CHAPTER 3

MMSE LINEAR PRECODER WITH IMPERFECT CSI

3.1 INTRODUCTION

Previously, Minimum Mean Square Error linear precoder for various wireless communication systems was designed by considering perfect CSI at the transmitter side. But in practice, the channel estimation errors which result during the channel estimation process and antenna (spatial) correlation are inevitable. Further, CSI available at the BS is often outdated, due to mobility of the receiver and the feedback delay. Hence, if temporal correlation coefficient is known at the BS, current CSI can be updated. To improve the performance of the system, future channel is predicted using two dimensional (2-D) MMSE prediction. In this chapter, the proposed MMSE linear precoder is designed by considering the imperfect CSI and 2-D predicted CSI.

3.2 LITERATURE REVIEW

The process of shaping the transmitted signal based on the knowledge of the CSIT is known as precoding. Nonlinear precoding techniques have been shown to offer superior performance although they complicate the receiver and the transmitter, since a modulo operation (which in general depends on the CSI) has to be implemented at both sides of the communication link (Fischer et al. 2003, Peel et al. 2005 & Windpassinger 2004). The linear precoding schemes have low complex transmitter and
receiver structures, but typically have performance degradation with respect to non-linear schemes. Previous work on linear and non-linear precoding schemes assume that perfect CSI is available at the transmitter. Thus in practice, in the channel estimation process, the errors are inevitable. This erroneous CSI at the transmitter degrades the performance, due to the difference between the true channel and the erroneous channel used for designing a precoder. Hence, it is necessary to design a precoder to incorporate the channel estimation error.

The impact of channel estimation errors on the performance of wireless communication systems is analyzed in (Vakili et al. 2006). There have been more practical designs that consider imperfect CSIT. Zero forcing linear precoder is proposed by Jindal (2006) in conjunction with finite rate feedback. Each mobile in the system performs vector quantization on its channel realization using random quantization code books. Two different models of partial CSI such as shape feedback model (normalized channel vector is available at the BS) and limited feedback model (each user quantizes its channel vector according to the code book) are considered in (Ding et al. 2007) for the robust design of precoders. The effects of channel estimation error on sum rate capacity of multiple antenna broadcast channels are studied by Ding et al. (2005). Under an i.i.d Rayleigh flat fading assumption, an achievable lower bound and cooperative upper bound for sum rate capacity have been derived to arrive at sum rate loss. The channel estimation error was considered in the uplink-downlink duality analysis (Ding & Blostein 2007). Khaled et al. (2004) has studied minimum total MSE design with outdated CSIT and perfect CSIR. Zhang et al. (2005&2006) assumed that CSIT is the channel mean information (CMI) and/or the channel correlation information (CCI), whereas the receiver has perfect CSI. The imperfect CSI is assumed at both ends in the thesis (Palomar 2003), but the authors have not considered channel correlation. Zhang et al. (2008) have
derived the closed-form robust designs (including minimum total MSE design) by assuming that imperfect CSI, including channel mean and receive correlation information, is available at both ends.

The effect of non idealities such as low rank channels and channel estimation error variance on the antenna selection in MIMO systems has been studied by (Molisch 2003). Narasimhamurthy & Tepedelenlioglu (2009) have studied the space time frequency coded MIMO OFDM system employing receiver antenna selection in the presence channel estimation error. The effect of channel estimation on antenna subset diversity (SSD) selection has been studied and performance loss of non-ideal SSD and ideal SSD was compared in the literature (Gifford 2008). Gucluoglu & Panayirci (2008), have analyzed the effect of channel estimation error on performance of space time coded systems with transmit and receive antenna selection over quasi static flat fading channel.

Dabbagh & Love (2008) have proposed the MMSE linear precoder design by considering imperfect CSI at the BS, in which the channel estimation error variance has been incorporated in the system design for multi user wireless communication system. The authors proved that the linear MMSE precoder designed by considering channel estimation error outperforms the regularized channel inversion precoder.

The effect of feedback delay on MIMO systems employing rate adaptation has been presented in the literature (Ramya & Srikrishna Bhashyam 2010). Annapureddy et al. (2009) have analyzed outage probability and symbol error rate of transmit antenna selection in the presence of feedback delay. The feedback delay and channel estimation errors are considered for an antenna selection system for some specific scenarios in the literature (Ramya & Srikrishna Bhashyam 2009). Kobayashi et al. (2006) proposed MMSE precoder with mode selection for MIMO systems to reduce
performance degradation due to feedback delay. Yadav et al. (2014) designed linear precoder to maximize the cut off rate expression for the doubly correlated partially coherent MIMO channel.

In the practical downlink systems, the mobile is often surrounded by the local scatterers and channels from different antennas tend to be uncorrelated, whereas the channels from different base station antennas are often correlated due to limited scattering. Yu et al. (2009) designed mean feedback precoder and covariance feedback precoder by considering channel estimation error variance, and antenna correlation. It is also well known that due to mobility of the vehicle and time varying nature of the channel, the CSI available at the BS has to be updated. Hence, it is necessary to predict future CSI. In the literature (Castro et al. 2008), the robust MMSE linear precoder for MU-MIMO system is designed using predicted CSI which is spatially and temporally correlated. In this chapter, two Dimensional (2-D) MMSE prediction proposed by Wong & Evans (2006) is used to find future CSI and the predicted CSI is used in MMSE precoder.

Hence, in this chapter, the imperfect CSI parameters such as channel estimation error variance, feedback delay and the spatial correlation are considered in the design of linear MMSE precoder. Further, linear precoding matrix is derived for the predicted CSI.

The structure of this chapter is organized as follows. The system model is presented in section 3.3. In section 3.4, the new MMSE based precoder is designed. The 2-D predicted MMSE precoder design is discussed in section 3.5. The performance analysis is derived in section 3.6. The simulation results are discussed in section 3.7, and section 3.8 concludes the chapter.
3.3 SYSTEM MODEL

Linear precoding is the linear transformation of data sequence before transmission. Figure 3.1 shows the general block diagram of linear precoding in downlink MIMO wireless communication system. Let $H$ be the flat fading channel with complex Gaussian i.i.d elements, with $N_t$ number of transmit antennas and $K$ number of users. The input-output system model for flat fading MIMO channel is given by $y = HTx + n$, where $y$ is the received signal vector of size $K \times 1$, $G$ is the precoder matrix of size $N_t \times B$, the number of information symbols is $B \leq \min(N_t, K)$, the transmitted vector of size $B \times 1$ is $x$, and $n(K \times 1)$ is additive white Gaussian noise vector. The vectors $x$ and $n$ are assumed to be independent. The constraint for the transmit energy is given as $E[Trx(Trx)^H] = Tr(Tr^H) = P_t$, where $P_t$ is the total transmit power. In this system model, the CSI available at the BS is assumed to be imperfect (i.e), the BS is assumed to have the information about channel estimation error, antenna correlation and temporal correlation.

