PART - II

MEDIA DECISION
METHODOLOGY FOR EVALUATION OF TELEVISION PLAN

3.1 Introduction:

After the optimum budget has been determined, the next problem in the proposed advertising decision hierarchy is the selection of the media to be used in the campaign. The problem of media planning is defined as the selection of media vehicles to deliver appropriate messages in appropriate copy form to target audience for influencing their knowledge, attitudes and actions (63).

The importance of the media planning problem in India can be observed from its usefulness to various business and non-business organisations and the amount of money spent on buying space and time in media vehicles as we have discussed in Chapter-1.

Though the problem has received considerable attention, it is complex because the choice has to be made not only from among a combination of major media classes, but within a class, a large number of media vehicles have to be considered. Realizing the
need for better methods of media selection, some of the Indian advertising agencies, in 1970 started work on development of such methods.

The earlier work was devoted to the development of an approach for selection of media vehicles in press medium and thereafter in cinema medium. They were aimed at providing aids to the advertiser in selection of appropriate publications in press medium and appropriate theatres in cinema medium for achieving the advertising objectives at minimum cost.

In fact, television has today emerged as the single most all-pervasive medium to reach the people with purchasing power, both in urban and rural areas. It cuts across barriers of literacy and reaches all classes of people, irrespective of their sex, age, education, income levels and cultural diversity. It entails, as a medium, the lowest cost per viewer of advertising by ensuring a wider reach. The recent data shows that there are on an average, 10 viewers a set according to a Government estimate, giving a theoretical viewership of more than 50 millions, nearly 4 times the audience that goes daily to cinema. The Government earnings from sponsored programmes and spot advertisements have exceeded Rs. 60 crores. Due to these facts,
we have given higher priority to research on selection of vehicles in the medium of television. The importance of the problem, the availability of data on television audiences, and the easy accessibility to powerful computers, have created the need for developing formal methodologies for solving television media planning problem which can handle the complexities faced by a media planner in India.

In this Chapter, we have discussed the method by which the given television plan can be evaluated and in the next Chapter, we have developed the methodology for preparing the best television plan for the given advertising objectives that the advertiser wishes to achieve, and the constraints within which he operates.

3.2 Literature Survey:

In order to develop an appropriate evaluation model* for media planning, we undertake the survey of literature to critically examine the existing work in the field.

* The models which only "evaluate" a media plan in terms of the goals set by the advertiser are termed as "Evaluation" models.
We have developed here 'Television Evaluation Models' and hence we discuss the existing Media Evaluation Models which can be classified into the following two categories.

3.2.1 Media Evaluation Non Response Models:

These models evaluate a given media plan in terms of the Reach and the Frequency attained by it in various target audience groups and also provide methods for estimating the Reach and the Frequency achieved by a media plan in a specified target audience group such as:

1. Empirical approach by Agostini(7) used data provided by an extensive French study about individual and common audiences of 15 media vehicles. The exact Reach was computed based on these data for different sets of vehicles. For each set the ratio of the total common audiences of all combinations of two vehicles to the total individual audience of all vehicles was plotted against the actual Reach. A curve was approximated to express this empirical relationship and using this curve a formula was developed to estimate the Reach.
The main limitation of this approach is that it becomes necessary to conduct similar studies for all the media vehicles used by a media planner in different regions over different time periods. Moreover, Reach obtained here is close to the actual value under the condition that the ratio of the total common audience combinations of two vehicles to the total audience of all vehicles is either very small or very large. When these conditions are not satisfied, the estimates provided by the method may not be close to the actual Reach. Even in those conditions where the empirical formula may result in good approximation of Reach, theoretical methods provide equally good estimates and at the same time they do not require expensive data collection.

2. The approach suggested by Kwerel (69) is superior to the empirical approach suggested by Agostini (7) as it does not require data collection and at the same time provides valid basis for generalisation. In most cases, the procedure suggested by Kwerel to estimate micro bounds on Reach provides better results as compared to the micro bounds.
Besides providing estimates for the Reach, Kwerel has developed an approach for determining the range within which the Reach of a media plan would be found. Though this is a useful concept for a media planner, it is not likely to be helpful when the range is too wide.

3.2.2. **Media Evaluation Response Models**: 

These models evaluate a given media plan in terms of audience response and also estimate the Reach and Frequency in various target audience groups such as:

1. Simulation is the major approach used in this category of media models. Chances of an individual in the target audience being exposed to the advertisement through different vehicles are estimated here using surveys. Significant contributions in this category have been given by Gensch (38) and Brown (14). All simulation models have a large sample representative of the target audience in terms of demographic, geographic, and socio-economic classifications. The sample consists of either real or hypothetical individuals. Probability of an individual in a target audience group
being exposed to a vehicle is estimated through surveys and it is used in the model as input. The media vehicle effectiveness weights as well as an individual's importance to the planner are also used as inputs to the model.

These models are mainly used to help the media planner to get some idea about the profiles of the audiences generated through his proposed media plan over the planning horizon. The only disadvantage of the model is as the number of details to be considered in the model increases, the cost of using the model rises disproportionately to the benefits of including such details.

2. The models considered here given the complete distribution of audience exposure which is achieved by fitting theoretical distribution to actual audience exposure data covering a small number of media vehicles. Then the theoretical distribution is used to predict the audience exposure pattern for schedules of a large number of vehicles.

For press medium the models are used to generate OTS distribution by getting the probability of exposure of an individual based
on survey data and then to use Binomial or Monte Carlo methods.

For TV medium, Friedman reported that the compound probability distribution namely the Negative Binomial can be used to compute the Reach and Frequency of a TV advertising plan. Later Headen developed a Beta Binomial Model for audience exposure to TV.

Leckenly J. D. and Kishi S. developed the model that estimates the proportion of target audience which will be delivered to mn times the combinations of n insertions in m vehicles. This estimated information, called an 'exposure distribution' provides the basis for the development of schedule evaluation criteria such as 'cost per thousand' and 'effective Reach' to target audience.

An empirical survey presented here, based on 515 observations shows that this model is more accurate than the previous models (Friedman, Headen, Leckenly).

It is well known that the response to advertising exposure is not necessarily linear but one in which the law of diminishing returns operates.
Hence in recent years, most media planning researchers have depended on non-linear responses based on Binomial, Multinomial, Beta and such other distributions of frequency of exposure for evaluation purposes.

3.3 Summary of the Present study:

From the above discussion it can be observed that for TV evaluation plan, all the models discussed require data regarding the probability of a person in the target audience being the viewer of either one or more programmes relayed on the channel or different channels. These data are not always readily available. In the development of newer approach for television planning, our motive has been to make underlying assumptions as realistic as possible and at the same time to ensure that the data required for the use of the models are such as can be easily obtained. Here we have evaluated the plan in terms of Reach and Frequency—the measures commonly used in practice. We have provided here, the Reach the Frequency generated by the plan and we leave on the user's judgement the task of assessing the response that will be generated by the plan.

In the evaluation of the plan, we have considered the first step to prepare a mathematical
foundation for the computation of Reach and Frequency. The problem of computing these measures is essentially probabilistic in nature. The uncertainty arises due to the fact that the number of TV channel operation made by a member of the target audience is not known with certainty. Even if it is known definitely, the manner in which the programme viewing is spread within a particular TV channel is not definitely known. The models that we have developed here are designed to account for these uncertainties.

The starting point in the development of the models is the probability distribution of the number of operation of TV channel made by a member of the target audience within a given time period. It is assumed that either such a distribution is given or can be derived. If it is available, it can be directly used. In certain situations where the data is given, with the assumption, these data can be used to derive the probability distribution of the number of programme viewings made in the given period.

We have proposed here two models to account for the uncertainties in the pattern of distribution of programme viewership among the potential
channels which are not certain even if the number of TV channel operations during the given time period is definitely known.

We have developed here two different models by changing the assumptions. These models can also be used for Multi-Media Evaluation plan which we have discussed in chapter-4. A model based on comparatively strong assumptions is presented in section (3.4) whereas the model based on less restrictive assumptions is discussed in section (3.5). The hypothetical problems are also designed here to test the practical utility of the models developed. Finally section (3.6) deals with concluding remarks based on the models offered.
3.4 Evaluation Model - 1 :

3.4.1 Assumptions :

(1) There are \( m \) TV channels in all. Probability that a person from the target audience will operate a particular channel is \( p_1 \) and it is assumed to be the same for all TV channels.

(2) There are in all \( k \) TV programmes* which are relayed on each channel. Probability that a person will see a particular TV programme on a given channel is \( p_2 \) which is assumed to be the same for all TV programmes.

(3) Probability that a person will operate one channel or see one programme is independent of the event that he will operate another channel or see another programme.

(4) Probability that a person will operate a particular channel or see a specific programme can be estimated from the past date**.

* Here 'Programme' is defined as a 'Popular Programme' along with which the different advertisements can be relayed to popularize the product. However a widely accepted term 'TV spot' can be also used here in place of 'TV programme'. 'TV spot' can be defined as the spot in the television schedule where the advertisement can be inserted.

** Many times ratings of TV Channels and TV programmes are readily available.\(^{94}\)
3.4.2 Development of the Model:

Let $E_i$ denote the event that a member of the target audience operates the $i^{th}$ channel, $i = 1, 2, \ldots, m$.

Hence we have $m$ independent events and we are interested in computing the probability that exactly $x$ among these $m$ events happen simultaneously.

The problem of obtaining OTS* Distribution in this case is here equivalent of finding the probability distribution of random variable $x$. That is the probability that exactly $x$ events among the $m$ events occur simultaneously. Hence, the solution of this problem, reduces to that of finding the value of $P(x)$, which is already available.

