 < \beta\right) \tag{4.19}
\]

where \( \beta = \frac{e^{R-1}}{\alpha\text{Tr}(GG^*)} \). For the Raleigh fading model with AWGN, \( \hat{H}_{\text{pred}} \) and \( H_{\text{act}} \) are both zero mean and jointly Gaussian, and are related as 

\[
H_{\text{act}} = \rho_c \hat{H}_{\text{pred}} + \sqrt{1 - \rho_c^2}V,
\]

where \( V \sim \mathcal{CN}(0, 1) \) and \( \rho_c \) is the correlation coefficient which is being obtained from 2-D prediction.

Now the outage probability for the given \( \hat{H}_{\text{pred}} \) is

\[
P_{\text{cut}} = P\left(\|\rho \hat{H}_{\text{pred}} + \sqrt{1 - \rho_c^2}V\|^2 < \beta\right) \tag{4.20}
\]

\[
= P\left(\frac{1 - \rho_c^2}{2} \left\| \sqrt{\frac{2\rho_c^2}{1 - \rho_c^2}} \hat{H}_{\text{pred}} + \sqrt{2}V \right\|^2 < \beta\right)
\]

Let 

\[
B = \left\| \sqrt{\frac{2\rho_c^2}{1 - \rho_c^2}} \hat{H}_{\text{pred}} + \sqrt{2}V \right\|^2
\]

and \( \mu = \frac{\rho_c^2}{1 - \rho_c^2} \). Hence, the Equation (4.20) is now written as

\[
P_{\text{cut}} = P\left(B < 2\beta(1 + \mu)\right) \tag{4.21}
\]

Since for a given \( \hat{H}_{\text{pred}} \), the elements of \( \sqrt{2}V \) are complex Gaussian with variance 1 per dimension, \( B \) is non central chi-square distributed with \( 2K \) Degrees of Freedom (DOF). Denoting 

\[
Z = \left\| \hat{H}_{\text{pred}} \right\|^2
\]

and the non centrality parameter \( \delta = 2\mu\left\| \hat{H}_{\text{pred}} \right\|^2 \). Hence, the outage probability for a given \( Z \) is

\[
P(B < 2\beta(1 + \mu)|Z) = F_{\text{nc-}\chi^2,2N_p,\delta}(2\beta(1 + \mu)) \tag{4.22}
\]
where \( F_{nc-X^2,2K,\delta}(\alpha) = \sum_{k=0}^{\infty} \frac{e^{-\delta/2(\delta/2)^k}}{k!} \gamma_k+K(\frac{\alpha}{2}) \) and \( \gamma_k(k) = \int_0^k e^{-y} y^{k-1} dy \) is the Incomplete Gamma function.

The overall probability of outage is given as

\[ P_{\text{out}} = \int_0^\infty F_{nc-X^2,2K,\delta}(2\beta(1+\mu)) f_z(Z) dz \]  \hspace{1cm} (4.23)

where \( f_z(Z) \) is the probability density function (pdf) of \( Z \). Since, \( Z = \tilde{H}_{\text{pred}}^\dagger \tilde{H}_{\text{pred}} \) has the Wishart distribution with pdf

\[ f_z(Z) = \frac{\det(Z)^{K-N-t-1}}{2^{KtN/2} \det(\Sigma)^{tK/2} \pi^{(K-N-t)/2}} \exp\left(\frac{1}{2} \text{tr}(\Sigma^{-1}Z)\right) \]

where \( \Sigma \) is a positive semi definite matrix of size \( N_t \times N_t \), \( |Z| \) is the determinant of \( Z \), \( \text{tr}(\cdot) \) is the trace of a matrix and \( \Gamma \) is the Gamma function. On substituting \( f_z(Z), \delta = 2\mu z \) and \( F_{nc-X^2,2K,\delta}(\alpha) \) in Equation (4.23), the outage probability is