![Figure 3.1 Linear Precoding System Model](image)

3.3.1 Channel Estimation Error

The channel estimation is performed on $H_w$ using the well-established orthogonal training method described in the literature (Musavian
et al. 2007, Hassibi & Hochwald 2003, Yoo et al. 2004). However, practical systems seldom have perfect channel information. The error due to estimation cannot be neglected even though the number of pilot symbols is increased. Now, consider the situation, where $\mathbf{H}_w$ is imperfectly known to the receiver, i.e., the receiver is provided with some partial information $\hat{\mathbf{H}}_w$ of channel with which the receiver performs MMSE estimation of $\mathbf{H}_w$. It is assumed that $\mathbf{H}_w$ and $\hat{\mathbf{H}}_w$ are jointly ergodic and stationary Gaussian process while the entries of $\hat{\mathbf{H}}_w$ are independent. Therefore, perfect CSI is not even available at the receivers and erroneous CSI is sent to the transmitter.

The MMSE estimation is defined as

$$\hat{\mathbf{H}}_w = \mathbf{E}(\mathbf{H}_w | \hat{\mathbf{H}}_w)$$

and the estimation error is

$$\mathbf{E}_w = \mathbf{H}_w - \hat{\mathbf{H}}_w$$

By property of estimation, $\hat{\mathbf{H}}_w$ and $\mathbf{E}_w$ are uncorrelated and entries of $\mathbf{E}_w$ are zero mean complex Gaussian (ZMCG) with variance

$$\sigma^2_c = \mathbf{E} \left[ \left[ |[\mathbf{H}_w]_{ij}|^2 - |[\hat{\mathbf{H}}_w]_{ij}|^2 \right] \right]$$

(3.3)

where $|[\mathbf{H}_w]_{ij}|$ and $|[\hat{\mathbf{H}}_w]_{ij}|$ represent the $(i,j)_{th}$ elements of $\mathbf{H}_w$ and $\hat{\mathbf{H}}_w$ respectively. The entries of $\hat{\mathbf{H}}_w$ are also i.i.d ZMCG with variance $1 - \sigma^2_c$. The parameter $\sigma^2_c$ captures the quality of estimation and is assumed to be known to both the transmitter and the receiver. However, since the focus of this chapter is to study the effect of channel estimation error, it is assumed that $\hat{\mathbf{H}}_w$ and $\sigma^2_c$ are known a priori.
3.3.2 Spatial Channel Correlation

MU-MIMO system provides significant capacity gain over an SISO system with the assumption that the channel coefficients are independent and identically distributed. Actually, this assumption does not always hold in practical situations. The rich scatterers in the propagation environment provide the i.i.d. channel coefficients. However, in many realistic environments, signal correlation among the antenna elements exists due to poor scattering conditions which may degrade the system performance significantly. This has given a motivation for analyzing MIMO systems in correlated fading environments.

When there is a spatial correlation, the channel model commonly used in the literature (Shiu et al. 2000 & Forenza et al. 2007) is Kronecker model and is rewritten here from the Equation (2.7)

\[
H = R_r^{1/2}H_wR_t^{1/2}
\]  

(3.4)

where \( R_r(K \times K) \) and \( R_t(N_t \times N_t) \) are receive and transmit correlation matrices respectively, and are calculated as in the Equation (2.4).

In the practical downlink channels, the mobile is likely to be surrounded by a large number of local scatterers. However, the BS antennas are located at highly sufficient elevation to limit scattering and thus channels arising from the downlink transmit antennas are highly correlated. Therefore, \( H = H_wR_t^{1/2} \) as mentioned in the literature (Gershman & Sidiropoulos 2005).

3.3.3 Temporal Channel Correlation

In a wireless communication system, the transmitter can acquire channel information indirectly. The CSI based on the channel estimates at the
receiver can be obtained either by using reciprocity principle or by using feedback. In both cases, there exists a delay, such as a scheduling or a feedback delay, between obtaining the channel information and using the channel information by the transmitter. The time varying nature of the wireless channel and the feedback delay together lead to a different channel at the time of transmission than the channel that is fed back. The channel exhibits the temporal correlation

\[
\rho = \frac{E[H_n H_{n+1}]}{\sqrt{E[H_n^2]E[H_{n+1}^2]}} = I_0(2\pi f_d T \Delta)
\]

(3.5)

modelled by the Bessel function, where \(\rho\) is the temporal correlation coefficient, \(H_n\) and \(H_{n+1}\) are the channel estimates at time \(n\) and \(n + 1\) respectively, \(I_0(x)\) is the zero\(^{th}\) order Bessel function, \(f_d\) is Doppler frequency, \(\Delta\) is feedback delay in number of frames, \(T\) is the frame duration and \(0 \leq \rho \leq 1\). Due to feedback delay, the precoding matrix generated at any instance is based on outdated CSI available at the BS. Using Gaussian channel vector model, the delay in the feedback is captured by the correlation co-efficient \(\rho\) between old channel and the actual channel. In the literature (Ramya & Srikrishna Bhashyam 2009) the current channel is related to the past channel as

\[
H_{\text{new}} = \rho H + \sqrt{1 - \rho^2} W
\]

(3.6)

where \(H\) is past CSI at the BS, \(H_{\text{new}}\) is current CSI, \(W\) is \(K \times N_t\) matrix, independent of \(H\), whose elements are i.i.d with \(\mathbb{C} \mathcal{N}(0,1)\).

### 3.3.4 Problem Formulation

Let \(\mathbf{x}\) be a \(K \times 1\) data vector consisting of \(K\) independent streams of data with i.i.d zero mean, unit covariance matrix and uncorrelated with
noise. The data vector is then fed into $N_t \times K$ precoder matrix $G$ and the precoded data vector is transmitted to $K$th user through a slowly time varying Rayleigh flat fading channel. The channel between the transmitter and the user $k$ is represented by the vector $h_k$ of size $1 \times N_t$ and the channel matrix $H$ of size $K \times N_t$ is written as $H = [h_1^H h_2^H \ldots h_K^H]^H$

