CHAPTER V

IMPLICATIONS OF NET SHIFT IN POPULATION DUE TO MIGRATION ON INTERCENSAL MORTALITY AND FERTILITY ESTIMATES

5.1 Introduction:

There is a reason to believe on the grounds of several studies conducted so far, that considerable shift in populations of an area due to migration, may cause structural changes in the socio-economic and demographic characteristics of the population of that area. Such characteristics are various in kind and nature. The present work, however, does
not seek to examine the extent of such changes in all such characteristics, but the same has been confined to only a few demographic determinants like mortality and fertility, which are mainly responsible to induce changes in the population in absolute terms.

Estimates of mortality and fertility are often needed in assessing the problems and formulation of plans for health, family welfare and many other important functions of government and private organisations. For the hearing needs of measure of these components, we often report to the quick estimates through the use of certain standard indirect techniques for the reasons already discussed in the earlier chapters of this thesis. Some of the most feasible indirect techniques which can be used under the situations of inadequate statistical systems for obtaining the estimates of important vital events have also been referred in Chapter III. An elaboration of these techniques is not, however, the very purpose of the present attempt, but
the basis on which such techniques have been developed will have to be referred at appropriate places in the present analysis for evaluating the implications of net shift on the rates of mortality and fertility of an area.

5.2 Mortality, Fertility and Migration:

As discussed earlier, mortality, fertility and migration are the three main components of population which determine the current and the future changes in the population. Of the above three components, migration not only determine change in the population but also affects the estimates of the first two components during a specified period.

It is therefore pertinent that, any indirect technique available for obtaining the estimates of vital rates, shall produce reasonable results only if the population under consideration is closed to migration. If there exists, however, a significant shift in population due to migration, the population is required to be adjusted to an
extent to which an amount of net shift is present in the population. Estimation of net shift, for the period for which the estimates of vital rates are desired, therefore becomes an essential part of the procedure.

In view of the problems, already discussed in Chapter II, about the availability of adequate and qualitative data for obtaining the necessary estimates of net shift, the important step of adjusting the population for net shift due to migration is generally surpassed while obtaining the desired estimates of vital rates through the indirect methods. This is normally done when the estimates are required on an urgent basis for our plan formulations.

In absence of any direct evidence and without having any reason to believe that the population under consideration has remained free from any amount of net shift, the estimates of vital rates obtained on the basis of unadjusted population, may sometime lead to very wrong conclusions. The usefulness of such estimates therefore should be well evaluated, keeping in view the purpose for which the same have to be made use of.
5.3 Present attempt:

In the present endeavour an attempt has, therefore, been made to highlight the implications of net shift on the estimates of mortality and fertility by means of developing certain mathematical inequalities. Before proceeding further in our attempt it is important to clarify at this stage that the terms mortality and fertility referred here, have particularly been used as general terms for the death rate and birth rate respectively.

For developing the proposed inequalities for evaluating the implications mathematically, at the first instance, I started with an assumption that the population, for which the estimates of mortality and fertility are to be determined, is in fact open to migrating and possesses an element of significant net shift due to migration. At the second instance, I obtained two sets of estimates of mortality and fertility for the same population based on two independent propositions as under:
(i) The population is closed to migration and may have only a negligible amount of net shift during the period under consideration so that the desired estimates of vital rates may be obtained without adjustment of population for migration or net shift due to migration.

(ii) The population is open to migration and possesses a significant amount of net shift, so that the desired estimates of vital rates can be obtained after adjustment of population for migration or net shift due to migration.

The basic concept used in developing the present analysis pertains to the fact that, the presence of net shift in the population an intercensal period \((0, t)\) may spuriously inflate or deplete the census survival rates \(\frac{L_x}{L_{x-t}}\); since the migrants are often concentrated in certain age and sex groups and as such a substantial volume of net shift may strongly bias the survival rates, which in turn, may significantly affect the mortality and fertility rates also.
5.4 Development of Mathematical Analysis

The main essence of my approach is, as said in section 4.3, to obtain mortality and fertility estimates using indirect methods for the population (open to migration) unadjusted and adjusted for net shift due to migration with two different propositions referred in section 5.3 and certain other assumptions narrated in the following sub-section.