It is wellknown that,

$$P(X = x) = \sum_{j=x}^{m} (-1)^{j-x} \binom{j}{x} S_j \quad \ldots (3.1)$$

* Here OTS can be defined as opportunity to operate a particular TV channel.
where \( S_j \) = sum of all the probabilities that a person operates \( j \)th specified channel out of total \( m \) channels.

\[
S_j = \sum_{i_j = i_{j-1} + 1}^{m} \sum_{i_{j-1} = i_{j-2} + 1}^{m-1} \sum_{i=1}^{m-j+1} \cdot P(A_{i_1}, A_{i_2} \ldots A_{i_j})
\]

\[
P(A_{i_1}, A_{i_2} \ldots A_{i_j}) = \prod_{i_1 < i_2 < \ldots < i_j} p_{i_1}p_{i_2}p_{i_3} \ldots p_{i_j}
\]

(since all the probabilities are independent).

Now, \( S_j \) is the sum of all the probabilities that out of total \( m \) channels, \( j \) channels are operated and the selection of \( j \) channels out of \( m \) can be made in \( \binom{m}{j} \) ways, hence \( S_j \) in this case is given by

\[
S_j = \binom{m}{j}p^j
\]

(since the probability of channel operation is the same). ....(3.2)

Considering (3.2) in (3.1), we get

\[
P(X=x) = \sum_{j=x}^{m} (-1)^{j-x} \binom{j}{x}(\binom{m}{j})p^j
\]

* \( \binom{m}{j} \) stands for \( \frac{m!}{j!(m-j)!} \).
\[
= \binom{m}{x} p_1^x + (-1)^{x+1} \binom{m}{x+1} p_1^{x+1} \\
+ (-1)^{x+2} \binom{m}{x+2} p_1^{x+2} + \ldots + \\
-(-1)^{m-x} \binom{m}{x} p_1^x \\
= \binom{m}{x} p_1^x (1 - \frac{(m-x)}{1} p_1 + \frac{(m-x)}{2} p_1^2 + \ldots + (-1)^{m-x}) \\
= \binom{m}{x} p_1^x (1 + \sum_{j=1}^{m-x} (-p_1)^j) \\
= \binom{m}{x} p_1^x (1 - p_1)^{m-x}, \quad \ldots (3.3)
\]

It can be observed that (3.3) is Binomial distribution with parameters \((m, p_1)\).

Similarly using the same logic, we can derive the distribution of TV programmes and show that the probability that a person will observe \(y\) TV programmes out of total \(k\) TV programmes is given by

\[
P(Y = y) = \binom{k}{y} p_2^y (1 - p_2)^{k-y}, \quad y = 0, 1, 2, \ldots, k, \quad \ldots (3.4)
\]

It can be observed that (3.4) is also Binomial with parameters \((k, p_2)\).
But the overall distribution of TV programmes within the channels is obtained by compounding the distribution of channels with the distribution of TV programmes (59).

To obtain probability density function of compound distribution:

The probability generating function * of compound distribution is given by

\[ G(z) = G_1(G_2(z)) \]

\[ = \sum_{x=0}^{m} p(x)(G_2(z))^x \quad \ldots \ldots (3.5) \]

where \( G_2(z) \) is probability generating function of distribution of TV programmes and is given by

\[ G_2(z) = E(z^Y) = \sum_{y=0}^{k} P(Y=y)z^y \]

\[ = (q_2 + p_2z), \quad \text{where} \quad q_2 = 1 - p_2. \quad \ldots \ldots (3.6) \]

Substitution of (3.6) in (3.5) yields

* Probability generating function of the variable \( Y \) is defined as

\[ G(z) = E(z^Y) = \sum_{y \in S} z^y p_y, \quad -\delta < \delta, \quad |z| < 1 \]

\[ \quad \text{where} \quad \delta > 0, \quad p_y > 0. \quad \ldots \ldots (33) \]
\[ G(z) = \sum_{x=0}^{m} \binom{m}{x} p_1^x q_1^{m-x} (q_2 + p_2 z)^k x \]

(using the result (3.3))

\[ \text{...(3.7)} \]

where \( q_1 = 1 - p_1 \).

Hence we obtain,

\[ G(z) = \sum_{x=0}^{m} \binom{m}{x} p_1^x q_1^{m-x} (q_2 + p_2 z)^k x \]

\[ = \sum_{x=0}^{m} \binom{m}{x} p_1^x q_1^{m-x} \sum_{r=0}^{k} \binom{kx}{r} p_2^r q_2^{kx-r} . \]

\[ \text{...(3.8)} \]

Now probability that a person will observe in all \( r \) TV programmes within all the channels is given by considering coefficient of \( z^r \) in (3.8), that is,

\[ P(r) = \sum_{x=0}^{m} \binom{m}{x} p_1^x q_1^{m-x} \sum_{r=0}^{k} \binom{kx}{r} p_2^r q_2^{kx-r} , \]

\[ r = 0, 1, 2, ..., mk. \]

\[ \text{...(3.9)} \]

Reach :

Here we define Reach of the distribution as probability that a person will observe atleast one TV programme and is given by

\[ \text{Reach}(R) = \sum_{r=1}^{mk} P(r) \]

\[ = 1 - P(r=0). \]

\[ \text{...(3.10)} \]
Considering \( r = 0 \), in (3.9) we get,

\[
P(r=0) = \sum_{x=0}^{m} \binom{m}{x} (p_1 q_2)^x q_1^{m-x}
\]

\[
= (q_1 + p_1 q_2)^m \quad \text{...(3.11)}
\]

Using (3.11) in (3.10) we get,

\[
R = 1 - (q_1 + p_1 q_2)^m \quad \text{...(3.12)}
\]

Frequency:

Here Frequency shows average number of TV programmes observed by a member of the target audience and it is given by

Frequency \( (F) = E(r) = \left[ \frac{d}{dz} G(z) \right]_{z=1} \).

From (3.8), probability generating function is given by

\[
G(z) = (q_1 + p_1(q_2 + p_2 z)^k)^m
\]

Now differentiating both the sides with respect to \( z \) we get,

\[
\frac{dG(z)}{dz} = \frac{d}{dz}(q_1 + p_1(q_2 + p_2 z)^k)^m
\]

\[
= [m(q_1 + p_1(q_2 + p_2 z)^k)^{m-1}
\]

\[
[p_1(q_2 + p_2 z)^{k-1} p_2] \].
\]
On making use of $p_i + q_i = 1$, $i = 1, 2$, we get
\[
[z=1] \frac{dG(z)}{dz} \bigg|_{z=1} = mkp_1p_2 \quad \text{and hence}
\]
\[
F = mkp_1p_2 . \quad \ldots (3.13)
\]

3.4.3 Sensitivity of Reach and Frequency to the number of TV channels and the number of TV programmes within the channel.

In many practical situations an advertiser is interested in knowing the sensitivity of Reach and Frequency of the plan with respect to the number of TV channels as well as the number of TV programmes within the channel.

3.4.3.1 Impact of Increase in the number of TV programmes in each channel on Reach and Frequency.

Reach:

From (3.12), we have,

\[
R(k) = 1 - (q_1 + p_1q_2)^m \quad \text{and}
\]

\[
R(k+1) = 1 - (q_1 + p_1q_2)^{k+1}m
\]

where $R(k)$ is Reach when $k$ TV programmes are relayed on each of the channels.
Now,

\[
R(k) - R(k+1) = \left( q_2 + p_1 q_2^{k+1} \right)^m - \left( q_1 + p_1 q_2^k \right)^m
\]

Since \( q_2 \ll q_2 \), \( R(k) - R(k+1) \ll 0 \).

That is, as the number of TV programmes within the channel increases, Reach increases.

\[
\ldots\ldots (3.14)
\]

Frequency:

From [3.13], we have,

\[
\text{Frequency (F)} = \frac{m k p_1 p_2}{r}
\]

Here it can be easily seen that if the number of TV programmes within the channels increases, Frequency also increases.

\[
\ldots\ldots (3.15)
\]

3.4.3.2 Impact of Increase in the Number of TV Channels on Reach and Frequency:

Reach:

From [3.12], we have

\[
R(m) = 1 - \left( q_1 + p_1 q_2^k \right)^m
\]

where \( R(m) \) is the Reach when \( m \) TV channels are utilized.
Let the number of TV channels be increased by 1 and hence
\[ R(m+1) = 1 - (q_1 + p_1 q_2^k) \]
Therefore, we have
\[ R(m) - R(m+1) = \left( q_1 + p_1 q_2^k \right)^{m+1} - \left( q_1 + p_1 q_2^k \right)^m \]
\[ = \left( q_1 + p_1 q_2^k \right)^m \left[ (q_1 + p_1 q_2^k) - 1 \right] \]
\[ = (q_2 + p_1 q_2^k)^m \left[ p_1 (q_2^k - 1) \right] \]
(Since \( p_1 + q_1 = 1 \)).
This difference is nonpositive since \( q_2 < 1 \), that is as the number of TV channels increases, Reach increases.
\[ \ldots (3.16) \]
In this case also the same interpretation as (3.15) can be made for Frequency.

3.4.4 Comparison of the Effectiveness of Advertising in Utilizing One Channel and More Than One Channels:

Case-I: There are \( m \) channels and we want \( k = m \) programmes to be relayed during a specified time period.
Situation 1: Usage of Single Channel:

All the $k=m$ programmes are relayed on one channel at different times in a specified time period i.e. we use one channel only.

Considering $m=1$ and replacing $k$ by $m$ in (3.9), (3.12) and (3.13) respectively, we get,

$$P_1(r) = \sum_{x=0}^{1} \binom{1}{x} q_1 x^{1-x} q_2^{mx-r}$$

$$r=0,1,2,\ldots,mk \quad \ldots(3.17)$$

(\text{Since } k = m)

$$R_1 = 1 - [q_1 + p_1 q_2^m] \quad (\text{Since } k=m) \quad \ldots(3.18)$$

$$F_1 = m p_1 p_2 \quad (\text{Since } k=m) \quad \ldots(3.19)$$

where $P_1(r)$ is the probability that a person will see $r$ programmes when only one channel is used. $R_1$ and $F_1$ are Reach and Frequency respectively for Situation-1.