Incorporating channel estimation error, spatial correlation among the channel taps and the temporal correlation due to feedback delay, the channel model $H_{\text{new}}$ is described as

$$H_{\text{new}} = \rho (\hat{H} + E) + \sqrt{1 - \rho^2} W$$  \hspace{1cm} (3.7)

where $H_{\text{new}}$ is the current channel and $H$ is the past channel matrix, and is given as $H = H_w R_{t}^{\frac{1}{2}} = (\hat{H}_w + E_w) R_{t}^{\frac{1}{2}} = \hat{H} + E$, $\hat{H} = \hat{H}_w R_{t}^{\frac{1}{2}}$ is the estimated channel matrix and $E = E_w R_{t}^{\frac{1}{2}}$ is the estimation error matrix. It is assumed that the channel estimation errors are independent of data and channel noises as assumed in the literature (Ding & Blostein 2007 and Dabbagh & Love 2008). The received signal vector at $K$ receivers is given by

$$y = H_{\text{new}} G x + n$$  \hspace{1cm} (3.8)

where $H_{\text{new}}$ is the current channel matrix and $n$ is AWGN with distribution $\text{CN}(0, \sigma^2 I_K)$. Using the CSI model in Equation (3.7), the received signal vector is now written as

$$y = \left( \rho (\hat{H} + E_w R_{t}^{\frac{1}{2}}) + \sqrt{1 - \rho^2} W \right) G x + n$$  \hspace{1cm} (3.9)

With this channel model assumption, the transmitter precoding matrix is designed based on the knowledge of estimated channel matrix $\hat{H}$. Due to transmit power limitation, the transmit power constraint states that the total
average transmit energy per symbol after transformed by precoding matrix $G$ is equal to $P$ and is expressed as $E[x^H G^H G x] = \text{Tr}(G^H G) \leq P$, where $P$ is the total transmit power. Given the outdated estimated channel matrix $\hat{H}$, the precoding matrix $G$ at the BS is designed so that the MSE signal at different users’ receivers is minimized. In this system model, a simple receiver structure is assumed, in which each receive antenna consists of a non-zero gain ‘$a$’ common to all received data stream. This common gain does not change the mutual information, since it acts both on signal and noise and interference. In fact, each user can have its own gain at the receiver. The assumption of the common gain is just a means to find a better transmit processing. The problem statement can now be expressed as

$$G_{opt} = \arg \min_{\begin{array}{c} \text{Tr}(G^H G) \leq P \\ a \in \mathbb{R}_+ \end{array}} \mathbb{E}[\|aH_{new}Gx + an - x\|^2 | \hat{H}]$$  \hspace{1cm} (3.10)

where $\|.|^2$ is the Euclidean norm.

### 3.4 Precoder Design

In this section, the linear MMSE precoder is designed by considering perfect CSI and imperfect CSI.

#### 3.4.1 Precoder Design with Perfect CSI

In this section, the precoder design for the perfect CSI is derived. Since the data vector $x$ is assumed to be i.i.d., zero mean complex random variables with unit variance and uncorrelated with the noise, the Equation (3.10) now can be rewritten as,

$$G = \arg \min_{\begin{array}{c} \text{Tr}(G^H G) \leq P \\ a \in \mathbb{R}_+ \end{array}} \mathbb{E}[\|aH_{act}Gx + an - x\|^2]$$  \hspace{1cm} (3.11)
\[
G = \arg\min_{\mathbf{G} \in \mathbb{C}^{N \times N}, \text{Tr}(\mathbf{G}^\dagger \mathbf{G}) \leq P} \left\{ \mathbb{E}[\| \mathbf{aH}_{\text{act}} \mathbf{G} \mathbf{x} - \mathbf{x} \|^2] + a^2 \mathbb{E}[\| \mathbf{n} \|^2] \right\}
\]

(3.12)

where \( \mathbf{G} \) denotes precoding matrix and \( \mathbf{H}_{\text{act}} \) denotes the perfect CSI.

It is assumed to have equality power constraint so that \( \text{Tr}(\mathbf{G}^\dagger \mathbf{G}) = P \) and if we let \( \mathbf{T} = a \mathbf{G} \), then \( a^2 = \frac{\text{Tr}(\mathbf{T}^\dagger \mathbf{T})}{P} \) and now the optimal precoding matrix \( \mathbf{G} \) is obtained from

\[
\mathbf{T} = \arg\min_{\mathbf{T} \in \mathbb{C}^{N \times N}, \text{Tr}(\mathbf{T}^\dagger \mathbf{T}) \leq a^2 P} \left\{ \mathbb{E}[\| \mathbf{H}_{\text{act}} \mathbf{T} \mathbf{x} - \mathbf{x} \|^2] + \frac{K_0 a^2 \text{Tr}(\mathbf{T}^\dagger \mathbf{T})}{P} \right\}
\]

(3.13)

where \( \sigma^2 \) is the noise variance. The first part of the Equation (3.13) is now derived as follows,

\[
\mathbb{E}[\| \mathbf{H}_{\text{act}} \mathbf{T} \mathbf{x} - \mathbf{x} \|^2] = \mathbb{E} \left[ (\mathbf{H}_{\text{act}} \mathbf{T} \mathbf{x} - \mathbf{x})^\dagger (\mathbf{H}_{\text{act}} \mathbf{T} \mathbf{x} - \mathbf{x}) \right]
\]

\[
= \mathbb{E} \left[ \mathbf{x}^\dagger (\mathbf{H}_{\text{act}} \mathbf{T} - I)^\dagger (\mathbf{H}_{\text{act}} \mathbf{T} - I) \mathbf{x} \right]
\]

\[
= \text{Tr} \left( (\mathbf{H}_{\text{act}} \mathbf{T} - I)^\dagger (\mathbf{H}_{\text{act}} \mathbf{T} - I) \right)
\]

\[
= \text{Tr} \left( \mathbf{T}^\dagger \mathbf{H}_{\text{act}} \mathbf{H}_{\text{act}}^\dagger \mathbf{T} \right) - \text{Tr} \left( \mathbf{T}^\dagger \mathbf{H}_{\text{act}} \mathbf{H}_{\text{act}}^\dagger \right) - \text{Tr} \left( \mathbf{H}_{\text{act}} \mathbf{T} \right) + I
\]

(3.14)

On substituting the Equation (3.14) in the Equation (3.13) the optimal precoding matrix can be obtained from

\[
\mathbf{T} = \arg\min_{\mathbf{T} \in \mathbb{C}^{N \times N}, \text{Tr}(\mathbf{T}^\dagger \mathbf{T}) \leq a^2 P} \left\{ \text{Tr} \left( \mathbf{T}^\dagger \mathbf{H}_{\text{act}} \mathbf{H}_{\text{act}}^\dagger \mathbf{T} \right) - \text{Tr} \left( \mathbf{T}^\dagger \mathbf{H}_{\text{act}} \mathbf{H}_{\text{act}}^\dagger \right) - \text{Tr}(\mathbf{H}_{\text{act}} \mathbf{T}) + I + \frac{K_0 a^2 \text{Tr}(\mathbf{T}^\dagger \mathbf{T})}{P} \right\}
\]