\[ P_{\text{out}} = \int_0^\infty \frac{\det(Z)^{K-N-t-1}}{2^{KtN/2} \det(\Sigma)^{tK/2} \pi^{(K-N-t)/2}} \sum_{j=0}^{\infty} \frac{\mu z^j e^{-\mu z}}{j!} \gamma_{K+j}(\beta(1+\mu)) dz \]  \hspace{1cm} (4.24)

\[ = \sum_{j=0}^{\infty} \frac{\det(Z)^{K-N-t-1}}{2^{KtN/2} \det(\Sigma)^{tK/2} \pi^{(K-N-t)/2}} \int_0^\infty \frac{\mu z^j e^{-\mu z}}{j!} \gamma_{K+j}(\beta(1+\mu)) dz \]  \hspace{1cm} (4.25)

\[ = \sum_{j=0}^{\infty} \left( \frac{\det(Z)^{K-N-t-1}}{2^{KtN/2} \det(\Sigma)^{tK/2} \pi^{(K-N-t)/2}} \right) \gamma_{K+j}(\beta(1+\mu)) \int_0^\infty \frac{\mu z^j e^{-\mu z}}{j!} dz \]  \hspace{1cm} (4.26)

It is a well known fact that \( \int_0^\infty y^j e^{-y} dy = j! \) and if we let \( \mu z = \lambda \) in the integrand of the Equation (4.26), then \( \int_0^\infty \frac{\mu z^j e^{-\mu z}}{j!} dz = \int_0^\infty \frac{\lambda^j e^{-\lambda}}{j!} d\lambda = 1/\mu \).
Now, the Equation (4.26) becomes

\[
P_{\text{out}} = \sum_{j=0}^{\infty} \left( \frac{\det(Z)}{2^{\frac{K-Nt-1}{2}} \pi^{\frac{KNt}{2}} \prod_{i=1}^{Nt} \frac{1}{r^{K+1-1/2}}} \right) \frac{\exp \left( \frac{1}{2} \text{tr}(\Sigma^{-1}Z) \right)}{\mu} \gamma_{K+j}(\beta(1 + \mu)) \tag{4.27}
\]

If the incomplete Gamma function \( \gamma_{K+j}(\beta(1 + \mu)) \) value is substituted in the Equation (4.27), then the overall outage probability is now written by letting \( \alpha = \beta(1 + \mu) \)

\[
P_{\text{out}} = \sum_{j=0}^{\infty} \left( \frac{\det(Z)}{2^{\frac{K-Nt-1}{2}} \pi^{\frac{KNt}{2}} \prod_{i=1}^{Nt} \frac{1}{r^{K+1-1/2}}} \right) \frac{\exp \left( \frac{1}{2} \text{tr}(\Sigma^{-1}Z) \right)}{\mu} \sum_{j=0}^{\infty} \frac{\alpha^{j} \Gamma(K+j-1)}{(K+j-1)!} \] \tag{4.28}

On exchanging integration and summation,

\[
= \int_{0}^{\alpha} \left( \frac{\det(Z)}{2^{\frac{K-Nt-1}{2}} \pi^{\frac{KNt}{2}} \prod_{i=1}^{Nt} \frac{1}{r^{K+1-1/2}}} \right) \frac{\exp \left( \frac{1}{2} \text{tr}(\Sigma^{-1}Z) \right)}{\mu} \sum_{j=0}^{\infty} \frac{\alpha^{j} \Gamma(K+j-1)}{(K+j-1)!} \] \tag{4.29}

