9.1 INTRODUCTION
Taguchi methods have become increasingly popular in the recent years. The sizable quality improvements that resulted from implementations of these methods have added curiosity among most of the American manufacturers. Taguchi methods belong to the class of approaches that attempt to ensure quality through design, in this case through the identification and control of critical variables (or noises) that cause deviations to occur in the process/product quality. Taguchi methods, developed by Dr. Genichi Taguchi, refer to techniques of quality engineering that embody both statistical process control (SPC) and new quality related management techniques. Most of the attention and discussion on Taguchi methods has been focused on the statistical aspects of the procedure; it is the conceptual framework of a methodology for quality improvement and process robustness that needs to be emphasized. The entire concept can be described in two basic ideas:

1. Quality should be measured by the deviation from a specified target value, rather than by conformance to preset tolerance limits
2. Quality cannot be ensured through inspection and rework, but must be built in through the appropriate design of the process and product. Taguchi methods emphasize the attainment of the specified target value and the elimination of variation. The main objective of this chapter is optimizing the tablet hardness by using the concept of Taguchi. This often had been one of the major problem during production process with the tablets not meeting the hardness specifications and finally resulting in lot of scrap and rework. The concept of Orthogonal Array has been used in this paper for identifying the critical factors that affect the response variable significantly. Here in this chapter the concept of noise (i.e.-uncontrollable variability) is taken into consideration since this variability is difficult to control, the product is designed in such a way that they are robust to noise factor. Once the critical factors have been identified, the final formulation can be defined by optimizing the levels of all critical factors to achieve the fixed goal. This chapter presents a systematic approach of optimizing the tablet hardness by using the concept of taguchi's S/N ratio in an orthogonal array design. This study confirms the efficiency and effectiveness of using design of experiments and other statistical tools for tablet hardness optimization. In order to have less number of tablets with tablet hardness lower than its target value during the manufacturing process an optimization process was obtained by using the concept of design of experiments and taguchi's
signal to noise ratio. In this chapter the concept of tablet hardness optimization has been explained by considering a hypothetical example. Here Signal is being referred to as Mean whereas Noise is being referred to as Variation.

9.2 APPLICATION

Let us consider a case of Pharmaceutical Industry where tablets of 100mg are being manufactured during the production process. The objective of this chapter is optimizing the tablet hardness that is being produced during the production process by taking into consideration the controllable and uncontrollable factors. Here in this experiment we have considered 7 controllable factors i.e. (A, B, C, D, E, F, and G) and 3 Noise (uncontrollable) factors i.e. (M, N, and P). Each of the factors mentioned above occurs at 2-levels, so the total number of trials to be carried out for controllable factors comes out to be eight whereas for noise factors the number of trials comes out to be four. Symbolically it is represented as OA_{e}(2^7) and OA_{4}(2^3). The design so formed is called a Orthogonal Array which was first introduced by Plackett and Burman in 1946 and the main purpose of this type of design was to study the effect of main factors. The Plackett and Burman designs are designs suitable for studying up to \( k = (N-1)/(L-1) \) factors each with \( L \) levels with the expense of \( N \) trials, that is, orthogonal arrays of the form \( OA_{N}(L^k) \).
The Orthogonal Array chart is as follows:

<table>
<thead>
<tr>
<th>Orthogonal Array</th>
<th>Number of Rows</th>
<th>Maximum Number of Factors</th>
<th>Maximum Number of Columns at These Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>L4</td>
<td>4</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>L8</td>
<td>8</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>L9</td>
<td>9</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>L12</td>
<td>12</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>L16</td>
<td>16</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>L32</td>
<td>16</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>L36</td>
<td>18</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>L25</td>
<td>25</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>L27</td>
<td>27</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>L32</td>
<td>32</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>L36</td>
<td>36</td>
<td>23</td>
<td>23</td>
</tr>
<tr>
<td>L50</td>
<td>36</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>L54</td>
<td>54</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>L64</td>
<td>64</td>
<td>26</td>
<td>26</td>
</tr>
<tr>
<td>L81</td>
<td>81</td>
<td>40</td>
<td>40</td>
</tr>
</tbody>
</table>

In this experiment 8 trials are run for the 7 controllable factors whereas 4 trials are run for 3 noise factors, as can be seen from the above OA chart. The four trials of the outer array represented four combinations of the noise factors, which were simulated at each of the 8 trials of the inner array. Four repetitions thus of each experimental trial took place simulating the noise space. The response values represent the tablet hardness measured in (kp) and these are shown in table.
Table-1: [Factors and their Levels]