(3.15)
In order to find the optimal precoding matrix $T$, the gradient of the Equation (3.15) with respect to the optimal precoding matrix $T$ is computed and is given as

$$
\nabla_T \left\{ \text{Tr} \left( T^H H_{\text{act}}^H H_{\text{act}} T \right) - \text{Tr} \left( T^H H_{\text{act}}^H \right) - \text{Tr} (H_{\text{act}} T) + \frac{K \sigma^2 \text{Tr}(T^H T)}{p} \right\}
$$

$$
= T^H H_{\text{act}}^H H_{\text{act}} - H_{\text{act}}^H + \frac{K \sigma^2 T^H}{p} = 0
$$

$$
T = \left( H_{\text{act}}^H H_{\text{act}} + \frac{K \sigma^2}{p} I \right)^{-1} H_{\text{act}}^H
$$

(3.16)

With the equality power constraint, the BS precoding matrix is given by

$$
G = \frac{p}{\sqrt{\text{trace}(T^H T)}} T = \left( H_{\text{act}}^H H_{\text{act}} + \frac{K \sigma^2}{p} I \right)^{-1} H_{\text{act}}^H
$$

(3.17)

### 3.4.2 Precoder Design with Imperfect CSI

In this section, the optimal precoding matrix is derived to minimize MSE objective function, given the outdated estimated channel matrix $\hat{H}$ at the BS. The modified MMSE based optimization criteria used in the literature (Choi & Murch 2003) is considered here. Hence, the optimization problem now becomes the unconstrained optimization one, which can be solved without the need of forming the Lagrangian function. Thus, the optimization criteria proposed in (3.10) is now given as,

$$
G_{\text{opt}} = \arg\min_{\text{Tr}(G^H G) \leq P, a \in \mathbb{R}_+} \mathbb{E} \left[ \left\| a \left( \rho (\hat{H} + E_w R_{\ell}^{-1}) \right) + \sqrt{1 - \rho^2} W \right) G x + an - x \right\|^2 \mid \hat{H}
$$

(3.18)
where, $a \in \mathbb{R}_+$ is an optimization variable which corresponds to adaptive gain control variable at different users receivers and is introduced in this system to simplify the receiver structure. In order to simplify the solutions, it is assumed that they are identical across the users. The noise vector $\mathbf{n}$ is independent of the data vector $\mathbf{x}$, and the channel estimate $\mathbf{\hat{H}}$. Hence, the Equation (3.18) can be simplified as

$$
\mathbf{G}_{\text{opt}} = \underset{a \in \mathbb{R}_+}{\text{arg min}} \mathbb{E} \left[ \| a \left( r \mathbf{\hat{H}} + \mathbf{E}_w \mathbf{R}_W \right) \mathbf{G} \mathbf{x} - \mathbf{x} \|^2 | \mathbf{\hat{H}} \right] + a^2 \mathbb{E}(\| \mathbf{n} \|^2)
$$

(3.19)

Since the equality power constraint is assumed in this work, $\text{Tr}(\mathbf{G}^H \mathbf{G}) = P$, and it can be obtained as $\frac{\text{Tr}(\mathbf{G}^H \mathbf{G})}{P} = 1$. Therefore, the Equation (3.19) is equivalent to

$$
\mathbf{G}_{\text{opt}} = \underset{\text{Tr}(\mathbf{G}^H \mathbf{G}) = P \quad a \in \mathbb{R}_+}{\text{arg min}} \mathbb{E} \left[ \| a \left( r \mathbf{\hat{H}} + \mathbf{E}_w \mathbf{R}_W \right) \mathbf{G} \mathbf{x} - \mathbf{x} \|^2 | \mathbf{\hat{H}} \right] + a^2 \frac{\text{Tr}(\mathbf{G}^H \mathbf{G})}{P} \mathbb{E}(\| \mathbf{n} \|^2)
$$

(3.20)

which can be simplified further as

$$
\mathbf{G}_{\text{opt}} = \underset{\text{Tr}(\mathbf{G}^H \mathbf{G}) = P \quad a \in \mathbb{R}_+}{\text{arg min}} \mathbb{E} \left[ \| a \left( r \mathbf{\hat{H}} + \mathbf{E}_w \mathbf{R}_W \right) \mathbf{G} \mathbf{x} - \mathbf{x} \|^2 | \mathbf{\hat{H}} \right] + a^2 \frac{\text{Tr}(\mathbf{G}^H \mathbf{G})}{P} \mathbb{E}(\| \mathbf{n} \|^2)
$$

(3.21)

By setting $\mathbf{T} = a \mathbf{G}$ and $(\| \mathbf{n} \|^2) = \sigma^2$, the Equation (3.21) is equivalent to
$$T_{opt} = \arg\min_{T^H(\tilde{T}) \leq \sigma^2 \rho, \ a \in \mathbb{R}_+} \left( E \left[ \left\| \left( \rho \left( \hat{H} + E_w R_t^{1/2} \right) + \sqrt{1 - \rho^2 W} \right) T x - x \right\|^2 | \hat{H} \right] + \frac{k \sigma^2 T^H(\tilde{T})}{p} \right)$$

(3.22)

The first part of the Equation (3.22) is now derived as follows,

$$E \left[ \left\| \left( \rho \left( \hat{H} + E_w R_t^{1/2} \right) + \sqrt{1 - \rho^2 W} \right) T x - x \right\|^2 | \hat{H} \right]$$

$$= E \left[ \left( \rho \left( \hat{H} + E_w R_t^{1/2} \right) + \sqrt{1 - \rho^2 W} \right) T x - x \right]^H \left( \rho \left( \hat{H} + E_w R_t^{1/2} \right) + \sqrt{1 - \rho^2 W} \right) T x - x \right]$$

$$= E \left[ \left( \rho x^H T^H \hat{H}^H \right) + \left( \rho x^H T^H R_t^{1/2} E_w^H \right) + \left( \sqrt{1 - \rho^2} x^H T^H W^H \right) - x^H \right] \left( \rho \hat{H} T x + \left( \rho E_w R_t^{1/2} T x + \sqrt{1 - \rho^2} W T x - x \right) \right]$$

(3.24)

