Chapter 3

Adaptive Filtering of Speech

3.1 Introduction

The properties of acoustic noise origin and channel is time dependent hence the factors like phase, sound velocity of irrational noise, content and frequency component are non-stationary. To cope these challenges the ANC system requires being adaptive in nature. Adaptive filters are nonlinear and time-varying stochastic systems due to their dependence on the input data. The adaptive filter systems can be configured in four configurations [63]. Least Mean Square is considered as the leading algorithm in popularity among adaptive filters. In this chapter the Normalized Least Mean Square algorithm is considered with comparison in their performance. The SNR and MSE values are calculated for both algorithms, based on tests performed in MATLAB. In signal processing, Wiener filter is used for the estimation of arbitrary method by linear time invariant filtering a noisy signal [64]. Wiener filter is employed to generate the estimation of desired method by linear time-invariant filtering assuming that the signals are stationary.

Filter can be defined as the circuits which are used to select meaningful information from the transmitted signal which is appeared as input to the filter it may be sometime a part of hardware like in case of analog signal the filter is called as frequency selective network which select a range of frequencies out of mix of different frequencies. Similarly when we are talking about the software then it may be defined as the loop of instructions which gives desirable results and blocks undesired data from given information.

In the past few decades a significant development is seen in the field to deduct noise signals from speech signals. Recent developments in the method of filtration are sourced from technical progress that interested the researchers. Today’s digital signal processing systems are high in performance with minimum consumption of power, lower in cost and flexibility towards various standards of test [65].

Adaptive filter is a digital filter which alters its characteristics iteratively till the desirable output will achieved from input signals. As the name suggest it can adapt the condition which
really need to get the desired output. There are number of adaptive filters available depends upon the applications and design criteria.

An adaptive filter is defined on following basis:

- The filter processes the signals.
- The filter coefficients and its formulation vary the input signal to respective filtered output signal.
- The filters offers iterative filtering which means the parameters of filters can be manipulated in desired order.
- The algorithm of filter decides their parameters adjustment for a given time of filtering. Figure 3.1 presents a block diagram of adaptive filter.

![Figure 3.1: Block Diagram of Adaptive filter](image)

The adaptive filters are based on time dependent, self-adjusting characteristics. The adaptive filters are generally created by enhancing the FIR filters using an adaptive algorithm that creates a new range of coefficients at every stage. The approach of these coefficients is to create a filter that outputs an error free signal and the noise content with lowest value in signal. The noise in signal is identified through the given signal flow diagram (each diagram is different for a system) and this signal evaluates the system’s closeness to optimum. Most adaptive filter algorithms are the generalized expression of conventional wiener filter and hence to gain the knowledge of adaptive filters, a study of wiener filter is necessary.
3.2 Adaptive Filtering System Configuration

Technical applications of adaptive filtering first surfaced in early 1960s. The first adaptive filter is the equalizer that neutralizes the effect of Inter-Symbol Interference (ISI) for information in telephone propagation channels [66]. Later, the configuration of adaptive filters enhanced and broad category of applications exploited their feasibility. The adaptive algorithms were developed mathematically for being compatible to communication systems and signal processing. Adaptive filtering is the analysed of four configurations: (a) System Identification (b) Noise cancellation (c) Linear Prediction (d) Inverse System. Though these algorithms resemble each other in implementation, the configurations are distinct enough to differentiate. The input signal $x(t)$, result (desired) $d(t)$, output $y(t)$, transfer function and error signal $w(t)$ and $e(t)$ respectively are used in all four systems. Here, error signal $e(t)$ is difference of desired and actual output $e(t) = y(t) - d(t)$. System identification and inverse system configuration have unknown linear constant $u(n)$ for generation of linear output to present input.

3.2.1 Adaptive System Identification Constant

System identification determines the discrete estimation of transfer function if the variables of system are undefined. The signal $x(t)$ is compared with output $y(t)$ for both the adaptive and system with undefined variables. The desired signal response $d(n)$ is the difference of output from adaptive system and output from unknown one. The convergence approach is used to shift $d(n)$ towards zero. The iterative process shifts the convergence close to unknown function and consequently minimizing the error function.