[Inner Array]

<table>
<thead>
<tr>
<th>Levels</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Low</td>
</tr>
<tr>
<td>B</td>
<td>Low</td>
</tr>
<tr>
<td>C</td>
<td>Low</td>
</tr>
<tr>
<td>D</td>
<td>Low</td>
</tr>
<tr>
<td>E</td>
<td>Low</td>
</tr>
<tr>
<td>F</td>
<td>Low</td>
</tr>
<tr>
<td>G</td>
<td>Low</td>
</tr>
</tbody>
</table>

[Outer Array]

<table>
<thead>
<tr>
<th>Levels</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>Low</td>
</tr>
<tr>
<td>N</td>
<td>Low</td>
</tr>
<tr>
<td>P</td>
<td>Low</td>
</tr>
</tbody>
</table>

Table:2 [Orthogonal Layout for Inner and Outer Array]

OA₈(2⁷)  [Inner Array]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

OA₄(2³)  [Outer Array]

<table>
<thead>
<tr>
<th>M</th>
<th>N</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>
The objective of a robust design is to find the controllable process parameter settings for which noise or variation has a minimal effect on the product's or process's functional characteristics. It is to be noted that the aim is not to find the parameter settings for the uncontrollable noise variables, but the controllable design variables. To attain this objective, the control parameters, also known as inner array variables and for each experiment of the inner array, a series of new experiments are conducted by varying the level settings of the uncontrollable noise variables, also known as outer array variables. The level combinations of noise variables are done using the outer orthogonal array.

Table:3 [Experimental Setup and Data for Case-Study]

<table>
<thead>
<tr>
<th>Trial</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>Tablet Hardness (kp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>7.3 6.5 6.8 7.5</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>6.5 6.7 6.7 6.8</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>7.2 7.2 7.9 7.3</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>6.9 7.6 7.3 6.4</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>6.5 6.9 6.4 6.3</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>7.6 6.8 7.9 7.2</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>7.0 7.3 7.7 6.7</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>8.0 7.8 8.0 7.7</td>
</tr>
</tbody>
</table>
Here S/N Ratio of the above data is computed by using the below mentioned formula:

\[ S/N = 10\log_{10}(\frac{\mu^2}{\sigma^2}) \]

Table: 4 [Computation of S/N Ratio]

<table>
<thead>
<tr>
<th>Trial</th>
<th>Rep-1</th>
<th>Rep-2</th>
<th>Rep-3</th>
<th>Rep-4</th>
<th>( \mu )</th>
<th>( \mu^2 )</th>
<th>( \sigma^2 )</th>
<th>S/N Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.3</td>
<td>6.5</td>
<td>6.8</td>
<td>7.5</td>
<td>7.03</td>
<td>49.35</td>
<td>0.2092</td>
<td>23.73</td>
</tr>
<tr>
<td>2</td>
<td>6.5</td>
<td>6.7</td>
<td>6.7</td>
<td>6.8</td>
<td>6.68</td>
<td>44.56</td>
<td>0.0158</td>
<td>34.49</td>
</tr>
<tr>
<td>3</td>
<td>7.2</td>
<td>7.2</td>
<td>7.9</td>
<td>7.3</td>
<td>7.40</td>
<td>54.76</td>
<td>0.1133</td>
<td>26.84</td>
</tr>
<tr>
<td>4</td>
<td>6.9</td>
<td>7.6</td>
<td>7.3</td>
<td>6.4</td>
<td>7.05</td>
<td>49.70</td>
<td>0.2700</td>
<td>22.65</td>
</tr>
<tr>
<td>5</td>
<td>6.5</td>
<td>6.9</td>
<td>6.4</td>
<td>6.3</td>
<td>6.53</td>
<td>42.58</td>
<td>0.0692</td>
<td>27.89</td>
</tr>
<tr>
<td>6</td>
<td>7.6</td>
<td>6.8</td>
<td>7.9</td>
<td>7.2</td>
<td>7.38</td>
<td>54.39</td>
<td>0.2292</td>
<td>23.75</td>
</tr>
<tr>
<td>7</td>
<td>7.0</td>
<td>7.3</td>
<td>7.7</td>
<td>6.7</td>
<td>7.18</td>
<td>51.48</td>
<td>0.1825</td>
<td>24.50</td>
</tr>
<tr>
<td>8</td>
<td>8.0</td>
<td>7.8</td>
<td>8.0</td>
<td>7.7</td>
<td>7.88</td>
<td>62.02</td>
<td>0.0225</td>
<td>34.40</td>
</tr>
</tbody>
</table>