On expanding the Equation (3.24),

$$= \rho^2 E[x^H T^H \hat{H}^H \hat{H} T x] + \rho^2 E[x^H T^H \hat{H}^H E_w R_t^{1/2} T x] +$$

$$\rho \sqrt{1 - \rho^2} E[x^H T^H \hat{H}^H W T x] - \rho E[x^H T^H \hat{H}^H x] +$$

$$\rho^2 E[x^H T^H R_t^{1/2} E_w^H \hat{H} T x] + \rho^2 E[x^H T^H R_t^{1/2} E_w^H E_w R_t^{1/2} T x] +$$

$$\rho \sqrt{1 - \rho^2} E[x^H T^H R_t^{1/2} E_w^H W T x] - \rho E[x^H T^H R_t^{1/2} E_w^H x] +$$

$$\rho \sqrt{1 - \rho^2} E[x^H T^H W^H \hat{H} T x] + \rho \sqrt{1 - \rho^2} E[x^H T^H W^H E_w R_t^{1/2} T x] +$$

$$(1 - \rho^2) E[x^H T^H W^H W T x] - \sqrt{1 - \rho^2} E[x^H T^H W^H x] - \rho E[x^H \hat{H} T x] -$$

$$\rho E[x^H E_w R_t^{1/2} T x] - \rho \sqrt{1 - \rho^2} E[x^H W T x] + E[x^H x]$$

(3.25)
The following assumptions have been made to solve the Equation (3.25).

i. The data vector \( \mathbf{x} \) is assumed to have independent streams of data with zero mean and unit covariance matrix. Hence, \( E[\mathbf{x}^H \mathbf{x}] = \mathbf{I_K} \).

ii. The channel estimate \( \hat{\mathbf{H}} \) and the estimation error matrix \( \mathbf{E_w} \) are independent. Hence, \( E[\hat{\mathbf{H}}^H \mathbf{E_w}] = 0 \).

iii. \( \mathbf{H}_{\text{act}} = \rho \mathbf{H} + \sqrt{1 - \rho^2} \mathbf{W} \) where \( \mathbf{W} \) is \( \text{CN} (0, \mathbf{I_K}) \) and independent of \( \mathbf{H} \). Hence, channel estimate \( \hat{\mathbf{H}} \) and the random matrix \( \mathbf{W} \) are independent. Hence, \( E[\hat{\mathbf{H}}^H \mathbf{W}] = 0 \).

iv. It is assumed that the entries of the channel estimation error matrix \( \mathbf{E_w} \) are i.i.d with \( \text{CN}(0, \sigma_c^2) \) and independent of data and channel noises. Hence, \( E[\mathbf{E_w}^H \mathbf{E_w}] = K \sigma_c^2 \mathbf{I_K} \) Therefore,

\[
\rho^2 E[\mathbf{x}^H \mathbf{T}^H \mathbf{R}_t^H \mathbf{E_w}^H \mathbf{E_w} \mathbf{R}_t^2 \mathbf{T} \mathbf{x}] = \\
\rho^2 E[\mathbf{x}^H \mathbf{T}^H \mathbf{R}_t^H \mathbf{E_w}^H \mathbf{E_w} \mathbf{R}_t^2 \mathbf{T} \mathbf{x}] = K \rho^2 \sigma_c^2 \text{tr}(\mathbf{T}^H \mathbf{R}_t \mathbf{T})
\]

v. Since the channel estimation error matrix is assumed to be zero mean \( E[\mathbf{E_w}] = 0 \).

vi. The entries of the random matrix \( \mathbf{W} \) defined in the para 3 of section 2 is assumed to be zero mean and unit variance. Hence, \( E[\mathbf{W}] = 0 \) and \( E[\mathbf{W}^H \mathbf{W}] = \mathbf{I_K} \).

After incorporating the assumptions (i) to (vi), the Equation (3.25) is now solved as
\[ \begin{align*}
&= \rho^2 E[x^H T^H \hat{H}^H H^H x] + \rho^2 E[x^H T^H R_t^H E_w^H E_w R_t x] - \rho E[x^H T^H \hat{H}^H x] - \\
&\rho E[x^H \hat{H} T x] + (1 - \rho^2) E[x^H T^H W^H W^H T x] + E[x^H x] \\
&= \rho^2 \text{Tr}(T^H \hat{H}^H \hat{H} T) + K \rho^2 \sigma_c^2 \text{Tr}(T^H R_t T) I - \rho \text{Tr}(\hat{H} T) - \rho \text{Tr}(T^H \hat{H}^H) + \\
&(1 - \rho^2) \text{Tr}(T^H T) + I_K \\
&= \rho^2 \text{Tr}(T^H \hat{H}^H \hat{H} T) + K \rho^2 \sigma_c^2 \text{Tr}(T^H R_t T) I - \rho \text{Tr}(\hat{H} T) - \rho \text{Tr}(T^H \hat{H}^H) + \\
&(1 - \rho^2) \text{Tr}(T^H T) + I_K + K \frac{\sigma^2 \text{Tr}(T^H T)}{p}
\end{align*} \]

(3.26)

On substituting the Equation (3.27) in the Equation (3.22), the optimization function becomes,

\[ \begin{align*}
T_{\text{opt}} &= \mathop{\text{arg min}}_{\text{Tr}(T^H T) = 0, \, \sigma \in \mathbb{R}^+} \left( \rho^2 \text{Tr}(T^H \hat{H}^H \hat{H} T) + K \rho^2 \sigma_c^2 \text{Tr}(T^H R_t T) I - \rho \text{Tr}(\hat{H} T) - \rho \text{Tr}(T^H \hat{H}^H) + \\
&(1 - \rho^2) \text{Tr}(T^H T) + I_K + K \frac{\sigma^2 \text{Tr}(T^H T)}{p} \right)
\end{align*} \]

(3.28)

To find the optimal precoding matrix \( T_{\text{opt}} \), the gradient of the Equation (3.28) with respect to the transmit precoding matrix \( T \) is computed and is given as

\[ \begin{align*}
\nabla_T \left\{ \rho^2 \text{Tr}(T^H \hat{H}^H \hat{H} T) + K \rho^2 \sigma_c^2 \text{Tr}(T^H R_t T) I - \rho \text{Tr}(\hat{H} T) - \rho \text{Tr}(T^H \hat{H}^H) + \\
&(1 - \rho^2) \text{Tr}(T^H T) + I_K + K \frac{\sigma^2 \text{Tr}(T^H T)}{p} \right\} = \rho^2 T^H \hat{H}^H \hat{H} T + K \sigma^2 \sigma_c^2 T^H R_t - \rho \hat{H} + (1 - \rho^2) T^H + K \frac{\sigma^2 T^H}{p} = 0
\end{align*} \]

(3.29)

The following facts are used to obtain (3.29):

(i) For the complex valued, \( \nabla_X \text{Tr}(AXB) = BA \), \( \nabla_X \text{Tr}(AX^H B) = 0 \), where \( X \) and \( X^H \) are treated as independent variable referred in the literature (Kay 1993 and Sampath et al 2001) and
(ii) Given an arbitrary matrix $\Lambda$ and the matrix variable, $\nabla_X \text{Tr}(AX) = \nabla_X \text{Tr}(XA) = \Lambda$. Hence, to find the gradient of (3.28) with respect to $T$, $T^H$ is treated as an independent matrix variable. Finally, the optimal precoding matrix $T_{opt}$ is determined as

$$T_{opt} = \left( \rho^2 \hat{H}^H \hat{H} + K \sigma^2 \mathbf{R}_t + (1 - \rho^2) + K \sigma^2 / P \right) I^{-1} \rho \hat{H}^H$$

(3.30)

With the equality power constraint, the BS precoding matrix is given by

$$G_{opt} = \sqrt{P} T_{opt} / \sqrt{\text{Tr}(T_{opt}^H T_{opt})}$$

(3.31)

When there is no feedback delay, $\rho = 1$, and when there is no spatial correlation among the channels, $\mathbf{R}_t = \mathbf{I}_K$, the optimal precoding matrix $G_{opt}$ exactly matches the solution in the literature (Dabbagh & Love 2008).