3.2.2 Noise Cancellation Configuration

The input signal $x(t)$ corrupted with noise $N_1(t)$ on comparison with $d(t)$ having $s(t)$ (corrupted with another noise $N_0(t)$). The adaptive filter coefficients adapt to cause the error signal to be a noiseless version of the signal $s(t)$.

It is necessary in this case that both noises are uncorrelated to signal $s(t)$ each other. However, noise correlation with each other is sustainable for system [67]. If $e(t) = s(t)$ holds true than other equations can be written as

$$y(t) \approx N_1(t) \quad (3.1)$$
\[ d(t) = s(t) + N_0(t) \]  \hspace{1cm} (3.2)

And

\[ e(t) = s(t) + N_0(t) - y(t) \]  \hspace{1cm} (3.3)

### 3.2.3 Linear Prediction

This method has two fold applications. First, if the output is generated from error signal \( e(t) \) then linear prediction is performed. The next input signals are predicted by adaptive filter coefficients from statistics of input signal \( x(t) \). In second application, this method filters noise; similar to adaptive noise cancellation if output is generated from \( y(t) \).

### 3.2.4 Inverse System Configuration

In this system, adaptive filter models the inverse of unknown system \( u(n) \). In adaptive equalization this method is efficient as spectral changes are eliminated caused by prior system or transmission line.

### 3.3 Adaptive Algorithm

The old method- steepest descent is believed to the origin for the optimization of gradient based adaptation algorithm [68]. This recursive algorithm initiates with an arbitrary initial value and with each step, the tap weigh vector updates. The resultant tap weight vector is the convergence of wiener filter solution. The least mean square algorithm is exhaustively studied in second chapter of this thesis under the topic ‘literature review’ and the analysis of various works from independent researchers states the miss-adjustment of filter with the wiener filter and adaptive filter. Another approach- fixed step size least mean square (FSS LMS) belongs the category of stochastic gradient algorithms. Stochastic gradient differentiates the FSS LMS algorithm from steepest descent that employs the deterministic gradient in an iterative process of Wiener filter designed for stochastic inputs. The algorithm is not based on the results of pertinent correlation function measurements, neither use inversion of matrix. Authors of [69] discuss the optimization issue for step size methods at every stage of performance enhancement.

Many researchers have drafted a number of algorithms out of the adaptive filter for active noise cancellation [70]. The first approach of enhancement was the deployment of LMS
algorithm along with adaptive filter. Though the LMS algorithm is easy to design for the programmers, the slow convergence rate does not make it an appreciated selection. A solution to this is normalized LMS filter algorithm and thus was accepted against it. NLMS algorithm provides powerful learning rate in comparison with LMS algorithm based filtering. Other adaptive filter combinations are described follow on.

3.3.1 Wiener Filter

Wiener filters belong to the category of linear optimum discrete time filters. The filters are the specific combination of transversal finite impulse response (FIR) filter that employs the Mean Square Error (MSE) for gaining the tap weight vector of optimum filter and minimize the MSE signal. Wiener filters are actually designed for the complicated case of time series with filter designed have specific impulse response to stimulate complex baseband signal in real application scenarios.

The wiener filter architecture seeks the prior knowledge about data statistics that would be input to the system [71]. The optimum nature of filter is seen only in cases where statistical features of noisy speech signal correlates with that provided as the prior information to the filter. In case if this information is absent, the design of wiener filter would be complicated and can never be stated as optimum. In such cases, ‘estimate and plug’ approach is generally followed where, the two-stage filtering process has first stage of estimating the statistical parameters followed by the ‘plugs’ to obtain results in non-recursive formula.