Situation-2: Usage of Multi-channels:

All the channels are utilised. That is only one programme is being shown on each channel at different times in a specified time period i.e. $k=1$. 
Considering \( k=1 \) in (3.9), (3.12) and (3.13) respectively, we get,

\[
P_2(r) = \sum_{x=0}^{m} \binom{m}{x} p_1^x q_1^{m-x} \binom{r}{x} p_2^x q_2^{r-x} \quad \ldots \ldots \text{(3.20)}
\]

\[
h_2 = 1 - (q_1 + p_1 q_2)^m \quad \ldots \ldots \text{(3.21)}
\]

\[
F_2 = m p_1 p_2 \quad \ldots \ldots \text{(3.22)}
\]

where \( P_2(r) \) is the probability that a person will see \( r \) programmes when all the channels are utilized. \( R_2 \) and \( F_2 \) are Reach and Frequency respectively for Situation-2.

Comparison of Reach of two Situations:

From (3.18) and (3.21),

\[
R_1 = 1 - (q_1 + p_1 q_2)^m
\]

\[
R_2 = 1 - (q_1 + p_1 q_2)^m
\]

and hence

\[
R_1 - R_2 = (q_1 + p_1 q_2)^m - (q_1 + p_1 q_2)^m.
\]

Let \( D(m) = (q_1 + p_1 q_2)^m - (q_1 + p_1 q_2)^m \).

where \( 1 \leq m < \alpha \).
We test behaviour of $D(m)$ with respect to $m$ in the following three cases.

(i) When $m=1$, $D(m) = (q_1 + p_1 q_2) - (q_1 + p_1 q_2) = 0$.

(ii) When $m \to \infty$, $D(m) \to -q_1$.

(iii) When $1 \leq m < \infty$, we have,

$$D(m) - D(m+1) = [(q_1 + p_1 q_2)^{m+1} - (q_1 + p_1 q_2^m)] - [(q_1 + p_1 q_2)^{m+1} - (q_1 + p_1 q_2^{m+1})]$$

$$= (q_1 + p_1 q_2)^m [1-(q_1 + p_1 q_2)] - p_1 q_2^m (1-q_2)$$

$$= (q_1 + p_1 q_2)^m [1-p_1 q_2] - p_1 q_2^m p_2$$

$$= (q_1 + p_1 q_2)^m [p_1 p_2 - p_1 p_2 q_2^m]$$

$$= p_1 p_2 [(q_1 + p_1 q_2)^m - q_2^m]$$

$$= p_1 p_2 q_2^m [(p_1 + q_2)^m - 1]$$

$$= p_1 p_2 q_2^m \left[ \left( \frac{1-p_1 + (1-p_2) p_1}{1-p_2} \right)^m - 1 \right]$$
Since \( p_1 p_2 < p_2 \), it can be easily shown that
\[
\left( \frac{1 - p_1 p_2}{1 - p_2} \right)^m > 1.
\]
Also \( p_1, p_2, q_2 \) are all positive and hence \( D(m) - D(m+1) > 0 \).

From results (i), (ii) and (iii) it can be observed that \( D(m) \) is a monotonic non-decreasing function of \( m \) and as \( m \) increases, \( D(m) \) decreases.

Therefore, the value of \( D(m) \) < 0 and hence \( R_1 - R_2 \leq 0 \).

From this it can be said that Reach is more when all the channels are utilized as compared to usage of only one channel.

Comparison of Frequency of two Situations:

From (3.19) and (3.22), we have,
\[
F_1 = m p_1 p_2, \quad F_2 = m p_1 p_2.
\]

This means that it does not make any difference whether the programmes are relayed through
one channel only or through more than one channel as far as Frequency in both the situations are concerned.

Case-II : The number of channels is greater than the number of programmes relayed in a specified time period i.e. \( m > k \).

Here the results are the same as those of the previous case because we can use only \( k \) channels and remaining \( m-k \) channels remain unutilised. Therefore both the situations are the same as mentioned earlier.

Case-III : The number of channels is less than the number of programmes relayed in a specified time period i.e. \( m < k \).

Situation-1 : Single Channel :

All the programmes are relayed on one channel in a specified time period.

Considering \( m=1 \) in (3.9), (3.12) and (3.13) respectively we have,

\[
P_1(r) = \sum_{x=0}^{\min(k, r)} \binom{1}{x} p_1^x q_1^{1-x} \binom{k}{r} p_2^r q_2^{k-x-r}
\]

\( r = 0, 1, 2, \ldots, k \) \( \ldots \ldots \) (3.23)
Situation-2 : Multi-Channels:

All the channels are utilised. That is total $k$ programmes are divided into $m$ channels. Suppose $k$ is a multiple of $m$ and hence $\frac{k}{m}$ programmes are relayed on each channel.

In order to obtain probability, Reach and Frequency of this case, we replace $k$ by $\frac{k}{m}$ in (3.9), (3.12) and (3.13) respectively which obviously yields

$$P_2(r) = \sum_{x=0}^{m} \binom{m}{x} p_{-}^{m-x} q_{1}^{x} \left( \frac{k}{m} \right)^{x} \left( \frac{1}{m} \right)^{m-x} \frac{k^{x}}{m^{x}}$$

$$R_2 = 1 - \left( q_{1} + p_{1} q_{2} \right)^{m}$$

$$F_2 = k \cdot p_{1} p_{2}$$

Comparison of Reach of two Situations:

From (3.24) and (3.27),

$$R_1 = 1 - \left( q_{1} + p_{1} q_{2}^{m} \right)$$
$a_2 = 1 - (q_1 + p_1 q_2^m)^m$

and hence

$$h_1 - R_2 = (q_1 + p_1 q_2^m)^m - (q_1 + p_1 q_2^k).$$

Let $\frac{k}{m} = h$ (a finite quantity)

and therefore

$$h_1 - R_2 = (q_1 + p_1 q_2^h)^m - (q_1 + p_1 q_2^{hm}).$$

We assume $D(m) = (q_1 + p_1 q_2^h)^m - (q_1 + p_1 q_2^{hm})$

and test behaviour of $D(m)$ with respect to $m$ in the following cases.

(i) When $m = 1,$

$$D(m) = 0.$$ 

(ii) When $m \to \infty,$

$$D(m) \to -q_1.$$ 

(iii) When $1 < m < \infty,$ we have,

$$D(m) - D(m+1) = [(q_1 + p_1 q_2^h)^m - (q_1 + p_1 q_2^{hm})] - [(q_1 + p_1 q_2^h)^{m+1} - (q_1 + p_1 q_2^{h(m+1)})].$$

On further simplification it can be shown that

$$D(m) - D(m+1) = p_1 P_2 [(q_1 + p_1 q_2^h)^m - q_2^{hm}].$$
Since we have $p_1 > 0$, $p_2 > 0$, $q_1 > 0$, and $q_2 > 0$, we have:

$$\frac{h_m}{q_2} = \frac{h_m}{q_2} \left[ \frac{q_1 + p_1 q_2}{q_2} \right] - 1$$

$$= p_1 p_2 q_2^h m \left[ \frac{1 - p_1 p_2}{1 - p_2} \right] - 1]$$

Hence from results (i), (ii) and (iii), we have $H_2 > H_1$.

Comparison of Frequency of two Situations:

From (3.25) and (3.28), we have,

$$F_1 = k p_1 p_2, \quad F_2 = k p_1 p_2.$$

Here it can be seen that Frequency in both the Situations is the same.

Even if $k$ is not a multiple of $m$ and $k > m$, these results will hold good because in this case also on all the channels at least one programme is being telecast.
From this, we can say that in all the three cases we get the same result which shows that rather than relaying all programmes on one channel, all the channels should be utilised.

3.4.5 Particular Case and Application of the Model:

Here we consider a particular case with two channels and two programmes.

We assume here that when a member of the target audience will see one programme he will see only the programme which is relayed first and we want to know whether the same person will prefer to see second programme on the same channel or prefer to see it on another channel.

The result that we have derived here theoretically is verified with the help of empirical results discussed by Goodhardt and Ehrenberg[42]. Let us suppose that there are two programmes which are relayed in a specified week on two different occasions. Now Situation-1 and Situation-2 can be defined as follows:
Situation-1: Both the programmes are relayed on only one channel on two different occasions.

Situation-2: There are two channels and one programme is relayed on each channel on two different occasions.

Let $P_i(1,2,\ldots,r)$ denote the probability that a person will see $1^{st}$, $2^{nd}$, $3^{rd}$, $\ldots, r^{th}$ programme in Situation-$i$, $i = 1,2$ which is same as $P_i(r)$ defined earlier. In particular, $P_i(1,2)$ is same as $P_i(2)$ but $P_i(2)$ means the person will see only $2^{nd}$ programme when $i^{th}$ situation is considered, (for $i = 1,2$) and it is not the same as $P_i(2)$ but we have $P_i = P_i(1)$.

To compute probabilities of Situation-1 and Situation-2:

This is a particular case of Case-I when there are two TV channels and two programmes as discussed in section (3.4.4).