3.5 PRECODER DESIGN WITH 2-D PREDICTION

Due to mobility of the vehicle and the time varying nature of the channel, the current CSI available in the BS is outdated. Hence, to improve the performance of the system, the future CSI is predicted by considering the current and $L-1$ previous estimates of CSI. The channel prediction algorithms for MIMO channel have been studied extensively in the literature (Zhou & Giannakis 2004 and Luo et al. 2004). However, these approaches are appealing when the channel responses at each antenna element in the transmit and receive arrays are uncorrelated. The linear prediction for the correlated MIMO channel using SVD of channel matrix has been proposed in the literature (Gillaud & Slock 2004) and is computationally complex. In this thesis, since, both the spatial and temporal correlation of the channel are
considered, the two dimensional (2-D) MMSE prediction filter is used for predicting the future CSI (Wong & Evans 2006).

![Diagram](image)

**Figure 3.2 MMSE Precoder with 2-D Prediction**

Due to the time varying nature of the channel, the channel model described in section 3.34 at time ‘n’ is represented as $H_{\text{new}}(n) = [(h_{\text{new}}(n))_1^H (h_{\text{new}}(n))_2^H \ldots (h_{\text{new}}(n))_K^H]^H$ where $(h_{\text{new}}(n))_j$ is the $1 \times N_t$ channel vector of $j^{th}$ user at time $n$. Let the vectorized version of the MIMO channel matrix be $\hat{h}(n) = \text{vec}(H_{\text{new}}(n))$, where, $\text{vec}(\cdot)$ is the operator which stacks the columns of its arguments one on top of the other into a vertical vector making $\hat{h}(n)$ the $KN_t \times 1$ vector. Let $\bar{h}$ be the vector which contains the current and $L - 1$ previous channel vector $\hat{h}(n)$ where $L$ is the number of delay vectors to be processed.

$$\bar{h} = [\hat{h}(n) \hat{h}(n-\Delta) \hat{h}(n-2\Delta) \ldots \hat{h}(n-(L-1)\Delta)]^H$$ (3.32)
where $\Delta$ is the sample duration. Now, the predicted channel vector $\bar{h}(n + \Delta)$ of size $KN_t \times 1$ is given as

$$\bar{h}(n + \Delta) = W_{2D}\bar{h}$$

(3.33)

where $W_{2D} = (r_t^T \otimes R_s)(R_{tx}^T \otimes R_s + \sigma_e^2I)^{-1}$ is the 2-D MMSE filter of size $KN_t \times LKN_t$, $r_t$ is the $L \times 1$ vector of correlation values and is given as $r_t^T = [r_t(\Delta) ... r_t(L\Delta)]$, $R_{tx}$ is the Hermitian symmetric and Toeplitz $L \times L$ temporal autocorrelation matrix, and is given as $R_{tx}(i,j) = r_t((i-j)\Delta)r_t(\Delta) = E[[H_w(n + \Delta)]_{ij}[H_w(n)]_{ij}]$ $\forall i, j$ where $[.]_{ij}$ indicates the $(i,j)$th component of the matrix argument and $R_s = R_t^T \otimes R_t$ is the spatial autocorrelation matrix. $R_t$ is the transmit correlation matrix of size $N_t \times N_t$ with entries $R_{t(i,j)} = O_t|i-j|$ and $R_r$ is the receive correlation matrix of size $K \times K$ with entries $R_{r(i,j)} = O_r|i-j|$ where $O_t$ and $O_r$ are the transmit and receive correlation coefficients respectively. Since no receive correlation is assumed in this thesis, the receive correlation matrix $R_r$ is made equal to the identity matrix $I_K$. The computation of $W_{2D}$ requires $O(N^3L^2)$ operation, where $N = KN_t$ (Castro et al. 2008).

The predicted channel vector of size $KN_t \times 1$ obtained from (3.33) is represented as $\bar{h}(n + \Delta) = [(h_{pred})_1(h_{pred})_2 ... (h_{pred})_K]^H$ and is rearranged to form the $K \times N_t$ predicted channel matrix $H_{pred} = [(h_{pred})_1^H(h_{pred})_2^H ... (h_{pred})_K^H]^H$. The predicted CSI $H_{pred}$ is sent through the feedback channel. Since the degradation due to feedback delay can be mitigated by the use of channel prediction, the predicted CSI $H_{pred}$ is used to generate precoding matrix. The MMSE precoding matrix is now generated by considering the predicted channel $H_{pred}$. Hence, to reduce the effect of feedback delay and mobility of vehicle, the predicted channel matrix
$H_{\text{pred}}$ is used in optimal precoding matrix generation. Therefore, the precoding matrix generated in (3.30) is rewritten by replacing $\hat{H}$ by $H_{\text{pred}}$.

$$T_{\text{opt}} = \left( \rho^2 H_{\text{pred}} H_{\text{pred}}^H + (K \rho^2 \sigma^2 R_t + (1 - \rho^2) + K \sigma^2 / P)I \right)^{-1} \rho \quad (3.34)$$

Since the MMSE precoder at the base station incorporates all the above said three parameters in the system design, the following two signalling overheads have to be considered. In case of TDD system, the BS has to compute the velocity of the mobile, feedback delay and the channel estimates. In case of FDD system, such information has to be transmitted to the BS through the feedback channel. Since the channel is spatially and temporally correlated, the prediction filtering operation itself requires $O(N^2 L)$, which is also quite complex.