In physical deployment of Wiener method, a major hindrance of high cost equipments is noted. Hence considering all the aspects, the use of wiener method can be entertained in most hostile conditions where no other possible solution is available. A solution to wiener filter is the adaptive algorithm that converge its filtration based on the characteristics of input signal. The adaptive filters use recursive algorithm and this algorithm powers the filter to produce the desired output in a vulnerable medium and where the statistical information of input signal is absent. The adaptive algorithm considers arbitrary points of characteristics and with increase in iteration the algorithm is modified. The optimum wiener solution in statistical cases can be obtained. In a non-stationary environment the adaptive algorithm tracks the input signal and the statistical data of it provided that variations are sufficiently low.
The direct result of recursive algorithm application depends on its iterative process where the parameters upgrade from one level to another. With increase in iteration steps the parameters obtain dependency on data. Hence, the adaptive filters in actual practice are non-linear in nature and do not follow the superposition principle. However, the adaptive filters are generally categories as linear and nonlinear filters. The adaptive filters are said to be linear in case when quantity estimate is computed in an adaptive manner for the linear combination of observed outputs is given as input. In the else case, the filter is termed as non-linear filter. A wide range of recursive algorithms are generally deployed as the solution for linear filter (adaptive) [71]. The choice of algorithm however is dependent on following points:

- Rate of Convergence: the number of iterations in a given algorithm is termed as rate of convergence. The stationary characteristics of input signal are converged ‘close enough’ to attain the optimum wiener solution. The convergence rate allows rapid adapting of arbitrary statics in a stationary environment.

- Miss-adjustment: The difference in the convergence rate creates a cumbersome state for filters. The parameters are scaled quantitatively to the amount equal to final MSE, if produced to the adaptive filter will direct the solution against the MMSE that was introduced by Wiener filter.

- Tracking: in adaptive filters, the filtering equations are operated in a shifting environment. The algorithms of system track the variation in statistical parameters of given input. The algorithm’s capability is tracked based on the ROC and steady state fluctuation of noise.

- Robustness: The robustness measure of an algorithm is dependent on the disturbances caused due to small packets of energy components and leads to small abrupt estimation errors. The source of these disturbances lies in number of factor, existing internally or externally to filter [72].

- Computational requirements: total operations for example: addition, subtraction, multiplication and division are counted for a single step of iteration. Secondly, the memory units required to save intermediate data and program is figured and last but not the least, investment in programming an algorithm for a given simulator system.
Structure: the information flow for a given algorithm is referred in this section to determine the hardware cost. The analog algorithm having higher modular structure and parallelism or concurrency that suits well to implant the system in VLSI.

Numerical Properties: the numerical implementation of algorithm produces higher level of accuracies reason being the quantization errors. These errors generate due to ADC conversion of input signals and digital form of system calculations. Though only the later source of quantization is mainly responsible for quantization errors in signal. The main concerning issues of a system are generally the numerical stability and accuracy [72]. The stability being an inherent characteristic cannot be controlled but the accuracy of system is generally given by the total bits used as representation of sample data and coefficients of filter. The adaptive approach of filtering algorithm is robust enough being insensitive to word-length variation in digital implementation.

These factors, in their own ways, also enter into the design of nonlinear adaptive filters, except for the fact that we now no longer have a well-defined frame of reference in the form of a Wiener filter. Rather, we speak of a nonlinear filtering algorithm that may converge to a local minimum or, hopefully, a global minimum on the error-performance surface.

3.3.1.1 Transversal FIR Filter

Transversal FIR filter characteristics are presented in form of vector with tap weights values known to the system [73]. These tap weights are responsible to determine filter’s performance of system. In form of column vector, the values are given by:

$$ w(n) = [w_0(n)w_1(n)w_2(n)\ldots\ldots\ldots\ldots w_{n-N+1}(n)]^T $$  \hspace{0.5cm} (3.4)

The above vector is nothing but the FIR filter’s impulse response. The elements in this vector correspond with filter order given by N. FIR usage are easy and the filter output for any given time n is the product’s sum of tap weight vector to the delayed values of input. The vector representation of this delayed input in form of column is:

$$ x(n) = [x(n)x(n-1)x(n-2)x(n-3)x(n-N+1)]^T $$ \hspace{0.5cm} (3.5)