(Rep: Replication)

We find that from the above table trial -2 is the optimum trial with least variation in it. \textbf{Taguchi refers Signal to Noise ratio as Noise Performance Measure.} Using the data from the experimental results we calculate the following terms:

S/N Ratio, Level Average, Grand Average and Level Effects.
### Table 5: Computation of Level Average, Grand Average, Level Effects, and Total Effects

<table>
<thead>
<tr>
<th>Inner Array [Controllable Factors]</th>
<th>Outer Array [Noise Factors]</th>
<th>Tablet Hardness (kp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trial</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>---------</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>LA1</td>
<td>26.92</td>
<td>27.46</td>
</tr>
<tr>
<td>LA2</td>
<td>27.63</td>
<td>27.10</td>
</tr>
<tr>
<td>GA</td>
<td>27.28</td>
<td>27.28</td>
</tr>
<tr>
<td>LE1</td>
<td>-0.355</td>
<td>0.184</td>
</tr>
<tr>
<td>LE2</td>
<td>0.355</td>
<td>-0.184</td>
</tr>
<tr>
<td>TE</td>
<td>-0.710</td>
<td>0.367</td>
</tr>
</tbody>
</table>

Where LA1 = Level Average of Factors at level-1 and
LA2 = Level Average of Factors at level-2
GA = Grand Average
LE1 = Level Effect of Factors at level-1
LE2 = Level Effect of Factors at level-2
TE = Total Effect.
9.3 Definitions:

**Computation of Grand Average:** The grand average is calculated by adding the S/N ratio from each control factor run and dividing the total number of runs.

\[
\text{Grand Average} = \frac{\text{Sum of Individual Observations}}{\text{Total Number of Observations}}
\]

**Computation of Level Average:** The level mean is calculated by summarizing the observations at each level of treatment and dividing by the number of observations in that level.

\[
\text{Level Mean} = \frac{\sum (\text{All Observations at Level } i)}{\text{Number of Observations at Level } i}
\]

**Computation of Level Effect:** The level effect is calculated by subtracting the grand average from the level mean.

\[
\text{Level Effects} = \text{Level Mean} - \text{Grand Average}
\]

**Computation of Total Effect:** The total effect is calculated by subtracting the least level mean from the greatest level mean.

\[
\text{Total Effects} = \text{Greatest Level Mean} - \text{Least Level Mean}
\]
### Table:6 [S/N Ratio Table]

<table>
<thead>
<tr>
<th>Trials</th>
<th>S/N Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23.73</td>
</tr>
<tr>
<td>2</td>
<td>34.49</td>
</tr>
<tr>
<td>3</td>
<td>26.84</td>
</tr>
<tr>
<td>4</td>
<td>22.65</td>
</tr>
<tr>
<td>5</td>
<td>27.89</td>
</tr>
<tr>
<td>6</td>
<td>23.75</td>
</tr>
<tr>
<td>7</td>
<td>24.50</td>
</tr>
<tr>
<td>8</td>
<td>34.40</td>
</tr>
</tbody>
</table>

Now from the above table-6 we compute the average effect of each factor.

### Table:7 [Average Effect of Factors]

<table>
<thead>
<tr>
<th></th>
<th>26.93</th>
<th>27.18</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(1)</td>
<td>26.93</td>
<td>E(1)</td>
</tr>
<tr>
<td>A(2)</td>
<td>27.64</td>
<td>E(2)</td>
</tr>
<tr>
<td>B(1)</td>
<td>27.47</td>
<td>F(1)</td>
</tr>
<tr>
<td>B(2)</td>
<td>27.10</td>
<td>F(2)</td>
</tr>
<tr>
<td>C(1)</td>
<td>29.28</td>
<td>G(1)</td>
</tr>
<tr>
<td>C(2)</td>
<td>25.28</td>
<td>G(2)</td>
</tr>
</tbody>
</table>

Here A(1) and A(2) are the two levels of factor A. Now computing the effect summary of the factors as shown in table-7.

**Effect Summary of factor-A = (27.64 - 26.93)/2 = 0.3551 (Using table-7)**
The negative sign indicates that as the level of factor increases from low to high the mean S/N Ratio decreases. With the help of table-5 and table-6 we find out the S/N Ratio for each of the factors.