### 3.6 PERFORMANCE ANALYSIS

In this section, the average SINR at the different users’ receivers is computed. The $j^{\text{th}}$ row of the matrix $H_{\text{pred}}$ denotes the channel vector corresponding to the $j^{\text{th}}$ user and is denoted as $(h_{\text{pred}})_j$. The received signal of the $j^{\text{th}}$ user is written as

$$y_j = (h_{\text{pred}})_j g_j x_j + (h_{\text{pred}})_j \sum_{k=1} \mathbf{g}_k x_k + \eta_j \quad (3.35)$$

where $g_j$ denotes the $j^{\text{th}}$ column vector of $G_{\text{opt}}$, $x_j$ is the symbol intended for user $j$ and $\eta_j$ is the noise of the $j^{\text{th}}$ user. Given the estimated channel matrix $\hat{H}$ conditioned on $e_w$, the $j^{\text{th}}$ user SINR is given as

$$\text{SINR}_j(e_w) = \frac{E\left[ \left\| (h_{\text{pred}})_j g_j x_j \right\|^2 \right]}{E\left[ \left\| (h_{\text{pred}})_j \sum_{k=1} \mathbf{g}_k x_k + \eta_j \right\|^2 \right]} \quad (3.36)$$
where $e_w$ is the vector of channel estimation error matrix $E_w$. The conditional expectation in the Equation (3.36) is evaluated and is given as

$$\text{SINR}_j(e_w) = \frac{\| \langle h_{\text{pred}} \rangle \theta_j \|^2}{\| \sum_{k \neq j} \langle h_{\text{pred}} \rangle \theta_k \|^2 + \sigma^2}$$  \hspace{1cm} (3.37)

It is well known from the optimization criteria (3.31) that,

$$g_j = \sqrt{\rho} t_j / \sqrt{\text{Tr}(T_{\text{opt}} H T_{\text{opt}})}$$ \hspace{1cm} (3.38)

where $t_j$ is the $j^{th}$ column vector of $T_{\text{opt}}$. On substituting the Equation (3.31) in the Equation (3.30) and let $\gamma = \frac{\rho}{\sigma^2}$ be the SNR, the SINR expression now becomes

$$\text{SINR}_j(e_w) = \frac{\gamma \| \langle h_{\text{pred}} \rangle \theta_j \|^2}{\gamma \| \sum_{k \neq j} \langle h_{\text{pred}} \rangle \theta_k \|^2 + \text{Tr}(T_{\text{opt}} H T_{\text{opt}})}$$ \hspace{1cm} (3.39)

At high SNR regime, when $\gamma \to \infty$, the Equation (3.39) is determined to be

$$\lim_{\gamma \to \infty} \text{SINR}_j(e_w) = \lim_{\gamma \to \infty} \frac{\gamma \| \langle h_{\text{pred}} \rangle \theta_j \|^2}{\gamma \| \sum_{k \neq j} \langle h_{\text{pred}} \rangle \theta_k \|^2 + \text{Tr}(T_{\text{opt}} H T_{\text{opt}})}$$ \hspace{1cm} (3.40)

$$= \frac{\| \langle h_{\text{pred}} \rangle \theta_{\text{inf}} \|^2}{\| \sum_{k \neq j} \langle h_{\text{pred}} \rangle \theta_{\text{inf}} \|^2}$$  \hspace{1cm} (3.41)

where $t_{\text{inf}}$ is the $j^{th}$ column vector of $T_{\text{opt}}$ at infinite SNR.

$$\lim_{\gamma \to \infty} T_{\text{opt}} = \left( \rho^2 H_{\text{pred}} H_{\text{pred}} + \left( K \rho^2 \sigma_t^2 \mathbf{R}_t + (1 - \rho^2) \mathbf{I} \right) \right)^{-1} \rho H_{\text{pred}}$$
which is independent of the SNR. Hence, the SINR of the \( j \)th user at infinite SNR is given as

\[
\text{SINR}_{j\text{inf}} = \mathcal{E} \left[ \frac{\left\| (\mathbf{h}_\text{pred})_j \mathbf{t}_{j\text{inf}} \right\|^2}{\left\| \sum_k x_k (\mathbf{h}_\text{pred})_k \mathbf{t}_{k\text{inf}} \right\|^2 | \mathbf{H} \} \right]
\]  

(3.42)

The SINR at infinite SNR in Equation (3.42) is a function of channel estimation error variance \( \sigma_e^2 \), the spatial and temporal correlation coefficients \( \mathbf{R}_t \) and \( \rho \) respectively and is independent of the SNR. Hence, if \( \mathbf{R}_t, \rho \) and \( \sigma_e^2 \) are fixed, the SINR of each user is upper bounded by a fixed value that does not depend on SNR.

### 3.7 SIMULATION RESULTS

In this section, the simulation results are discussed. In this simulation, a system with four transmit antennas at the BS and four users each with single receive antenna, is considered and the system employs QPSK modulation. It is assumed that the channel estimation error is known in the BS and it can be appropriately chosen depending on the channel dynamics and channel estimation schemes. Since in this thesis, the channel is assumed to be a block Rayleigh fading channel, the estimation error variance is calculated using \( \sigma_e^2 = \frac{1}{1 + (\mathbf{R}_t / N_t) \gamma_t} \), where \( \mathbf{T}_t \) is the training period, \( N_t \) is the number of transmit antenna and \( \gamma_t \) is the received SNR (Hassibi & Hochwald 2003). Hence, in this thesis, the optimum \( \sigma_e^2 \) values are assumed to be 0.1 and 0.01.

If the feedback delay is known, the temporal correlation coefficient \( \rho \) is easily calculated from \( \rho = \int_0 (2\pi f_t T \Delta) \). Throughout this simulation, the frame duration of 2 ms and one frame delay is assumed. The Doppler frequency is assumed to be 15 Hz and 50 Hz. Hence, the temporal correlation coefficient will have the values \( \rho = 0.9 \) and \( \rho = 0.99 \) respectively. In order
to validate the software, the black box testing is used to check whether the expected output data is obtained or not, for a given set of sample data in the design of proposed system.

It is also assumed that the transmit and receive correlation coefficients $O_t$ and $O_r$, whose values always lie between 0 (un correlation) and 1 (full correlation), are known at the BS. In this thesis, the average and poor scattering environments are considered and hence the spatial correlation coefficient values are assumed to be $O_t = 0.5$ and $O_t = 0.1$ respectively. The transmit and receive correlation matrices are defined as $R_{t(i,j)} = O_t|i - j|$ and $R_{r(i,j)} = O_r|i - j|$ respectively. The simulation is done using MATLAB with 100 symbols and the results are averaged over 1000 channel realizations.

For Minimum Mean Square Error design, the relevant performance measure is the Bit Error Rate (BER). If the CSI is assumed to be perfect, SINR calculated using Equation (3.39) is equal to SNR ($\gamma$). However, if the imperfect CSI is available at the transmitter side, the value of SINR is reduced by the factor $\|((h_i)t_i)\|^2$. The improvement in SINR is obtained if the future CSI is predicted. If the prediction is perfect, then the value of SINR is improved and hence the BER improvement is obtained.