The output of the filter at time n is expressed by equation 3.7. The vector containing the time delayed input values at time n is referred to as the input vector x(n). In adaptive filtering the
tap weight values is time varying so for each at each time interval a new FIR tap weight vector must be calculated, this is denoted as the column vector

$$w(n) = [w_0(n)w_1(n)w_2(n) ...... w_{N-1}(n)]^T$$

(3.6)

So the generalized output is given as:

$$y(n) = \sum_{i=0}^{N-1} w_i(n)x(n - i)$$

(3.7)

![Linear Combiner of adaptive filters (Wiener Filters)](image)

**Figure 3.2: Linear Combiner of adaptive filters (Wiener Filters) [72]**

To compare the IIR and FIR filters, adaptive filters are drawn with varying coefficients [72]. The iterative method of tap adapts filtering use weight control mechanism or transversal filter with updating weights.

The objective function of the Wiener filter is written as:

$$E[e^2(k)] = \xi (k)$$

(3.8)

$$\xi = E[d^2(k)] - 2w^T E[d(k)x(k)] + w^T E[x(k)x^T(k)]w$$

$$= E[d^2(k)] - 2w^T p + w^T R w$$

(3.9)
Where,

\[ p = E[d(k)x(k)] \] is cross correlation vector between desired and input signals.

\[ R = E[x(k)x^T(k)] \]

In matrix form it is can be written as:

\[
\begin{bmatrix}
  r_{00} & r_{01} & r_{0,N-1} \\
  r_{10} & r_{11} & r_{1,N-1} \\
  \cdot & \cdot & \cdot \\
  \cdot & \cdot & \cdot \\
  r_{N-1,0} & r_{N-1,1} & r_{N-1,N-1}
\end{bmatrix}
\]

(3.10)

Gradient Vector of MSE function is given by:

\[
g_w = \frac{\delta \xi}{\delta w} = \begin{bmatrix}
  \delta x \\
  \delta w_0 \\
  \delta w_1 \\
  \delta w_N
\end{bmatrix}^T
\]

(3.11)

### 3.3.2 Least Mean Square Filtering Algorithm

In Least Mean Square algorithm, adaptive filter weights are employed in accordance with input speech signals. The algorithm being adaptive in nature holds valid for a number of signal types. The algorithm computes output of linear filter response for the given noise reference signal and generates the errors that are estimated in terms of theoretical response and actual output. The signal flow graph demonstrating the LMS algorithm is:
The matrix $R$ (in previous section) denoted by $\tilde{R}(k)$, and of vector $p$, denoted by $\tilde{p}(k)$, can be used to find Wiener solution of following equation [73]:

$$w(k+1) = w(k) - \mu \hat{g}_w(k)$$

$$= w(k) + 2\mu (\tilde{p}(k) - \tilde{R}(k)w(k))$$

(3.12)

Where, $\hat{g}_w$ is estimation of gradient vector in respect to filter coefficients.

A solution to estimate the gradient vector is given by:

$$\hat{g}_w(k) = -2d(k)x(k) + 2x(k)x^T(k)w(k)$$

$$= 2x(k)(-d(k) + x^T(k)w(k))$$

$$= -2e(k)x(k)$$

(3.13)

The resulting gradient-based algorithm is called least-mean-square (LMS) algorithm with update equation

$$w(k+1) = w(k) + 2\mu e(k)x(k)$$

(3.14)
3.3.3 Normalized LMS Algorithm

Normalized LMS (NLMS) falls in the category of adaptive algorithms that uses the training of filter coefficients of adaptive filter. The algorithms mark the diversion in input signal and the output of filter. The normalized step size parameter is selected for a very stable and high convergence rate algorithm.

The Normalized LMS algorithm is the modified form of LMS algorithm. The update equation for the gradient based algorithm is given by [14]:

\[
\mathbf{w}(k + 1) = \bar{\mathbf{w}}(k) + \mu \cdot e(k) \frac{x(k)}{||\bar{\mathbf{w}}(k)||^2} 
\] (3.15)

There is only single difference between the functioning of LMS and NLMS algorithm. The NLMS uses a time varying step (\(\mu (k)\)). This step size maintains the signal strength along with enhancement in rate of convergence. Opposite to LMS algorithm, NLMS algorithm offers comparatively smaller error signal. It is also seen that NLMS algorithm provides higher convergence rate than the conventional LMS algorithm due to its multiplication algorithm. The standard LMS filter applies the adjustment to tap-weigh vector at (k+1) iteration of filter as the product of three terms: step size parameter, tap-input vector and estimation error.