Since, \( \sum_{j=0}^{\infty} \frac{\alpha^{j} \Gamma(K+j-1)}{(K+j-1)!} = e^{\alpha} - \sum_{j=0}^{\infty} \frac{\alpha^{j}}{j!} \), Equation (4.29) is simplified as

\[
= \int_{0}^{\alpha} \left( \frac{\det(Z)}{2^{\frac{K-Nt-1}{2}} \pi^{\frac{KNt}{2}} \prod_{i=1}^{Nt} \frac{1}{r^{K+1-1/2}}} \right) \frac{\exp \left( \frac{1}{2} \text{tr}(\Sigma^{-1}Z) \right)}{\mu} \left[ e^{\alpha} - \sum_{j=0}^{\infty} \frac{\alpha^{j}}{j!} \right] \] \tag{4.30}

\[
= \left( \frac{\det(Z)}{2^{\frac{K-Nt-1}{2}} \pi^{\frac{KNt}{2}} \prod_{i=1}^{Nt} \frac{1}{r^{K+1-1/2}}} \right) \left[ \int_{0}^{\alpha} \frac{dy}{\mu} - \sum_{j=0}^{\infty} \frac{\alpha^{j}}{j!} \int_{0}^{\alpha} \frac{e^{-y}}{\mu} dy \right] \tag{4.31}
\]

On simplifying the Equation (4.31), the overall outage probability for the predicted channel is
\[
\begin{align*}
P_{\text{out}} &= \left( \frac{\det(Z)}{(KN)}^{\frac{1}{2}} \exp\left( \frac{1}{2} \text{tr}(\Sigma) \right) \right) \left( \beta(1 + \mu) - \sum_{j=0}^{Nt-2} \gamma_{1+j} \left( \beta(1 + \mu) \right) \right) \\
&= P \left( \log \left( \frac{1 + \frac{P_e}{K}}{\sigma^2 + \sigma_n^2} \right) < R \right)
\end{align*}
\] (4.32)

### 4.4.2 Outage Analysis for the prediction Error Corrected Channel Precoder

The prediction error corrected channel \( \mathbf{H}_{\text{act}} \) obtained from the Equation (4.5) is used for the precoding matrix construction. The outage probability derived for the prediction error corrected channel is as follows,

\[
P_{\text{out}} = P \left( \log \left( \frac{1 + \frac{P_e}{K}}{\sigma^2 + \sigma_n^2} \right) < R \right)
\] (4.33)

By letting \( \alpha = \frac{P_e}{K} \sigma^2 + \sigma_n^2 \), and \( \beta = \frac{e^{R-1}}{\alpha \text{tr}(\mathbf{G}^H \mathbf{G})} \), Equation (4.33) is written as \( P_{\text{out}} = P (||\mathbf{H}_{\text{act}}||^2 < \beta) \). Since, \( ||\mathbf{H}_{\text{act}}||^2 = \sum_{j=1}^{Nt} \sum_{i=1}^{Nc} h_{ij}^2 \), which is chi square distributed with \( 2KN_c \) DOF and the outage probability for a predicted error corrected channel is now given as

\[
P_{\text{out}} = \gamma_{2KN_c}(\beta)
\] (4.34)

### 4.5 RESULTS AND DISCUSSION

In this section, the proposed precoding system, which is designed by prediction error corrected CSI, is compared with the precoding systems in the literature which are designed by predicted CSI. For the case of an urban wireless environment, the receiver is always surrounded by rich scattering object, and hence the channel is assumed to be flat fading with Rayleigh distribution, whose elements are i.i.d. zero mean complex Gaussian random variables with unit variance.
Figure 4.2 Outage Performance of Proposed System

It is assumed that the channel estimation error is known in the BS and it can be appropriately chosen depending on the channel dynamics and channel estimation schemes. Since in this thesis, the channel is assumed to be block Rayleigh fading channel, the estimation error variance is calculated using \( \sigma_c^2 = \frac{1}{1 + (\gamma_t/N_t)} \), where \( T_t \) is the training period, \( N_t \) is the number of transmit antenna and \( \gamma_t \) is the received SNR (Hassibi & Hochwald 2003). Hence, in this thesis, the optimum \( \sigma_c^2 \) values are assumed to be 0.1 and 0.01.