5.4.1 Assumptions:

For carrying out the proposed mathematical analysis, the following additional assumptions with regard to the availability of certain basic data and life tables etc. are made:

1. Availability of populations classified by age and sex for the two censuses covering the intercensal period under consideration.
(ii) Availability of smoothed distribution of populations by age and sex in order to minimise the errors due to age-misreporting.

(iii) Availability of a set of Model Life Tables with an assumption that abridged form of these tables can be logically employed to construct an approximate schedule of mortality.

(iv) Estimate of total net shift in population for the intercensal period \((o, t)\) of the area for which the implications are to be evaluated.

5.4.2 Estimation of Net shift, Mortality and Fertility Rates:

Since initially we have assumed that the population under consideration is open to migration, we are required to obtain two sets of estimates of mortality and fertility rates in relation to the population unadjusted for net shift and the population adjusted for net shift due to migration respectively.
For obtaining the second set of estimates with population adjusted for the net shift the principal requirement at the beginning would be the estimate of net shift in population for the area under consideration. Since net shift takes place at different rates in different age-groups, the same may be obtained by age and sex groups through the survival rate method as discussed in Chapter-I.

5.4.3 Unadjusted and Adjusted Estimates of Mortality and Fertility:

The underlying rationale of the techniques for obtaining the mortality and fertility rates are based on the United Nations recommended method and does not require much emphasis here. It is, however, necessary to peep into its conceptual aspects, so that the same can be made use of in our present analysis.

The basic literature, which is required for the above purpose is a set of model life tables and the records of population growth and its distribution by age. Initially,
the estimates are obtained generally for one sex and then
the estimates for the other sex and for the total persons
are generated using the sex ratio at birth and the mean sex
ratio of the total population. In my discussions for compu-
tational procedure, I have dealt with only one sex viz;
female in the beginning.

The essence of computation is to find a life table,
from among the model tables, which may be used to project
the population at initial census to produce population at
terminal census most consistent with the recorded one. The
appropriate life table may be located by trial and error and
by using the interpolation techniques. Mathematically the
procedural steps can be elaborated by assuming the following
terms:

Let

(1) \[ n_{x-t}^P \] and \[ n_x^P \] = The female populations at initial
and terminal censuses respectively in the age-group
\((x, x+n)\), aparted by an intercensal period \((0, t)\).
(ii) $n^*_x = \text{Adjusted female population at the terminal census.}$

(iii) $n^M_x = \text{Estimated net shift in the female population due to migration in the age group } (x, x+n).$

(iv) $n^Q_x = \text{Mean unadjusted female population in age-group } (x, x+n), \text{ such that:}

$$n^Q_x = \frac{1}{2} (n^{P_o}_x - n^{P_t}_x) \quad \ldots (5.1)$$

(v) $n^*_x = \text{Mean adjusted female population in age-group } (x, x+n), \text{ such that:}

$$n^*_x = \frac{1}{2} (n^{P_o}_x - n^*_x) \quad \ldots (5.2)$$

(vi) $L_m$ and $L^*_m = \text{Unadjusted and adjusted median levels of mortality respectively.}$

(vii) $n^m_x$ and $n^*_m = \text{Unadjusted and adjusted life table mortality rates for females as implied by the census survival rates in the age-group } (x, x+n)\text{ based on the median levels of mortality } L_m \text{ and } L^*_m \text{ respectively.}$
(viii) \( n_s^{x,u} \) = Life table survival rate at mortality level 
(u) in the age-group \((x, x+n)\) for females.

(ix) \( n_p^{x,u} \) = Projected female population at mortality 
level (u) for the ages \(t\) years and above.