Situation-1:

Considering $m = 2$ in (3.17) we have,
\[ P_1(r) = \frac{1}{\Sigma} \binom{r}{x} p_1^x q_1^{1-x} (2^x) p_2^r q_2^{2x-r} \]  

\[ \cdots\cdots(3.29) \]

Considering \( r = 1 \) in the above expression, we get,

\[ P_1^{(1)} = \frac{1}{\Sigma} \binom{1}{x} p_1^x q_1^{1-x} (2^x) p_2 q_2^{2x-1} \]

\[ = 2 p_1 p_2 q_2. \]  

\[ \cdots\cdots(3.30) \]

Here \( P_1^{(1)} \) denotes probability that a person will see the 1st programme when both the programmes are relayed on only one channel.

Considering \( r = 2 \) in (3.29), we get,

\[ P_1^{(2)} = \frac{1}{\Sigma} \binom{2}{x} p_1^x q_1^{1-x} (2^x) p_2 q_2^{2x-2} \]

\[ = p_1 p_2^2. \]  

\[ \cdots\cdots(3.31) \]

where \( P_1^{(2)} \) denotes probability that a person will see 1st and 2nd programme both relayed on same channel.

Situation-2:

Considering \( m = 2 \) in (3.20) we get,
\[ P_2(r) = \sum_{x=0}^{r} \binom{2}{x} p_1^x q_1^{2-x} \binom{x}{r} p_2^r q_2^{-r} \]

\[ \ldots \ldots (3.32) \]

Considering \( r = 1 \) in (3.32), we get,

\[ P_2^{(1)} = \sum_{x=0}^{1} \binom{2}{x} p_1^x q_1^{2-x} \binom{1}{1} p_2^1 q_2^{-1} \]

\[ = 2p_1p_2 (q_1 + p_1q_2) \]

\[ \ldots \ldots (3.33) \]

where \( P_2^{(1)} \) shows probability that a person will see 1\textsuperscript{st} programme when on each channel one programme is relayed.

Considering \( r = 2 \) in (3.32), we get,

\[ P_2^{(2)} = \sum_{x=0}^{2} \binom{2}{x} p_1^x q_1^{2-x} \binom{2}{2} p_2^2 q_2^{-2} \]

\[ = p_1^2 \]

\[ \ldots \ldots (3.34) \]

where \( P_2^{(2)} \) shows probability that a person will see both the programmes relayed on different channels on two different occasions.
To compute the preference for channels:

Now according to the duplication of viewing law* stated by Goodhardt G. J. and Ehrenberg A. S. C., (42) we have,

\[ P(2) = kP(1)p(2) \]

\[ P(2) = kP_1 p_2 q_2 \]

where \( k \) is a constant and value of \( k \) in Situation-1 is greater than that in Situation-2.

In the model, let us test this result.

**Situation-1:**

From (3.30) and (3.31) we have,

\[ P_1 = 2p_1 p_2 q_2 \quad \text{and} \quad P_2 = p_1 p_2, \]

and from (3.35) we have,

* Duplication of viewing law: The general algebraic form of the law is given by \( r_s t = k r_s r_t \), where \( k \) is a constant for both the programmes, \( r_s \) is rating for 1st programme, \( r_t \) is rating for 2nd programme shown on different days. Generally, it has been observed by conducting field work that the value of \( k \) in Situation-1 is greater than 1 (i.e., it varies between 1.4 to 1.5) and in Situation-2 it is approximately equal to 1. Thus on the basis of field work they have shown that the percentage of the audience of any TV programme who watch a second TV programme on the same channel is greater than that of watching the same programme on another channel.
\[ P_1(2) = k_1 P_1(1) P_1(2) \quad \ldots \ldots (3.36) \]

where \( P_1(2) \) is the probability that a person will see the 2nd programme in Situation-1 and \( k_1 \) is constant of Situation-1.

Let us assume that \( P_1(1) \) is same as \( P_1(2) \).

That is the probability that a person will see the first programme is the same as the probability that he will see the second programme (according to the assumption stated in section (3.4.1)).

Considering (3.30) and (3.31) in (3.36), we obtain,

\[ p_1 p_2^2 = k_1 (2 p_1 p_2 q_1) (2 p_1 p_2 q_1) \]

\[ = 4 x_1 p_1 p_2^2 q_1^2 \]

which evidently reduces to

\[ k_1 = \frac{1}{4 p_1 q_1^2} \quad \ldots \ldots (3.37) \]

Situation-2:

From (3.33) and (3.34) we have,
\[ P_2^{(1)} = 2p_1p_2 (q_1 + p_1q_2) \]

\[ P_2^{(2)} = p_1^2 P_2 \]

From (3.35) we have,

\[ P_2^{(2)} = k_2^2 P_2^{(2)} \]

where \( P_2^{(2)} \) is the probability that a person will see the second programme in Situation-2 and \( k_2 \) is constant of Situation-2. Here also we assume that 

\[ P_2^{(1)} = P_2^{(2)} \]

Considering (3.33) and (3.34) in (3.38) we get,

\[ p_1^2 p_2^2 = k_2^2 (2p_1p_2(q_1 + p_1q_2)) (2p_1p_2(q_1 + p_1q_2)) \]

and hence \( k_2 = \frac{1}{4(q_1 + p_1q_2)^2} \)

Dividing constant \( k_1 \) by \( k_2 \), from (3.37) and (3.39) we get,

\[ \frac{k_1}{k_2} = \frac{(q_1 + p_1q_2)^2}{p_1q_1} \]
\[= \frac{1}{p_1} \left(1 + \frac{p_1q_2}{q_1}\right)^2.\]

Since \(o < p_1 < 1, o < q_1 < 1, o < q_2 < 1\), we have

\[\frac{1}{p_1} \left(1 - \frac{p_1q_2}{q_1}\right)^2 > 1.\]

Hence it is obvious that

\[\frac{k_1}{k_2} > 1 \quad \text{i.e.} \quad k_1 > k_2.\]

This shows that constant of Situation-1 is greater than that of Situation-2 which is the same as Goodhardt's \([42]\) result.

That is the probability that a person will observe the second programme relayed on the same channel is greater than the probability that he will observe the second programme relayed on different channels. This result shows channel loyalty since the duplication constant \(k\) is larger for two programmes relayed on the same channel than for two programmes on different channels. This means that, a viewer of a programme on a channel is more likely to watch a programme on the same channel on different occasions than to watch an equally popular programme, on a different channel and hence we conclude that there is a channel loyalty in the viewers.
3.5 Evaluation Model - 2

In practice, the data for TV evaluation plan is available in terms of TV viewing habits of the people and therefore, the TV viewing habits should be expressed in terms of the choice that the viewers make with regard to the TV channel and TV programmes. Hence this model attempts to translate the TV-viewing habits in terms of the choice people make in selecting the different TV channels and the different TV programmes within the channels. Thus Model - 2 is more practicable in the sense of resemblance to real life situation as compared to Model - 1.

3.5.1 Approach for Development of the Model

The development of the model is done for helping the planning process involved in the television medium. The necessary steps of this process are mentioned below:

1) Listing of all the available television channels which are suitable for advertising the product under consideration.
ii) Choosing from the list of television channels, a set of channels where the advertisement is to be relayed.

iii) Listing the popular TV programmes of each selected television channel within specific time periods which are suitable for inserting the advertisement.

iv) Choosing from the list of available TV programmes, a set of TV programmes where the advertisement can be inserted.

In the problem of evaluation, the decisions involved in all above steps have already been made and the planner is now interested in knowing the Reach and the Frequency that can be achieved on the basis of the plan prepared.

The approaches that have been developed for obtaining the Reach and the Frequency of a plan take into consideration the manner in which the members of the target audience operate TV channels and watch a particular programme.

The model assumes that the TV watching habits are known and can be expressed in terms of probability distribution of the number of TV programmes seen in a given time-span.
3.5.2 Assumptions

1. All the TV channels have equal chances of being operated by the member of the target audience.

2. All the TV programmes have equal chances of being viewed by the members of target audience.

3. A person's choice of a particular TV channel or a specific TV programme will not affect his choice of another TV channel or TV programme.

4. The probability distribution of the TV channel selection and TV programmes viewing by a member of the target audience is known.

5. The planning horizon is for a specified time period.

With these assumptions, we develop the methodology to compute the Reach and the Frequency of a given plan.

3.5.3 Notations

Let

\[ M \]

denote the total number of TV channels,
denote the number of TV channels selected for relaying the advertisement,

denote the total number of TV programmes available within each channel,

denote the number of TV programmes selected for relaying the advertisement from each selected channel,

denote the random variable, the number of channels operated by a member of the target audience, \( i = 0,1,2,\ldots,M \),

denote the probability that a member of the target audience operates exactly \( i \) channels, \( i = 0,1,2,\ldots,M \),

\[
\sum_{i=0}^{M} f_i = 1.
\]

Similarly, let

denote the random variable, the number of TV programmes viewed by a member of the target audience within each channel, \( j = 0,1,2,\ldots,N \),

denote the probability that a member of the target audience views exactly \( j \) programmes within each channel, \( j = 0,1,2,\ldots,N \),

\[
\sum_{j=0}^{N} g_j = 1.
\]
3.5.4 Development of the Model

3.5.4.1 Development of OTS Distribution:

To obtain Channel Distribution:

Let $A_h$ denote the event that a member of the target audience operates $h^{th}$ channel from the $m$ selected channels for relaying the advertisement, $h = 1, 2, \ldots, m$.

We want to compute here the probability that exactly $'x'$ events out of these $'m'$ events happen simultaneously same as in Model-1.

Hence, from (3.1), the expression for OTS distribution can be written as

$$P[X = x] = \sum_{h=x}^{m} (-1)^{h-x} \binom{h}{x} S_h \quad \ldots\ldots(3.40)$$

where $S_h$ is the sum of the probability of $h$ events.

i.e. $S_h = \sum_{i_1<i_2<\ldots<i_h} P_r(A_{i_1}, A_{i_2}, \ldots, A_{i_h})$

$$= \sum_{i_1<i_2<\ldots<i_k} P_{i_1, i_2, \ldots, i_h} \quad \ldots\ldots(3.41)$$
We first develop here the expression for $S_h$ as follows:

Let $E_i$ denote the event that a person from the target audience operates exactly $i$ channels during the planning horizon, $i = 0, 1, 2, \ldots, M$.