The frame duration and feedback delay is assumed as 1 ms and one frame respectively. The carrier frequency is assumed to be \( f_c = 2.5 \) GHz. The desired rate \( R \) is 2 nats/sec/Hz and the mobile velocity is assumed to have slow and average values of 10 km/h and 30 km/h. The temporal correlation coefficient is calculated from the Bessel formula \( J_0(2\pi f_d T \Delta) \). The Doppler frequency \( f_d \) is easily calculated from the mobile velocity and the carrier frequency.
In this thesis, the best and poor scattering environments are considered and hence the spatial correlation coefficient values are assumed to be $O_t = 0.9$ and $O_t = 0.1$ respectively.

Figure 4.2 depicts the outage performance of the proposed system which compensated the prediction error with the predicted system described in chapter 3 and the simulations are carried out using MATLAB 7.7 software. The channel parameters are assumed as $\sigma^2_e = 0.01$, $\varphi_t = 0.1$ and velocity of the vehicle is 10 Km/h. As shown, the outage performance of the proposed system is better than the predicted channel precoder for all SNRs. The outage probability derived for predicted channel precoder and prediction error corrected channel precoder are validated using the simulations. To attain the outage probability of 0.0002, the predicted system needs 16 dB SNR, while the proposed system needs only 10 dB SNR. Hence, 6 dB SNR gain is achieved with the proposed system.

![Comparison of Outage Performance for Different Velocity](image)

**Figure 4.3 Comparison of Outage Performance for Different Velocity**

Due to mobility of the vehicle, the temporal correlation coefficient of the channel is varying, which in turn varies the prediction error value.
Hence, in Figure 4.3, the probability of outage performance of the proposed system is compared for mobile velocities 10 Km/h and 30 Km/h and the other channel parameters such as channel estimation error variance $\sigma_\varepsilon^2$ and spatial correlation coefficient $\phi_\ell$ are kept constant with values $\sigma_\varepsilon^2 = 0.01$ and $\phi_\ell = 0.1$. If the velocity of the vehicle is decreasing, the temporal correlation coefficient $\rho$ is increasing which reduces MSE of the prediction filter. Hence, it is quite obvious that the outage performance is better when the velocity of the mobile is reduced. The outage performance of the proposed precoder outperforms the predicted channel precoder for all mobile velocities and also for all SNRs.

![Figure 4.4 Outage Performance for Different Error Variance](image)

In the next simulation, the robustness of the proposed system to the imperfect CSI is shown. In Figure 4.4, outage probability simulation is done for two different channel estimation error variance i.e $\sigma_\varepsilon^2 = 0.1$ and $\sigma_\varepsilon^2 = 0.01$ by keeping the velocity and the spatial correlation coefficient are constant with values 10 Km/h and $\phi_\ell = 0.1$ respectively. It is evident from Figure 4.4 that outage performance at high SNR region is better than at the low SNR region for both the values of channel estimation error variances. This is due to the fact that the noise variance $\sigma_n^2$ is high for low SNR values.
and hence the effect of channel estimation error variance is negligible. When the SNR value increases, the noise variance decreases and hence the effect of channel estimation error variance increases which reduces the outage performance at high SNR regime.

![Figure 4.5 Outage Performance for Different Spatial Correlation Coefficient](image)

In Figure 4.5, the outage probability performance of the proposed system is compared for various values of spatial correlation coefficient by assuming velocity = 10 Km/h and $\sigma^2_e = 0.01$. If the spatial correlation coefficient is high i.e $\theta_t = 0.9$, the outage performance is deteriorated when compared with the outage performance of the system with spatial correlation coefficient $\theta_t = 0.1$. From the figures 4.3, 4.4 and 4.5, it can be easily observed that the outage performance is degraded when any one of the channel parameters is assumed to be in the poor channel condition. The following table gives the effect of interdependency between the velocity, error variance and spatial correlation for SNR = 10dB.

The following observations are noted when one of the channel parameters is assumed to be in poor channel condition and other two
parameters are in good condition with SNR = 10 dB.