The tap-input vector \(u(k)\) is directly proportional with adjustment of filter. With increase in size of \(u(k)\) gradient noise amplifies in the LMS filter. To eradicate such problem, NLMS is deployed. The adjustment of \(u(k)\) at given iteration is said to be ‘normalized’ with respect to squared Euclidean norm. In terms of structure both LMS and NLMS replicate each other. The only difference of both the algorithm is mechanism of weight controller.

3.4 Performance of Adaptive Algorithm

The determining factors that affect the performance of algorithm are given in following points. Here, only major points are focused:

- Convergence Rate: The convergence rate of adaptive filter resembles the ROC of wiener filter. It is the rate of convergence to attain steady state mean square error.
• Miss-adjustment: the steady state behaviour of algorithm is described by miss adjustment. This quantitative measure ensembles the averaged the output of MSE that exceeds MMSE. The low is this parameter; the high is the performance rate of algorithm.

• Numerical Robustness: The adaptive filtering algorithms are implemented first as the simulation system on a personal computer that has finite word-lengths and produces high rates of quantization error. Sometimes, these errors are responsible for instability in adaptation algorithm. The algorithm is numerically robust if stability can be assured in actual implementation.

• Computational Requirements: from the perspective of practice point, computational parameter is an important factor. A number of operations are included in parameter during the full length of iteration cycle along with requirement of memory to store the intermediate data and program. These parameters are the costing factors of the system in which system is deployed [65].

• Stability: The stability of an algorithm is given in terms of the mean-square error of the signal obtained during the convergence cycle of a given finite input value. Though, a designer seeks a simple stable and more importantly the computationally robust algorithm as the solution of adaptive filter in their approach, the factors can lead to small miss-adjustment introduction with high convergence rate. The DSP application includes adaptive echo cancellation [74]

3.5 Applications of Adaptive Filtering

Adaptive filters are used extensively in various operations which has a wide range reason behind this is the simplicity and problem oriented nature of adaptive filter. Especially in case of communication and instrumentation it becomes popular because of correlation with various simulation techniques [70]. The discussion of some application is necessary to explain the problem of acoustic echo cancellation and related issue. The following are some general applications of adaptive filter:
3.5.1 System Identification

The system in figure 3.4 have a ‘black box’ presented in the dashed line that represents no one can see the quantities of the system from outside. The unknown system in this box provides a general input-output relationship with observation signal \( \eta(n) \) that creates hindrance in observed signal at the output of above system. If \( \hat{d} \) is the output for a given input \( x(n) \), the response of system is:

\[
d(n) = \hat{d}(n) + \eta(n)
\]  

(3.16)

In this system, the adaptive filter precisely formulates the input signal at output. In case if we assume that \( y(n) = d(n) \), the accurate modeling of adaptive filter is given in terms of unknown system driven by input signal.

3.5.2 Channel Identification

The transmission of useful signals from transmitter to receiver is offered through communication systems. The medium of transmission can either be solid (wire), liquid (medium) or gas (air). Since the system operates in real conditions, the channel show non-idealistic properties and distort the signal of transmitter before being received by receiver. The corrupted signal is difficult to decipher and the distortion effect is modeled through a linear filter that results in ‘smearing’ of some symbols commonly termed as the inter-symbol interference (ISI). The adaptive filter models the effect of channel ISI to produce output in an optimal manner. In such case of problem domain, the sender the sample signal whose
parameters are known to transmitter and receiver both. The receiver models the input signal via adaptive filtration for a given transmitted sequence. Through approach of suitable adaptation, adaptive filter parameters are stabilized and same parameters are used to decode further signals upon reception [71].