Therefore, it can be observed that

$$A_h = \bigcup_{i=1}^{M} (A_h \cap E_i), \quad \text{Since } E_i \cap E_i' = \emptyset$$

for $i \neq i'$.

Hence, we have

$$P(A_h) = \sum_{i=1}^{M} P(A_h \cap E_i)$$

$$= \sum_{i=1}^{M} P(A_h / E_i) P(E_i) \quad \text{.....(3.42)}$$

where $P(A_h / E_i)$ is the probability that a person operates $h$th specific TV channel given that he operates $i$ channels during the given time period.

Given the information that a person has operated $i$ different channels where all channels have equal chances of being operated, the probability of choosing $h$th channel out of total $M$ channels is given by
Similarly if we consider the probability that a person operates \( i_1, i_2, \ldots, i_n \) specified channels, given that he operates total \( i \) channels, we have by using the result (3.43),

\[
P(\bigcap_{k=1}^{i} A_k / E_1) = \mathbb{P}(A_1 \cap A_2 \cap \cdots \cap A_i / E_1)
\]

\[
= \frac{(i)}{M}.
\]

\[\text{(3.44)}\]

Here (3.44) is the conditional probability, and now the unconditional probability that a person operates \( h \) specified channels is given by

\[
P_{i_1, i_2 \ldots, i_h} = \mathbb{P}\left(\bigcap_{k=1}^{h} A_k\right)
\]

* The logic used here is the same as discussed by Sarla Nath (90).
\[
S_h = \sum_{i_1 < i_2 \ldots < i_h} p_{i_1, i_2, \ldots, i_h}
\]

\[
= \sum_{i_1 < i_2 \ldots < i_h} p(\bigcap_{l=i_1}^{i_h} A_l)
\]

which denotes the sum of all the probabilities that a person operates \( h \) specified channels out of \( m \) channels chosen for relaying the advertisement.

The choice of \( h \) channels out of \( m \) can be made in \( \binom{m}{h} \) ways \( \ldots (3.47) \)

Hence using results (3.45) and (3.47) is (3.46), we get,

\[
S_h = \left( \binom{m}{h} \right) \sum_{i > h} \left( \binom{M}{h} \right) f_i
\]
and hence using the result (3.48) in (3.40), we get,

\[ P(X=x) = \frac{m}{\frac{M}{i-h}} \left[ \prod_{i=h}^{M} \frac{(-1)^{h-x}}{M-i} \prod_{i=h}^{M} f_{i(M)} \right] \]

... (3.49)

The expression (3.49) can be further simplified so that the computation involved can be easily carried out.

**Theorem 1**: To show that OTS Distribution in this case is given by

\[ P(X=x) = \sum_{i=x}^{M} \frac{f_{i}}{M-i} (M-m)(m-x) \]

**Proof**: From (3.49), we have,

\[ P(X=x) = \sum_{i=x}^{M} \left[ \prod_{i=h}^{M} \frac{(-1)^{h-x}}{M-i} \prod_{i=h}^{M} f_{i(M)} \right] \]

... (3.50)

Here \( m \leq M \), and since \( m \) is a positive integer, we have,
\[
\binom{m}{h} = \begin{cases} 
0, & \text{if } h > m \\
\frac{m!}{h!(m-h)!}, & \text{if } h \leq m.
\end{cases}
\]

Using expression (3.51) in (3.50), we get,
\[
P(X=x) = \sum_{h=x}^{m} (-1)^{h-x} \binom{h}{x} \sum_{i=h}^{m} \frac{\binom{M-h}{i-h}}{\binom{M}{i}} f_i(\binom{m}{h})
\]

which is same as
\[
P(X = x) = \sum_{h=x}^{m} (-1)^{h-x} \binom{h}{x} \sum_{i=h}^{M} \binom{M-h}{i-h} f_i(\binom{m}{h}).
\]

Now interchanging the summation sign, the above expression can be rewritten as
\[
P(X=x) = \sum_{i=x}^{m} f_i(\binom{m}{i}) \sum_{h=x}^{M} (-1)^{h-x} \binom{h-x}{x} (\binom{m-x}{i-h}) (\binom{M-h}{i})
\]
\[
= \sum_{i=x}^{m} f_i(\binom{m}{i}) \left[ \sum_{h=x}^{M} (-1)^{h-x} (\binom{m-x}{h-x}(\binom{M-h}{i-h}) \right]
\]
\[
= \sum_{i=x}^{m} f_i(\binom{m}{i}) \left[ \sum_{h=x}^{M} (-1)^{h-x} (\binom{m-x}{h-x}(\binom{M-h}{i-h}) \right]
\]
Applying \((-1)^t\) \((\binom{q}{t}) = \binom{-q+t-1}{t}\), in the above expression,

\[
P[X=x] = \sum_{i=x}^{m} \frac{f_i}{M} \binom{M}{i} (-1)^{i-x} \sum_{h=x}^{i-x} \binom{m-x}{h-x} (-M+h+i-1),
\]

and by considering \(h-x = k\), we get,

\[
P[X=x] = \sum_{i=x}^{m} \frac{f_i}{M} \binom{M}{i} (-1)^{i-x} \sum_{k=0}^{i-x} \binom{m-x}{k} (-M+i-1).
\]

On making use of the result

\[
\sum_{t=0}^{k} \binom{q_1}{t} \binom{q_2}{k-t} = \binom{q_1 + q_2}{k},
\]

the above expression simplifies to

\[
P[X=x] = \sum_{i=x}^{m} \frac{f_i}{M} \binom{M}{i} (-1)^{i-x} \binom{m-x}{i-x} (-M + i - 1 + m - x)
\]

which obviously reduces to

\[
P[X=x] = \sum_{i=x}^{m} \frac{f_i}{M} \binom{M-m}{i-x} \binom{m}{i-x} \ldots (3.53)
\]
where \( P[X=x] \) shows the probability that a person will operate \( x \) channels, where \( m \) channels are selected for relaying the advertisement out of total \( M \) available channels.

Corollary: To show that \( P[X=x] \) is a probability density function.

Proof: From (3.53), we have,

\[
P[X=x] = \sum_{i=x}^{m} \frac{f_i}{\binom{M}{i}} \binom{M-m}{i-x} \binom{m}{x}
\]

which is non-negative, since \( f_i \geq 0 \). \( \ldots \ldots \) (3.54)

Also from (3.53), it can be observed that

\[
\sum_{x=0}^{M} P[X=x] = \sum_{x=0}^{M} \sum_{i=x}^{M} \frac{(M-m)}{\binom{M}{i}} \frac{(M)}{\binom{M}{i}} f_i
\]

which reduces to

\[
\sum_{x=0}^{M} P[X=x] = \sum_{i=a}^{M} f_i \sum_{x=0}^{\min(i, M-m)} \frac{(M-m)}{\binom{M}{i}} \frac{(M)}{\binom{M}{i}} f_i
\]

by interchanging the summation sign and using the result

\[
\sum_{t=0}^{k} \binom{q_1}{t} \binom{q_2}{k-t} = \binom{q_1 + q_2}{t}
\]
Thus from results (3.54) and (3.55) it can be said that $P[X=x]$ is a probability density function.

To obtain Programme Distribution:

Similarly the probability that a person will observe $y$ programmes where $n$ programmes are selected for relaying the advertisement out of total $N$ programmes is obtained by using the result (3.53) and it is given by

$$P[Y = y] = \sum_{j=y}^{n} \frac{\binom{n}{j} \binom{N-n}{j-y}}{\binom{N}{y}} g_j. \quad ...(3.56)$$

To Obtain Compound Distribution:

Same as discussed in section (3.4), the overall distribution of TV programmes within the channels is obtained by compounding the distribution
of TV channels with the distribution of TV programmes.

That is from (3.5), we have the probability generating function of compound distribution as

\[ G(z) = \sum_{x=0}^{m} P(X=x) (G_2(z))^x \quad \ldots \ldots (3.57) \]

where

\[ G(z) = E(z)^Y = \sum_{y=0}^{n} \sum_{j=y}^{n} \frac{(n)}{(j)} \frac{(N-n)}{(N)} g_j z^y \quad \ldots \ldots (3.58) \]

and hence using (3.53) and (3.58) in (3.57), we get,

\[ G(z) = \sum_{x=0}^{m} \sum_{i=x}^{m} \frac{(m)}{(i)} \frac{(M-m)}{(i-x)} \frac{(N-n)}{(N)} g_{j-x} z^x \]

which shows probability generating function of OTS Distribution. \ldots \ldots (3.59)

3.5.4.2 To Compute Advertising Effectiveness Measures:

Reach:

Here Reach is defined as the probability that a person from target audience will observe advertisement.
programme on television during the specified time period.