On the basis of the above defined terms, the unadjusted 
and adjusted rates of mortality and fertility can be 
obtained as discussed in the following sub-sections:

5.4.3.1 Unadjusted Estimates:

The initial population \( n_p^0 \) can be projected to 
the date of terminal census by applying \( n_s^{x,u} \) and thereby 
we get a set of projected values of the population at the 
terminal census with respect to different levels of morta­

\[ P_u = \sum m_p^{x,u} = \sum (n_p^0 \cdot n_s^{x,u}) \quad \ldots (5.3) \]

where \( P_u \) is the projected population at level of morta­

\( \ldots \) (u). On the basis of a set of projected values \( P_u \) at 
different values of level (u), it is possible to determine
at least two mortality levels, such that the actual census population \( \sum n^x P_x \) for \( x \geq t \), can be bracketed between the projected population \( P_u \) at these mortality levels say \( u = c \) and \( u = d \), such that we should have the following conditions

\[
P_c \leq \left( \sum n^x P_x \right) \leq P_d
\]

Hence on the basis of the two values of the projected population \( n^x P_x, c \) and \( n^x P_x, d \) and the levels of mortality \( c \) and \( d \), the mortality level \( L_1 \) implied by the census survival rate can be obtained through the process of interpolation as under:

\[
L_1 = (d-c) \left( \frac{P_x^t - n^x P_x, c}{n^x P_x, d - n^x P_x, c} \right)^{-1}; x \geq t \quad (5.5)
\]

\( L_1 \) is determined for population with age \( t \) years and above. Similarly we can determine appropriate levels of mortality for population aged \( (t+5) \) years and above, \( (t+10) \) years and above and so on and in this manner we get a sequence of mortality levels say \( L_1, L_2, L_3 \ldots \), for the
proportions surviving from the earlier census \( n_x^0 \) to age \( t \) and above, \( t+5 \) and above and so on at the latter census. The most satisfactory level of mortality according to the United Nations \( 58 \) Manual IV, to be selected out of the sequence of levels obtained above, is the median level of mortality \( (L_m) \) among the first nine levels.

Once \( L_m \) is determined, the death rate can be estimated by applying the age specific mortality rates \( n_x^m \) in the selected model table at median level of mortality \( (L_m) \) to the estimated mean or mid-period population \( Q_x \). The total number of female deaths \( D \) (say) during the intercensal period can therefore be obtained thereafter with the help of the following equation:

\[
D = \sum (Q_x \cdot n_x^m) \quad \text{... (5.6)}
\]

On the basis of the value of \( D \) and the total estimated mid-period population \( \sum n_x^0 \), the death rate \( d_x \) (say) per person per year can be obtained as under:
or \[ d_r = D \cdot \left( \sum n_j q_{nx} \right)^{-1} \]

\[ \text{or } d_r = \left( \sum n_j q_{nx} \cdot n_x \right) \left( \sum n_j q_{nx} \right)^{-1} \] ... (5.7)

If \( r \) is the exponential growth rate of the female population, then the female birth rate \( (b_r) \) can be estimated as under:

\[ b_r = r + d_r \] ... (5.8)

The birth rate for the male population and the total population can now be obtained on the basis of the sex ratio at birth and the sex ratio of the total mean population respectively. The death rates for males and the total population can thereafter be determined with the help of the growth rates and the estimated birth rates.

It may be noted here that the estimates of vital rates obtained above have not been subjected to any adjustment for an amount of net shift which has been assumed to be existing in the population under consideration.
5.4.3.2 Adjusted Estimates:

The adjusted estimates of mortality and fertility can be obtained on the lines similar to those discussed for unadjusted estimates, except that the population of the terminal census requires to be adjusted by age and sex for an amount of net shift present in the population for the period under consideration.