### 3.5.3 Plant Identification

In some of the control tasks, the transfer function knowledge for a given linear equation is used by physical controller to produce an efficient control signal. In the scenarios given above, transfer function is characterized using a known input signal and further matching the response of system with linear adaptive filter. After sufficient iterations, the system results in coefficients that are used to control overall closed-loop system for a specific desired system [71].

### 3.6 Noise Cancellation Model

The noise cancellation model for the filtration of noisy signals (Gaussian Noise) from the data signals is given in figure 3.5. The noise reduction problem to recover the speech signal of interest (SOI) \( x(k) \), by observation signal \( y(k) \) having the noise \( v(k) \)

\[
y(k) = x(k) + v(k) \tag{3.17}
\]

Where, \( v(k) \) is the additive noise, which is a Gaussian random process. It is assumed that the noise \( v(n) \) is uncorrelated with the SOI signal \( x(k) \) in figure 3.5. By applying the discrete Fourier transform (DFT) to equation 3.17, we can get the relationship of the signal model in discrete frequency domain.

![Figure 3.5: System Model Block Representation for Industrial Noise Cancellation](image-url)
3.7 Simulation Parameters

Table 3.1: Simulation Parameters for simulations in MATLAB

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Record Length (sec)</th>
<th>Filter Length</th>
<th>Filter Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>8000</td>
<td>2</td>
<td>0.02</td>
<td>24</td>
</tr>
<tr>
<td>11025</td>
<td>5</td>
<td>0.02</td>
<td>24</td>
</tr>
<tr>
<td>22050</td>
<td>5</td>
<td>0.02</td>
<td>24</td>
</tr>
<tr>
<td>44100</td>
<td>5</td>
<td>0.02</td>
<td>24</td>
</tr>
</tbody>
</table>

3.7.1 Mean Square Error

Mean Square error is the calculation of difference between the calculated output and the true output. For the data signals $x(k)$ and the calculated output after filtration $y(k)$ the Mean Square Error is given by the expression:

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (y(k) - x(k))^2$$

(10)

Where, $N=$number of iterations $y(k) =$output after noise signal filtration $x(k) =$ original input signal without noise element

3.7.2 Signal Noise Ratio

Signal-to-noise ratio (SNR) is the ratio of maximum power of the input signal and corrupting noise present in the system. The SNR can also be defined by the mean square error as:

$$SNR = 10 \log_{10} \left( \frac{Max_i^2}{MSE} \right)$$

(11)

Where $Max_i^2 = $ Maximum strength of data signal
3.8 Simulation Results

Figure 3.6: Input Data signals under test

Figure 3.7: Input Data signal input in the test using MATLAB simulations
Figure 3.8: Mixture of Gaussian Noise in the input signal

Figure 3.9: Filtration using Wiener filter
Figure 3.10: Filtration using LMS based adaptive filter

Figure 3.11: Filtration using NLMS based adaptive filter
Fig 3.12: PSNR and MSE value comparison of Wiener filter, LMS and NLMS.

Table 3.2: SNR and MSE values of filters

<table>
<thead>
<tr>
<th></th>
<th>LMS</th>
<th>NLMS</th>
<th>Weiner Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR (dB)</td>
<td>11.8259</td>
<td>12.9326</td>
<td>11.0788</td>
</tr>
<tr>
<td>MSE</td>
<td>0.0100214</td>
<td>0.00776714</td>
<td>0.0119027</td>
</tr>
</tbody>
</table>

3.9 Summary

Adaptive filtering has been used for speech denoising in the time domain. During the last decade, wavelet transform has been developed for speech enhancement. A noise reduction method for audio and speech signals is processed by applying adaptive linear filtering technique. The noise reduction problem has been formulated as a filtering problem which is
efficiently solved by using the NLMS method. In addition, the method pays attention to the non-stationary nature of some audio signal. Simulation results indicate that the proposed method can improve the performance of noisy audio signal. Through computer simulations, we have demonstrated that the proposed method is quite effective in noise reduction by comparing the MSE and PSNR values, especially in the case of stationary white Gaussian noise. The limitations of adaptive filters are illustrated through Empirical mode decomposition (EMD) that performs very well in such environments.