In Figure 3.3, the Bit Error Rate (BER) performance vs Signal to Noise Ratio (SNR) is compared for the existing system (which considers only the channel estimation error variance in the system design) proposed by Dabagh & Love (2008), the proposed system (which includes channel estimation error, feedback delay, and spatial correlation coefficient) and the proposed system with the predicted CSI. In this simulation, it is assumed that $\sigma_e^2 = 0.1$, $\rho = 0.9$ and $O_t = 0.5$. It is easily observed from the Figure 3.3 that the proposed system outperforms the existing system in the high SNR
regime and the proposed system with predicted CSI outperforms all the systems in the high SNR regime. The proposed system with prediction is more robust than the proposed system without prediction and existing system. In low SNR regime, the effect of noise in the precoding matrix nullifies the effect of channel estimation error, feedback delay and the spatial correlation and hence the precoding matrix is equivalent to regularized channel inversion. Further, as discussed in section 3.6, the ceiling effect is noted in all the precoding systems i.e. the average BER flattens for high SNR. The BER of the proposed system with prediction and without prediction at 25 dB SNR is 0.005 and 0.01 respectively. Hence, 50% performance improvement is obtained with the proposed system with prediction when compared to system without prediction. In the next simulation, as shown in Figure 3.4, the values assumed are $\sigma_e^2 = 0.01, \rho = 0.9$ and $\Omega_t = 0.5$. If the channel estimation error is reduced to 0.01, the BER performance is improved in all the precoding systems. Hence, it is evident that the better channel estimation results better BER performance.

![Figure 3.3 BER Performance of MMSE Precoding with $\sigma_e^2 = 0.1, \rho = 0.9$ & $\Omega_t = 0.5$](image-url)
Figure 3.5 compares the BER performance for the values $\sigma_e^2 = 0.01, \rho = 0.99$ and $O_t = 0.5$. It is observed from Figure 3.5 that the BER performance can further be improved if the temporal correlation is high ($\rho = 0.99$). In addition, it can be noted that there would be negligible performance difference between the existing system and the proposed system. This is an interesting fact, because, for the low channel estimation error variance and high temporal correlation, the optimal precoding matrix value derived in the Equation (3.30) is almost equal to the existing precoding matrix value which is derived in the literature (Dabbagh & Love 2008). Since the correlation between the current CSI and the future CSI is high, the prediction of future CSI is better. Hence, there is a significant BER performance improvement in this simulation.

![Figure 3.4 BER Performance of MMSE Precoding with $\sigma_e^2 = 0.01$, $\rho = 0.9$ & $O_t = 0.5$](image-url)
Figure 3.5  BER Performance of MMSE Precoding with $\sigma_e^2 = 0.01$, $\rho = 0.99$ & $\Omega_t = 0.5$

Figure 3.6  BER Performance of MMSE Precoding with $\sigma_e^2 = 0.1$, $\rho = 0.9$ & $\Omega_t = 0.9$

The simulation result shown in Figure 3.6 depicts the poor scattering environment with the worst case scenario. When compared to Figure 3.3, Figure 3.6 describes the effect of spatial correlation in the precoder performance. The BER performance is drastically reduced if the
transmit antenna correlation is high \( \rho_t = 0.9 \). Around 6 dB loss is incurred when the spatial correlation coefficient value increases from 0.5 to 0.9. Hence to attain the desirable performance, the proposed system with prediction is preferable. Figure 3.7 depicts the BER performance of all the precoders if \( 6 \times 4 \) MIMO system is used with the channel parameters \( \sigma_e^2 = 0.1, \rho = 0.9 \) and \( \rho_t = 0.5 \) as assumed in the simulation of Figure 3.3. It is observed from the above simulations that the proposed system with prediction outperforms the other two systems at all scenarios. Also, the performances of all the precoders can be improved by increasing the number of transmit antennas. In Figure 3.8, the BER performance of the proposed system with and without prediction for different transmit correlation coefficients by having constant values of channel estimation error variance and the temporal correlation coefficient is compared. Here, it is assumed that \( \sigma_e^2 = 0.01, \rho = 0.9 \). The BER performance comparison is studied for \( \rho_t = 0.5 \) and 0.9. If \( \rho_t = 0.5 \) i.e. the spatial correlation between the transmit antenna is less, then the BER performance is better than the system with \( \rho_t = 0.9 \). The effect of transmit correlation is reduced by predicting the future CSI which is evidenced in Figure 3.8. Since the noise is dominant in the different user receivers, the effect of change in the spatial correlation is zero in the low SNR regime.

Similarly, in Figure 3.9, the BER performance of the proposed system with and without prediction is compared for different temporal correlation coefficients such as \( \rho = 0.9 \) and \( \rho = 0.99 \). Here, the channel estimation error variance and the transmit correlation coefficient are kept constant and are assumed as \( \sigma_e^2 = 0.01 \) and \( \rho_t = 0.5 \). It is easily observed from Figure 3.9 that the proposed system with prediction outperforms the system without prediction.
Figure 3.7  BER Performance of MMSE Precoding with $\sigma_c^2 = 0.1$, $
\rho = 0.9$, $O_t = 0.5$ & $Nt = 6$.

Figure 3.10 depicts the BER comparison for different channel estimation error variance by keeping $O_t = 0.5$ & $\rho = 0.99$. Here again, it is quite obvious that the system with prediction outperforms the system without prediction.

Figure 3.8  BER Performance Comparison for Different Transmit Antenna Correlation
Figure 3.9 BER Performance Comparison for Different Temporal Correlation

Figure 3.10 BER Performance Comparison for Different Channel Estimation Error Variance

It is evident from Figure 3.10 that the ceiling effect is reduced when the predicted CSI is used to construct the precoding matrix. Hence, from the
above simulations, it is observed that the effect of feedback delay, channel estimation error variance and spatial correlation are reduced by incorporating all these parameters in the precoder design and by predicting the future CSI.

3.8 SUMMARY

The downlink MMSE precoder is designed by considering the channel estimation error variance, spatial correlation and feedback delay as the integral parts of the system design. Further, the future CSI is predicted using 2-D MMSE predictor. Then, the predicted CSI is used in the proposed precoder design. The proposed system with predicted CSI outperforms the previously existing precoders in the multi user scenario. The results fit those in the literature which considered the channel estimation error variance only. It is observed that the channel estimation error causes an error floor at high SNR and large performance degradation over entire SNR range. Similarly, high spatial correlation and feedback delay cause large impact on the system performance. The effect of feedback delay is compensated with the help of two dimensional prediction. The individual effect of channel estimation error, temporal correlation and spatial correlation on the system performance is also observed.