The adjustment for net shift in the population at the terminal census can be done with the help of the estimates $n_{Mx}^M$ of net shift on an assumption that the rate of net shift has remained uniform during the intercensal period. The adjustment can therefore be done as under:

Since $n_{Qx}$ and $n_{Qx}^*$ have been defined as the unadjusted and adjusted mean population, the adjusted total population at terminal census in the age-group $(x, x+n)$ can easily be obtained in terms of $n_{Qx}$ and $n_{Qx}^*$ through the following equation

$$n_{Px}^* = n_{Px} \cdot n_{Qx}^* (n_{Qx})^{-1} \quad \ldots (5.9)$$
But \( \hat{n}_x \) is the adjusted mean population for an amount of net shift \( n^M_x \), and can be obtained on an assumption of uniform rate of net shift as under:

\[
\hat{n}_x = n_x - \frac{1}{2} n^M_x
\]  \( \ldots (5.10) \)

Substituting the value of \( \hat{n}_x \) from above in equation (5.9) we get

\[
\hat{n}_x = n_x ( n_x - \frac{1}{2} n^M_x ) ( n_x )^{-1} \ldots (5.11)
\]

5.4.4 Set of Conditions:

The net shift in population may be on either direction i.e. its value may have either a negative or a positive sign depending upon an excess of either inflow or outflow of population over the other. Hence on the basis of the direction of net shift \( n^M_x \), I have obtained below a set of conditions showing various inequalities between the adjusted and unadjusted estimates of population and its components, on an assumption that the rate of net shift has remained uniform during the intercensal period under consideration.
A : Conditions for Population

Examining the inequalities between \( n^p_x \) and \( n^q_x \) & \( n^q_x \) and \( n^p_x \), we have the following set of conditions:

(i) If \( n^M_x \leq 0 \)

Then \( (n^Q_x - \frac{1}{2} n^M_x)(n^Q_x)^{-1} \geq 1 \)

and therefore

(a) \( n^p_x \geq n^t_x \)

(b) \( n^q_x \geq n^Q_x \)

and therefore

(c) \( p \geq p^t \) and \( q \geq Q \)

(ii) If \( n^M_x \geq 0 \)

Then \( (n^Q_x - \frac{1}{2} n^M_x)(n^Q_x)^{-1} \leq 1 \)

so that

(a) \( n^p_x \leq n^p_x \)

(b) \( n^q_x \leq n^Q_x \)

and therefore

(c) \( p \leq p^t \) and \( q \leq Q \)
B: Conditions for Mortality Rates

Examining the inequalities between $m^*_x$ and $m^x$, we obtain the necessary set of conditions as under:

(iii) If $M^x \leq 0$

Where the net shift is negative, it has been pointed out under conditions i(a) and i(b), that the adjusted values of the populations would be higher than the actual values as returned at the terminal census. Therefore, on the basis of this conclusion and the equation (5.5) meant for finding the appropriate level of mortality, it is clear that the level of mortality $L^*_m$ in case of adjusted population shall be higher than the level $L_m$ for unadjusted population. Secondly when the level of mortality is higher, the rate of mortality is lower. We have therefore, the following set of conditions:

(a) $L^*_m \geq L_m$ (due to condition i(a))

(b) $n^*_x \leq n^x$ (due to condition iii(a))
If $n^M_x \geq 0$

We have the following set of conditions based on the similar considerations as above.

(a) $L^*_m \leq L_m$

(b) $n^*_x \geq n^m_x$

The above inequalities can be more clearly explained on the basis of the fact that higher terminal population would generate higher level of mortality and higher survival rates, while higher survival rates would generate lower life table mortality rate. Therefore a negative netshift in population would cause a lower death rate as compared to the death rate estimated on the basis of unadjusted population, and similarly a positive net shift would generate an inflated death rate.

C: Conditions for Growth Rates

The exponential growth rate $r$ per person per year during the period $(0, t)$ can be obtained for the adjusted
and unadjusted population with the help of the following equations:

\[ \tilde{P} = P_0 e^{\tilde{r}t} \quad \ldots \quad (5.12) \]
\[ P^t = P_0 e^{rt} \quad \ldots \quad (5.13) \]

where \( \tilde{r} \) and \( r \) = adjusted and unadjusted exponential growth rates respectively.

\[ \tilde{P} = \sum_n \tilde{P}_x \text{ = total adjusted population at the terminal census} \]
\[ P^t = \sum_n P^t_x \text{ = total unadjusted population at the terminal census} \]
\[ P^0 = \sum_n P^0_x \text{ = total initial population} \]