Now the probability that a person will see no programme is given by

\[ P(0) = \left. G(z) \right|_{z=0} \]

But from (3.59), we have

\[
G(z) = \sum_{x=0}^{m} \sum_{i=x}^{m} \binom{m}{x} \binom{M-m}{i-x} f_i \left[ \sum_{y=0}^{n} \sum_{j=y}^{n} \binom{n}{y} \binom{N-n}{j-y} g_j z^y \right]^x
\]

\[
= \sum_{x=0}^{m} \sum_{i=x}^{m} \binom{m}{x} \binom{M-m}{i-x} f_i \left[ \sum_{j=0}^{n} \binom{N-n}{j} g_j z^j \right]^x
\]

which reduces to,

\[
G(0) = \sum_{x=0}^{m} \sum_{i=x}^{m} \binom{m}{x} \binom{M-m}{i-x} f_i \left[ \sum_{j=0}^{n} \binom{N-n}{j} g_j \right]^x
\]

\[ \cdots (3.60) \]

by considering \( z=0 \) and therefore we have
Reach \( (R) = 1 - \sum_{x=0}^{m} \sum_{i=x}^{m} \frac{\binom{m}{x} \binom{M-m}{i-x}}{\binom{M}{i}} f_i \left[ \sum_{j=0}^{n} \frac{\binom{n}{j} \binom{N-n}{j-y}}{\binom{N}{j}} g_j z^y \right]^x. \)

\[ \ldots (3.61) \]

Frequency:

Frequency shows here the average number of TV programmes observed by a member of the target audience within the specified time period which can be given by

\[
\text{Frequency} \ (F) = \left[ \frac{d}{dz} G(z) \right]_{z=1}.
\]

From (3.59), we have

\[
G(z) = \sum_{x=0}^{m} \sum_{i=x}^{m} \frac{\binom{m}{x} \binom{M-m}{i-x}}{\binom{M}{i}} f_i \left[ \sum_{j=0}^{n} \frac{\binom{n}{j} \binom{N-n}{j-y}}{\binom{N}{j}} g_j z^y \right]^x
\]

and hence

\[
\frac{dG(z)}{dz} = \left[ \sum_{x=0}^{m} \sum_{i=x}^{m} \frac{\binom{m}{x} \binom{M-m}{i-x}}{\binom{M}{i}} f_i \right] \left[ \sum_{y=0}^{n} \frac{\binom{n}{j} \binom{N-n}{j-y}}{\binom{N}{j}} g_j z^y \right]^{x-1}.
\]
\[
\begin{align*}
\frac{dG(z)}{dz} \bigg|_{z=1} &= \left[ \sum_{x=0}^{m} \sum_{i=x}^{m} \frac{\binom{m}{x} \binom{M-m}{i-x}}{\binom{M}{i}} f_i \right] \\
[\sum_{y=0}^{n} \sum_{j=y}^{n} \frac{\binom{n}{y} \binom{N-n}{j-y}}{\binom{N}{j}} g_j]^{x-1} \\
[\sum_{y=0}^{n} \sum_{j=y}^{n} \frac{\binom{n}{y} \binom{N-n}{j-y}}{\binom{N}{j}} y g_j].
\end{align*}
\]

But we have \( \sum_{y=0}^{n} \sum_{j=y}^{n} \frac{\binom{n}{y} \binom{N-n}{j-y}}{\binom{N}{j}} g_j = 1 \) and hence the expression (3.62) will evidently reduce to

\[
\text{Frequency (F)} = \left[ \sum_{x=0}^{m} \sum_{i=x}^{m} \frac{\binom{m}{x} \binom{M-m}{i-x}}{\binom{M}{i}} x f_i \right] \\
\left[ \sum_{y=0}^{n} \sum_{j=y}^{n} \frac{\binom{n}{y} \binom{N-n}{j-y}}{\binom{N}{j}} y g_j \right].
\]

\[\cdots(3.63)\]
Lemma 1: To show that Maximum Reach

\[ R = 1 - \sum_{i=1}^{M} f_i(g_o). \]

We define here the Maximum Reach as the Reach achieved by including all the TV Programmes within all the TV channels for relaying the advertisement which can be obtained by considering \( m = M \), and \( n = N \).

Hence from (3.61), we have

\[ R = 1 - \sum_{x=0}^{m} \sum_{i=x}^{m} \binom{m}{x} \binom{M-m}{i-x} f_i L^x \]

where \( L = \sum_{j=0}^{n} \binom{N-n}{j} g_j \) \( \ldots (3.64) \)

Now by interchanging the summation sign

\[ \sum_{x=0}^{m} \sum_{i=x}^{m} \binom{m}{x} \binom{M-m}{i-x} f_i L^x \]

can be written as

\[ \sum_{i=0}^{M} \frac{f_i}{\binom{M}{i}} \sum_{x=0}^{i} \binom{m}{x} \binom{M-m}{i-x} L^x \] \( \ldots (3.65) \)
which can be simplified to:

\[
\left[ \frac{f_0}{M_0} + (M_1) (M_2-m) \right] L \frac{f_1}{M_1} + (M_2) \left( L^2 \frac{f_2}{M_2} \right)
\]

\[
+ \ldots + L^i
\]

(by considering \( m = M \))

and can be rewritten as

\[
\sum_{i=0}^{M} f_i (L_i^i)
\]

\[\ldots (3.66)\]

where

\[
L = \sum_{j=0}^{N-n} \left( \begin{array}{c} N-n \\ j \end{array} \right) g_j
\]

\[= g_0 + \sum_{j=1}^{N-n} \left( \begin{array}{c} N-n \\ j \end{array} \right) g_j
\]

which reduces to \( g_0 \) if \( n = N \).

\[\ldots (3.67)\]

Therefore if we consider \( m = M \) and \( n = N \), using (3.67) in (3.66), the value of expression (3.65) can be given by

\[
\sum_{i=0}^{n} f_i (g_i^c)^i
\]

\[\ldots (3.68)\]
and hence,

\[ \text{Maximum Reach} = 1 - \sum_{i=0}^{M} f_i \left( g_o \right)^i \quad \ldots \ldots \ldots (3.69) \]

(by using (3.68) in (3.64)).

Lemma 2: To show that Maximum Frequency

\[ F = \left[ \sum_{i=1}^{M} i f_i \right] \left[ \sum_{j=1}^{N} j g_j \right] \]

Similarly Maximum Frequency can be obtained if the advertiser relays the advertisement for all the TV programmes within all TV channels during the planning horizon.

That is from (3.63), we have,

\[ \text{Frequency (F)} = \left[ \sum_{x=0}^{m} \sum_{i=x}^{m} \frac{\binom{x}{i} \binom{M-m}{i-x}}{\binom{M}{i}} \times f_i \right] \]

\[ \left[ \sum_{j=0}^{n} \sum_{y=j}^{n} \frac{\binom{n}{y} \binom{N-n}{j-y}}{\binom{N}{j}} \times y g_j \right] \]

Now expression

\[ \sum_{x=0}^{m} \sum_{i=x}^{m} \frac{\binom{x}{i} \binom{M-m}{i-x}}{\binom{M}{i}} \times f_i \]
can be rewritten as
\[ \frac{m}{\sum_{i=1}^{m} \frac{f_i}{\binom{M}{i}}} = \frac{m}{\sum_{x=1}^{m-1} (\frac{M-m}{x-1}) (\frac{M-m}{1-x})} \]
by interchanging the summation sign which obviously reduces to
\[ M \left[ \frac{f_1}{\binom{M}{1}} + \frac{(M-1)}{\binom{M}{2}} f_2 + \frac{(M-1)}{\binom{M}{3}} f_3 + \ldots \right. \]
\[ \left. + \frac{(M-1)}{\binom{M}{M}} f_M \right]. \]
If we consider \( m = M \) in the above expression and on further simplification, we get the value of the expression as
\[ \left[ \sum_{i=1}^{M} i f_i \right]. \] \[ \ldots (3.70) \]
Similarly it can be shown that when \( n = N \), the value of expression
\[ \sum_{j=0}^{n} \sum_{j=y}^{n} \frac{(n)}{(y)} (j-y) (\binom{N}{j}) y g_j \]
\[ = \left[ \sum_{j=1}^{N} j g_j \right] \ldots (3.71) \]
and hence using (3.70) and (3.71), in (3.63), we get

Maximum Frequency

\[ M_N = \left( \sum_{i=1}^{M} i f_i \right) \left( \sum_{j=1}^{N} j g_j \right). \]

\[ \text{......(3.72)} \]

Therefore, Maximum OTS per person reached

\[ = \frac{\text{Maximum Frequency}}{\text{Maximum Reach}} \]

\[ = \frac{M \left( \sum_{i=1}^{M} i f_i \right) \left( \sum_{j=1}^{N} j g_j \right)}{1 - \sum_{i=1}^{M} f_i g_i}. \]

\[ \text{......(3.73)} \]

3.5.4.3 Explanation of the Model through a Numerical Problem:

Let us assume that the TV channel operating habits of a specified target audience are as follows:

\[
\begin{array}{cc}
  i & f_i \\
  0 & 0.531 \\
\end{array}
\]
Total number of TV channels available = \( M = 3 \).

TV channels chosen for relaying the advertisement = \( m = 2 \).

Let us assume that the TV programme viewing habits of the same target audience are as follows:

\[
\begin{array}{c|c}
 j & g_j \\
 0 & 0.535 \\
 1 & 0.191 \\
 2 & 0.160 \\
 3 & 0.086 \\
 4 & 0.028 \\
\end{array}
\]

Number of TV programmes within each TV channel = \( N = 4 \).