Table:9 [S/N Ratio of the Factors]

<table>
<thead>
<tr>
<th>Factors</th>
<th>A(1)</th>
<th>A(2)</th>
<th>E(1)</th>
<th>E(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(1)</td>
<td>107.71</td>
<td>110.55</td>
<td>108.73</td>
<td>109.54</td>
</tr>
<tr>
<td>A(2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B(1)</td>
<td>109.87</td>
<td>108.40</td>
<td>108.67</td>
<td>109.59</td>
</tr>
<tr>
<td>B(2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(1)</td>
<td>117.13</td>
<td></td>
<td>94.64</td>
<td></td>
</tr>
<tr>
<td>C(2)</td>
<td>101.14</td>
<td></td>
<td>123.63</td>
<td></td>
</tr>
<tr>
<td>D(1)</td>
<td>102.97</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D(2)</td>
<td>115.30</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now we make use of above data as shown in table-9 for computation of ANOVA to find out which of the factors are showing significant effects and which of them are not.
Table:10 [ANOVA Table]

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>d.f</th>
<th>S.S</th>
<th>M.S.</th>
<th>F-Ratio</th>
<th>.p-value</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>1.009</td>
<td>1.009</td>
<td></td>
<td>**</td>
<td>**</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>0.270</td>
<td>0.270</td>
<td></td>
<td>**</td>
<td>**</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>31.963</td>
<td>31.963</td>
<td>87.160</td>
<td>0.0007</td>
<td>Significant</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>19.019</td>
<td>19.019</td>
<td>51.862</td>
<td>0.0020</td>
<td>Significant</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
<td>0.083</td>
<td>0.083</td>
<td></td>
<td>**</td>
<td>**</td>
</tr>
<tr>
<td>F</td>
<td>1</td>
<td>0.105</td>
<td>0.105</td>
<td></td>
<td>**</td>
<td>**</td>
</tr>
<tr>
<td>G</td>
<td>1</td>
<td>105.085</td>
<td>105.085</td>
<td>286.558</td>
<td>7.13984E-05</td>
<td>Significant</td>
</tr>
<tr>
<td>Error</td>
<td>4</td>
<td>1.467</td>
<td>0.367</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>7</td>
<td>157.534</td>
<td>22.505</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

9.4 ESTIMATION OF ERROR VARIANCE

The error variance which is equal to the error mean square, can be estimated as follows:

\[
\text{Error Variance} = \frac{\text{Sum of Squares due to Error}}{\text{Degree's of freedom for Error}}
\]

The error variance is denoted by \( \sigma_e^2 \).

Here from the ANOVA it can be seen that no degree's of freedom is left to estimate error variance. In such situation, we cannot directly estimate the error variance.

However, an approximate estimate of error variance can be obtained by pooling the sum of squares corresponding to the factors having lowest mean square. As a rule of thumb, we suggest that the sum of squares
corresponding to those factors whose contribution to the total sum of squares was found to be less than 5% have been used to estimate the error mean square or error variance. By this rule as can be seen from the ANOVA that factors A, B, E and F fall under this category. Error variance computed in this manner is called pooling. By traditional statistical assumptions pooling gives a biased estimate of error variance. To obtain a better estimate of error variance, a significantly larger number of experiments would be needed, the cost of which is usually not justifiable compared to added benefit. Due to the above mentioned reason only those factors whose percentage contribution to the total sum of squares was found to be less than 5% only those were considered as the factors which do not have much effect on the response variable and hence in the ANOVA table they are denoted by (**) and those factors sum of squares have been added up to form Mean Error Sum of Squares value.

9.5 Plot of Factor Effects at Different Levels.
9.6 CONCLUSIONS

Now form the above plot we can make following Inferences:

1. As seen in the ANOVA Factor-D and Factor-G showed significant effect and their mean S/N ratio was found to be maximum at 2\textsuperscript{nd} level of both the factors (also seen clearly in the factor effect plot). Hence we prefer to keep Factor-D and Factor-G at Level-2.

2. Inspite of Factor-C showing significant effect the mean S/N Ratio decreases as the level of factor increases so we prefer to keep the level of Factor-C at Level-1.

3. Remaining factors, which didn't have much significant effect on the response variable their levels, were adjusted according to the maximum S/N Ratio.

Considering the above inferences based upon the application studied by us we finally conclude that the optimum level of the factors obtained is as follows as far as the variability is concerned:

A2, B1, C1, D2, E2, F2, G2.

Further the manufacturing process should be run for the above optimal settings and the problem of tablet hardness can be improved largely to ass great extent as compared to the initial settings. This can have better achievement as compared to the usual traditional approach.