Dividing the equation (5.12) by equation (5.13) we get the following equation

\[ \tilde{P} (P^t)^{-1} = \frac{\text{d} \tilde{P}}{\text{d} \tilde{r}} (\tilde{r} - r) \]

or \( \tilde{r} = r + \frac{1}{2} \log \frac{\tilde{P}}{P^t} \cdot (P^t)^{-1} \quad \ldots \quad (5.14) \)
Now examining the inequalities between $\hat{r}$ and $r$ on the basis of above equation (5.14) and the conditions i(a) and ii(a) obtained earlier, we obtain the following set of conditions:

(v) If $\sum_{n} M_{X} \leq 0$

When $\sum_{n} M_{X}$ is negative or zero the second term on the right hand side of the above equation (5.14) shall have a $+$ve sign, since according to condition i(a) $\frac{\hat{r}}{\mu}$ is positive and greater than one. Hence we have the following conditions:

(a) $\frac{1}{t} \log_{e} \left( \frac{\hat{r}}{\mu} \right) > 0$ (due to condition i(c))

(b) $\hat{r} \geq r$ (due to condition v(a)).

(vi) If $\sum_{n} M_{X} > 0$

When $\sum_{n} M_{X}$ is positive, the conditions can similarly be obtained as under:
(a) \( \frac{1}{t} \log_e \left( \frac{P^*}{P^t} \right) \leq 0 \) (due to condition ii(c) )

(b) \( r^* \leq r \) (due to condition vi(a) )

Conditions v(b) and vi(b) show that the value of growth rate in population depends on the amount and direction of the net shift present in the population. The value of the adjusted growth rate would be higher than the unadjusted one when the population has experienced a negative net shift during an intercensal period \((o, t)\) and the same would be lower than the unadjusted one when there has been a positive net shift in the population.

D. Conditions for Death Rate and Birth Rate

If \( d_r^* \) and \( d_r \) are the adjusted and unadjusted death rates, the value of the same can be obtained as under

\[
d_r^* = \frac{m}{n} \cdot \frac{q}{n} \cdot \left( \sum \frac{q}{n} \right)^{-1}
\]

... (5.15)

and \( d = \frac{m}{n} \cdot \frac{q}{n} \cdot \left( \sum \frac{q}{n} \right)^{-1}
\]

... (5.16)

Dividing the equation (5.15) by the equation (5.16) we get
\[
\hat{d}_r = \frac{\mathcal{L}(m_x, q_x)(\sum q_x)^{-1}}{\mathcal{L}(m_x, q_x)}
\]

or

\[
\hat{d}_r = d_r \frac{\mathcal{L}(m_x, q_x)}{\mathcal{L}(m_x, q_x)} \cdot \frac{Q}{Q} \quad \ldots \quad (5.17)
\]

Now examining the inequalities between \( \hat{d} \) and \( \hat{d} \), on the basis of the above equation, we obtain the following set of conditions:

(vii) If \( \sum M_x \leq 0 \)

When \( \sum M_x \leq 0 \), then on the basis of the conditions already established earlier, the coefficient of \( d_r \) on the right hand side of the above equation (5.17) shall be normally less than unity, since the aggregated term in the numerator is lesser than the similar term in denominator. This can be explained more clearly in view of the fact that \( n^{Q_x} \) is greater than \( n^{Q_x} \) according to condition i(b) and \( n^{m_x} \) is lesser than \( n^{m_x} \) according to condition iii (b)
so that denominator on the right hand side of the above equation (5.17) would either be greater or equal or lesser than its denominator depending upon the values of the product terms $\sum (n^m_x, n^\ell_x)$ and $\sum (n^m_x, n^Q_x)$. But, since $n^\ell_x$ and $n^Q_x$ are the adjusted and unadjusted mean populations, the variation between the two may not be as high as the variation between the values of $n^m_x$ and $n^m_x$ which are based on the terminal populations, and therefore the coefficient of $d_r$ in equation (5.17) would be less than unity, and thus, we have the following condition:

$$\hat{d}_r \leq d_r$$

Similarly when the net shift is positive the inequality between $\hat{d}$ and $d$ can be obtained as given under condition (viii) below:

(viii) If $\sum n^M_x \geq 0$

$$\hat{d}_r \geq d_r$$
Conditions (vii) and (viii) show that if there exists a negative or positive net shift in the population and if the estimate of mortality rate is determined on the basis of unadjusted population, the same shall have a inflated or deflated value as the case may be.