Number of TV programmes selected for relaying the advertisement = \( n = 3 \).
Reach:

In this case, using (3.61), the Reach can be obtained as

\[ R = 1 - \sum_{i=0}^{2} \frac{f_i}{\binom{3}{i}} \sum_{x=0}^{2} \frac{x}{1-x} L^x \]

\[ = 1 - \left( f_0 - \frac{f_1}{3} (1 + 2L) + \frac{f_2}{3} (2L + \frac{2}{3}) \right) \]

where \( L = \sum_{j=0}^{3} \frac{1}{\binom{4}{j}} g_j \)

\[ = g_0 + \frac{1}{4} g_1. \]

Using the given information, we have \( L = 0.2512 \) and hence Reach is given by

\[ R = 1 - (0.531 + 0.1516 + 0.0692) \]

\[ = 0.2482. \]

From (3.69), we have,

Maximum Reach = \( 1 - \sum_{i=0}^{3} \frac{f_i}{\binom{1}{i}} (g_0)^i \)

\[ = 1 - (0.5310. + 0.1124 + 0.0395 + 0.0185). \]

\[ = 0.2986. \]
Frequency:

From (3.73), we have

\[
F = \left[ \sum_{x=0}^{m} \sum_{i=x}^{m} \frac{(m)}{(i)} \frac{(m-m)}{(i-x)} \right] \times f_i \left[ \sum_{y=0}^{n} \sum_{j=y}^{n} \frac{(n)}{(j)} \frac{(N-n)}{(j-y)} \right] \ y \ g_j
\]

\[
= \left[ \sum_{i=0}^{m} \frac{f_i}{(M_i)} x \sum_{x=0}^{(m)} \frac{(m)}{(i)} \frac{(m-m)}{(i-x)} \right]
\]

\[
\left[ \sum_{j=0}^{n} \frac{\xi_j}{(N_j)} \frac{(n)}{(j)} \frac{(N-n)}{(j-y)} \ y \right]
\]

\[
= mn \left[ \sum_{i=1}^{m} \frac{f_i}{(M_i)} x \sum_{x=1}^{(m-1)} \frac{(m-1)}{(i-x)} \frac{(M-m)}{(i-x)} \right]
\]

\[
\left[ \sum_{j=1}^{n} \frac{\xi_j}{(N_j)} \frac{(n-1)}{(j-1)} \frac{(N-n)}{(j-y)} \ y \right].
\]

\[\cdots (3.74)\]

In this case, using the given information, the Frequency can be obtained as

\[
F = (2) (3) \left[ \sum_{i=1}^{3} \frac{f_i}{(3)} x \sum_{x=1}^{1} \frac{(1)}{(1-x)} \frac{(1)}{(1-x)} \right]
\]

\[
\left[ \sum_{j=1}^{3} \frac{g_j}{(4)} x \sum_{y=1}^{2} \frac{(2)}{(j-1)} \frac{(1)}{(j-1)} \right].
\]
Considering the values of \( f_i \), \( i = 1, 2 \) and \( g_j \), \( j = 1, 2, 3 \) in the above expression we get

Frequency as

\[
F = 0.1867.
\]

From (3.72), we have,

Maximum Frequency

\[
= \left[ \sum_{i=1}^{3} i f_i \right] \left[ \sum_{j=1}^{4} j g_j \right]
\]

\[
= \left[ 1(0.210) + 2(0.138) + 3(0.121) \right]
\]

\[
\left[ 1(0.191) + 2(0.160) + 3(0.086)
+ 4(0.028) \right]
\]

\[
= \left[ 0.849 \right] \left[ 0.881 \right]
\]

\[
= 0.7480.
\]

From (3.73), we have,

Maximum OTS per person reached

\[
= \frac{\text{Maximum Frequency}}{\text{Maximum Reach}}
\]

\[
= \frac{0.7480}{0.2986}
\]

\[
= 2.5050.
\]
3.5.5 Sensitivity of Reach and Frequency to selected Parameters of a Plan:

In many practical applications an advertiser is interested in knowing the sensitivity of Reach and Frequency of the plan with respect to the number of TV channels and the number of TV programmes used in the plan. If the Reach and the Frequency turn out to be insensitive to the number of selected parameters, a great deal of effort need not be spent on determination of the parameters of a plan. On the other hand, if the Reach and the Frequency be very sensitive to this parameter, it is necessary to be careful in the determination of this number.

For these reasons, the sensitivity of Reach and Frequency to a change in the number of TV channels and a change in the number of TV programmes within the channel selected for relaying the advertisements have been examined here.

3.5.5.1 Change in the Number of TV channels:

Impact on Reach:

From result (3.61), we have
\[ R(M) = 1 - \sum_{x=0}^{m} \sum_{i=x}^{m} \binom{m}{x} \frac{(M-m)}{(i-x)} f_i L^x \]

where \( L = \sum_{j=0}^{n} \binom{N-n}{j} g_j \)

which denotes the Reach achieved if the number of TV channels are \( M \).

Let the number of TV channels be increased to \( M+1 \).

Therefore, \( R(M+1) = 1 - \sum_{x=0}^{m} \sum_{i=x}^{m} \binom{m}{x} \frac{(M+1)-m)}{(i-x)} f_i L^x \]

and hence,

\[ R(M+1) - R(M) = \sum_{x=0}^{m} \sum_{i=x}^{m} \binom{m}{x} f_i L^x \left[ \frac{(M-m)}{(i-x)} - \frac{(M+1-m)}{(i-x)} \right] \]

\[ = \sum_{x=0}^{m} \sum_{i=x}^{m} \binom{m}{x} f_i L^x h_i \]

\[ h_i = \frac{(M-m)}{(i-x)} - \frac{(M+1-m)}{(i-x)} \]

\[ = \frac{(M-m)}{(i-x)} - \frac{(M+1-m)}{(i-x)} \left[ 1 - \frac{(M+1 - m)(M+1 - i)}{(M+1)[(M+1)-m]-(i-x)} \right] \]
The expression (3.75), shows that value of $h_1$ depends on $[x(M+1) - im]$ since we have

$$\frac{(M+1 - m)}{i-x} \cdot \frac{(M+1 - i)}{(M+1 - m)} \cdot [x(M+1) - im].$$

...(3.75)

This means that no general conclusion can be drawn regarding the Reach of a plan. For some values of $M$ Reach increases and for some other values of $M$ Reach decreases. Table 3.1 shows change in Reach by changing the number of TV channels considering the numerical example discussed in section (3.5.4).
<table>
<thead>
<tr>
<th>Number of TV channels $M$</th>
<th>Reach $R(M)$</th>
<th>First Difference $\Delta R(M)$</th>
<th>Second Difference $\Delta^2 R(M)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 ( - )</td>
<td>0.2594 ( - )</td>
<td>- 0.0410</td>
<td>0.0248</td>
</tr>
<tr>
<td>4 (33.33 %)</td>
<td>0.2134 ( - 18.77%)</td>
<td>- 0.0162</td>
<td>0.0026</td>
</tr>
<tr>
<td>5 (66.67 %)</td>
<td>0.2022 ( - 08.01%)</td>
<td>- 0.0136</td>
<td></td>
</tr>
<tr>
<td>6 (100 %)</td>
<td>0.1836 ( - 07.21%)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note that $\Delta R(M) = R(M+1) - R(M)$ and $\Delta^2 R(M) = [R(M+2) - R(M+1)] - [R(M+1) - R(M)]$.

The figures in the brackets show the percentage increase or decrease. From the above table it can be observed that 33.33 % increase in the number of TV channels decreases Reach by 18.77 %.
Impact on Frequency:

From (3.63), we have

\[
F(M) = \sum_{x=0}^{m} \sum_{i=x}^{m} \frac{\binom{m}{i-x} \binom{M-m}{i}}{\binom{M}{i}} f_i x L_1
\]

where

\[
L_1 = \sum_{y=0}^{n} \sum_{j=y}^{n} \frac{\binom{n}{j-y} \binom{N-n}{y}}{\binom{N}{j}} y g_j
\]

which denotes the average frequency achieved if the number of TV channels are \( M \).

Let

\[
F(M+1) = \sum_{x=0}^{m} \sum_{i=x}^{m} \frac{\binom{m}{i-x} \binom{M+1-m}{i-x}}{\binom{M+1}{i}} x f_i L_1
\]

and hence from (3.76), we have,

\[
F(M+1) - F(M) = \sum_{x=0}^{m} \sum_{i=x}^{m} \left( \binom{M-1-m}{i-x} \frac{f_i x L_1}{\binom{M+1}{i}} - \binom{M-m}{i-x} \frac{f_i x L_1}{\binom{M}{i}} \right)
\]
\[ \sum_{x=0}^{m} \sum_{i=x}^{m} f_i x_i (\text{L}_1(-h_i)) \quad \ldots (3.77) \]

where

\[ h_i = \frac{(M - m)}{(i - m)} - \frac{(M - 1 - m)}{(i + 1)}, \]

Hence from expression (3.77), it can be said that the difference is positive if \( \frac{x}{i} < \frac{m}{(M + 1)} \) and the difference is negative if \( \frac{x}{i} > \frac{m}{(M + 1)} \). This means that no general conclusion can be drawn regarding the Frequency of a plan. Table 3.2 shows change in Frequency by changing the number of TV channels considering the numerical problem discussed in section (3.5.4).

<table>
<thead>
<tr>
<th>Number of TV Channels</th>
<th>Frequency</th>
<th>First Difference ( \triangle F(M) )</th>
<th>Second Difference ( \triangle^2 F(M) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 ((-))</td>
<td>0.1867((-))</td>
<td>-0.0465</td>
<td>0.0184</td>
</tr>
<tr>
<td>4 (33.33%)</td>
<td>0.1402((-33.17%))</td>
<td>-0.0281</td>
<td>0.0095</td>
</tr>
<tr>
<td>5 (66.67%)</td>
<td>0.1121((-25.07%))</td>
<td>-0.0186</td>
<td></td>
</tr>
<tr>
<td>6 (100 %)</td>
<td>0.0935((-19.89%))</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We see have that 33.33\% increase in the number of TV channels reduces the average Frequency by 33.17\%. 
3.5.5.2 Change in the Number of TV Programmes:

Impact on Reach:

From (3.61), we have,

\[ \text{Reach } R(N) = 1 - \sum_{x=0}^{m} \sum_{i=x}^{m} \frac{(m)(m-m)}{(M)} \frac{(M-m)}{(i-x)} f_1(L(N))^x \]

where \( L(N) = \sum_{j=0}^{n} \frac{(N-n)}{(N)} g_j \)

which denotes the reach achieved if there are total \( N \) TV Programmes available for relaying the advertisement.