By keeping channel estimation error variance and spatial correlation coefficient are in good condition ($\sigma^2_{\phi} = 0.01$ and $\varphi_t = 0.1$) and the change in velocity of the vehicle from 30 Km/h to 10 Km/h gives 10% improvement in terms of outage probability. When channel estimation error variance and velocity of the vehicles are kept constant ($\sigma^2_{\phi} = 0.01$ and velocity = 10Km/h), the change in spatial correlation coefficient value from 0.9 to 0.1 provides 7.5% improvement in the outage performance. If spatial correlation coefficient and velocity of the vehicle are kept constant ($\varphi_t = 0.1$ and velocity = 10Km/h) and the channel estimation error variance is varied from 0.1 to 0.01, then the improvement of 8% is obtained. Hence, if there is a change in any one of the channel parameters, there will be a significant change in the outage performance.

In the following discussions, the BER performance of the Prediction error corrected CSI precoding system is compared with the predicted CSI precoding system. The simulation parameters assumed for the outage analysis is assumed here.

![Figure 4.6 BER Performance of Proposed System](image-url)
The BER performance of the predicted system and the proposed system is simulated in Figure 4.6. In this figure, it is evident that the proposed system outperforms the predicted system. Hence, if the prediction error is compensated, the BER performance is also better than the predicted system and SNR gain of 6 dB is achieved to attain the BER of 0.001. In this simulation, the velocity of the vehicle is assumed to be 10 km/h and error variance is $\sigma_e^2 = 0.01$ and $\varphi_t = 0.1$. The BER performance of the proposed precoding system for the mobile velocity of 10 km/h and 30 km/h is compared in Figure 4.7 by keeping $\sigma_e^2 = 0.01$ and $\varphi_t = 0.1$ constant. It is also observed that the BER performance of the prediction error corrected channel precoder is deteriorated when the velocity of the mobile unit is increased. This is due to the fact that if velocity increases, the channel varies rapidly and hence the prediction error is high.

![Figure 4.7 BER Performances for Different Velocity](image)

The BER performance of the proposed precoder which compensated the prediction error is compared for different spatial correlation coefficient values $\varphi_t = 0.9$ and $\varphi_t = 0.1$, with the constant values of velocity 10 Km/h and channel estimation error variance $\sigma_e^2 = 0.01$, in Figure
4.8. It is quite obvious that the system with low spatial correlation coefficient outperforms the system with high spatial correlation coefficient.

![Graph showing BER performances for different spatial correlation coefficients](image)

**Figure 4.8 BER Performances for Different Spatial Correlation Coefficient**

![Graph showing BER performance for different error variances](image)

**Figure 4.9 BER Performance for Different Error Variance**

Figure 4.9 describes the BER performance of the proposed precoding system for the channel estimation variance $\sigma_0^2 = 0.1$ and $\sigma_0^2 = 0.01$ by assuming the mobile velocity as 10km/h and the spatial correlation...
coefficient $\phi_{t}$ as 0.1. Here also it is seen that the BER performance is better when the channel estimation error is low. Also, the ceiling effect is seen where BER of all the precoding techniques flattens for high SNR and does not improve by increasing the SNR. The ceiling effect occurs at high SNR regime in Figures 4.6, 4.7, 4.8 & 4.9 because of the channel estimation error. Thus, the results suggest that the proposed system provides possible improvement in the BER performance and the outage performance.

4.6 SUMMARY

In this chapter, the problem of linear precoder for MIMO wireless communication systems is addressed, when there are prediction error and outdated CSI, due to mobility of the vehicle. MMSE based linear precoder for prediction error corrected channel is designed. The two dimensional Wiener filter has been used for prediction and the temporal correlation coefficient is calculated from the NMSE obtained. Using this correlation coefficient the actual channel can be modelled and hence the precoder is designed. The outage probability has been derived for the linear precoding system with predicted channel and the prediction error corrected channel. The BER performance as well as the outage performance of the prediction error corrected channel precoder is found better than the precoder designed using predicted channel. Hence, this proposed concept can be applied for slow time varying channels in a MIMO wireless communication system to have better performance. The performance can be improved if the channel estimation is perfect.