Now, if $\hat{b}_r$ and $b_r$ are the adjusted and unadjusted birth rates, the same can be expressed through the following equations:

$$1 \quad \hat{b}_r = \hat{r} + \hat{d}_r \quad \ldots \quad (5.18)$$

and

$$b_r = \hat{r} + \hat{d}_r \quad \ldots \quad (5.19)$$

Substituting the value of $\hat{r}$ from equation (5.14) in equation (5.18) above, we get the following equation:

$$\hat{b}_r = r + \frac{1}{t} \log_e \hat{p} (p^t)^{-1} + \hat{d}_r \quad \ldots \quad (5.20)$$

Now substituting the value of $r$ from equation (5.19) in equation (5.20) above we get:
\[
\dot{b}_r = (b_r - d_r) + \frac{1}{t} \log_e \bar{P} (P^t)^{-1} + \hat{d}_r
\]
or \[
(\dot{b}_r - \hat{d}_r) = (b_r - d_r) + \frac{1}{t} \log_e \bar{P} (P^t)^{-1} \quad \ldots \quad (5.21)
\]
Hence from equation (5.21) we obtain the following conditions for the natural growth rate (i.e. birth rate minus death rate) and the birth rate:

(ix) If \( \sum M_x \leq 0 \)

(a) \((\dot{b}_r - \hat{d}_r) \geq (b_r - d_r)\)

Since \( \bar{P} > P^t \) according to condition i(c) and as such \( \log_e \bar{P} (P^t)^{-1} \geq 1 \).

Now since \( \hat{r} \geq r \) according to condition v(b) and \( \hat{d}_r \leq d_r \) according to condition (viii), we get the condition ix (b) as under:

(b) \( \dot{b}_r \leq b_r \)

Similarly when there is a positive net shift in the population, we have the following conditions:
(x) If \( \sum_{n}^{M} x \geq 0 \)

(a) \( (b_r - d_r) \leq (b_r - d_r) \) (due to condition vii (b))

and (b) \( b_r \geq b_r \).

Hence conditions ix (a) and ix (b) show that, in case the population which has experienced a negative net shift during a specified period, we may obtain a deplated rate of natural growth and an inflated birth rate if the population is not adjusted for an amount of net shift present therein, and similarly in case of a positive net shift, the unadjusted terminal population may provide an inflated natural growth rate and an deplated birth rate as has been shown under conditions x (a) and x (b) above.

The above conditions can also be extended to examine similar variations between an adjusted and unadjusted expectation of life at birth and at other ages, general and total fertility rates, gross reproduction rates and the net reproduction rates etc. For example, the value of the general
fertility rate depends on the total number of births and the female population count in the age-group (15-44), the unadjusted population may provide a depleted estimate of fertility rate if there is an overall net shift in the positive direction of the total population and also a positive net shift in the female population in the age-group (15-44). In case, however, there is an overall positive net shift in total population but a negative net shift in female population in the age-group (15-44), the extent of inflation or deflation would depend on the extent of net shifts present in both the sexes. Thus the different sets of conditions in the form of mathematical inequalities obtained in respect of mortality rate and birth rate, very specifically indicate about the implications of net shift and caution us about the far-reaching and indirect consequences in using the unadjusted estimates without having a proper access to the extent of possible net shift in population during the specified period.
An application of the above developed inequalities have been made on the population of Gujarat, India to analyse such implications during the period 1961-71 as a case study and which is incorporated in the next chapter of the present thesis.
Sketch Map of Gujarat State showing geographic sub-divisions adopted for the present net shift analysis.