Let the number of programmes within the TV channel be increased by \( N + 1 \).

Therefore, Reach is given by

\[ R(N+1) = 1 - \sum_{x=0}^{m} \sum_{i=x}^{m} \frac{(m)(m-m)}{(M)} \frac{(M-m)}{(i-x)} f_1[L(N+1)]^x \]

where \( L(N+1) = \sum_{j=0}^{n} \frac{(N+1-n)}{(N+1)} g_j \)

and hence,
\[ R(N) - R(N+1) = \sum_{x=0}^{m} \sum_{i=x}^{m} \frac{(m)_{i}^{M-m}}{i!} \left[ [L(N)]^x - [L(N+1)]^x \right]. \]

\[ \ldots \ldots (3.78) \]

To show that \( L(N+1) > L(N) \):

Let us consider the difference

\[ L(N+1) - L(N) = \sum_{j=0}^{n} \left( \binom{N+1}{j} - \binom{N}{j} \right) g_j \]

\[ \sum_{j=0}^{n} g_j d_j \]

\[ \ldots \ldots (3.79) \]

where

\[ d_j = \frac{\binom{N+1}{j} - \binom{N}{j}}{\binom{N+1}{j}} \]

\[ = \frac{\left( \frac{N+1 - j}{N+1} \right)^{-1} - \frac{N-j}{N}}{\binom{N+1}{j}} \]

\[ = \left( \frac{N+1 - j + 1 - n}{N+1 - j + 1} \right) - \left( \frac{N - n}{N - 1} \right) \frac{N-j-n-1}{N-j+1} \]

\[ \ldots \ldots \ldots \left( \frac{N-j-n+1}{N-j+1} \right) \]
But \( N + 1 > N \), and hence \( \frac{n}{N + 1} < \frac{n}{N} \).

i.e. \( 1 - \frac{n}{N + 1} > 1 - \frac{n}{N} \).

Using similar logic, we have

\[
1 - \frac{n}{(N + 1) - J + 1} > 1 - \frac{n}{N - J + 1}
\]

i.e. \( (1 - \frac{n}{N + 1})(1 - \frac{n}{N + 1 - 1}) \cdots (1 - \frac{n}{N + 1 - J + 1}) \cdots (1 - \frac{n}{N - J + 1}) \).

This implies that \( d_j \) is positive for all \( j = 1, 2, \ldots, n \), and hence the expression (3.79) is positive which obviously yields \( L(W + 1) > L(N) \).

Since \( x \) assumes all positive values from 0, 1, \ldots, \( m \), it can be said that

\[
[L(N + 1)]^x > [L(N)]^x \quad \text{for} \quad x = 1, 2, \ldots, m.
\]

This shows that the expression (3.78) is non-positive.
since \( f_i > 0 \). This can be verified from Table 3.3. That is any increase in the number of programmes within the TV channel decreases the Reach.

For the numerical problem discussed in section (3.5.4), if we change the number of programmes, the result indicates that the second difference is negative for all values of \( N \). Thus, the Reach decreases at a decreasing rate.

Table 3.3
Number of TV Programmes and Reach.

<table>
<thead>
<tr>
<th>Number of TV Programmes</th>
<th>Reach ( R(N) )</th>
<th>First Difference ( \Delta R(N) )</th>
<th>Second Difference ( \Delta^2 R(N) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 (-)</td>
<td>0.2594 (-)</td>
<td>-0.0133</td>
<td>-0.0111</td>
</tr>
<tr>
<td>5 (25%)</td>
<td>0.2461(-5.4%)</td>
<td>-0.0244</td>
<td>-0.0120</td>
</tr>
<tr>
<td>6 (50%)</td>
<td>0.2217(-11.01%)</td>
<td>-0.0364</td>
<td></td>
</tr>
<tr>
<td>7 (75%)</td>
<td>0.1853(-19.64%)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Here it can be observed that 25\% increase in the number of programmes results in decrease of 5.4\% in Reach. We note that all the first differences are negative and the second differences are also negative.
Impact on Frequency:

After examining the impact on Reach, we will now examine the impact on average Frequency by increasing the total number of TV Programmes within each channel.

From (3.63), we have,

\[
F(N) = \sum_{x=1}^{m} \sum_{i=x}^{m} \frac{(m)(M - m)}{i - x} f_i x(L_1(N))
\]

where

\[
L_1(N) = \sum_{y=0}^{n} \sum_{j=y}^{n} \frac{(n)(N - n)}{j - y} yg_j
\]

\[
= n \sum_{j=1}^{n} \sum_{y=1}^{j} \frac{(n - 1)(N - n)}{j - y} g_j
\]

which denotes the Frequency achieved if there are total N TV Programmes available for relaying the advertisement.

Hence Frequency \( F(N + 1) = \sum_{x=1}^{m} \sum_{i=x}^{m} \frac{(m)(M - m)}{i - x} f_i x(L_1(N + 1)) \)
where,
\[ L_1(N+1) = \sum_{y=0}^{n} \sum_{j=y}^{n} \binom{n}{y} \binom{N+1-n}{j-y} y g_j \]

which yields
\[ F(N+1) - F(N) \leq \sum_{m=1}^{M} \sum_{i=x}^{m} \frac{(m)(M-m)}{(i-x)} f_i x L_1(N+1) - L_1(N) \].

Now \( L_1(N+1) - L_1(N) \)
\[ = \sum_{y=0}^{n} \sum_{j=y}^{n} \binom{n}{y} \binom{N+1-n}{j-y} y g_j \]
\[ - \sum_{y=0}^{n} \sum_{j=y}^{n} \binom{n}{y} \binom{N-n}{j-y} y g_j \]
\[ = \sum_{y=0}^{n} \sum_{j=y}^{n} \binom{n}{y} y g_j [\binom{N+1-n}{j-y} - \binom{N-n}{j-y}] \]
\[ = \sum_{y=0}^{n} \sum_{j=y}^{n} \binom{n}{y} y g_j [h_j'] \]

where
\[ h_j' = \frac{\binom{N-n}{j-y}}{\binom{N}{j}} - \frac{\binom{N+1-n}{j-y}}{\binom{N+1}{j}} \].
Hence, from the earlier argument, it can be observed that no general conclusion can be made regarding the increment of the number of TV programmes on Frequency. But from the earlier expression (3.77), it can be observed that the difference is positive if

\[ \frac{\gamma}{j} < \frac{n}{(N+1)} \]

and, the difference is negative if

\[ \frac{\gamma}{j} > \frac{n}{(N+1)} \]

Table (3.4) shows change in Frequency by changing the number of TV programmes within the channel considering the same numerical example discussed in section (3.5.4).
Table 3.4
Number of TV Programmes and Frequency.

<table>
<thead>
<tr>
<th>Number of TV programmes (N)</th>
<th>Frequency F(N)</th>
<th>First Difference ΔF(N)</th>
<th>Second Difference Δ²F(N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4  ( - )</td>
<td>0.1867 (-)</td>
<td>- 0.0372</td>
<td>0.0122</td>
</tr>
<tr>
<td>5  (25 %)</td>
<td>0.1495 (- 24.88%)</td>
<td>- 0.0250</td>
<td>0.0073</td>
</tr>
<tr>
<td>6  (50 %)</td>
<td>0.1245 (- 20.08%)</td>
<td>- 0.0177</td>
<td></td>
</tr>
<tr>
<td>7  (75 %)</td>
<td>0.1063 (- 16.57%)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We see that the average Frequency is decreased by 24.38 % for an increase of 25 % in the number of programmes in each TV channel.

The numerical example discussed here illustrates, the sensitivity of Reach and Frequency with respect to the change in the number of TV channels and TV programmes which shows that they are sensitive to the number of TV channels and programmes used in the planning horizon. Hence the great amount of efforts should be put up in determination of the number of
TV channels and the number of TV programmes for relaying the advertisement which we have discussed in Chapter-4.

3.6 **Conclusion:**

In India TV has emerged as a powerful advertising medium having the choice of multiple TV channels in near future. Hence, it has become important to know the way in which viewers use television, the extent to which they identify with particular channels or programmes and the resulting impact on them.

From the results of Model-1, it can be observed that people are loyal to a particular channel and there is very little tendency to switch-over to a different channel.

It has also been shown that the probability that a person will view at least one programme, when we utilize all the channels is greater than that of using only one channel whereas average number of programmes seen by a person in both the situations are the same. This means that when we utilise more channels, more persons will be exposed to the advertisements
because the persons are loyal to the channel and hence if we use only one channel for showing the advertisement, only those persons will get advantage of seeing the advertisement who are operating that particular channel and the remaining are not able to see it. Therefore, if we want that maximum number of persons should see the advertisement atleast once, we should arrange the number of programmes in such a way that at least one advertisement is relayed on each of the available channels.

But for Model-1, the data on either the probability that the person from target audience will operate a particular TV channel or see the particular programme within the channel are not easily available. To obtain these data it is necessary to assess the programme-viewing habits of the target group. Therefore, for the resolution of the problem, we have decided to develop a more general approach in Model-2 which starts with the data on TV channel-operation and programme-viewing habits of the target audience and ends by providing for a given plan, the Reach and Frequency it achieves. This methodology has provided a base for preparation of an optimal plan according to a chosen criterion which has been discussed in the next chapter.
Though proposed models provide methodology for solving TV planning problem, they are based on strong assumptions. The assumption that each channel has the same probability of being operated is unrealistic since it rises and falls on the basis of their relative ratings. Similarly the assumption that each programme has the same probability of being viewed is also unrealistic because the programme ratings are very dissimilar. Also the assumption that probability of viewing different programmes and channels is independent is not practical since people are more likely to watch a particular set of shows than others. Thus a more realistic model can be developed by relaxing the above discussed assumptions.