Cognitive Algebra for Prediction of Performance

Task and Age Differences

One principal finding of the present research is that cognitive algebra underlying prediction of performance varies as a function of nature of task and age of the judges. In the present research, three tasks, namely, exam performance, performance in puzzle competition, and performance in music competition were studied. Factorial plots of the Motivation x Ability effect for these tasks yielded seemingly different patterns in judgments. The three tasks are considered separately below.

Exam performance. In prediction of exam performance, all the five groups of children had a similar pattern of near-parallelism. Such a pattern can be accounted for by either an adding or an averaging rule. Distinguishing tests between the two rules, however, rejected the adding rule but supported the averaging rule. So prediction of exam performance may be represented by an averaging rule:
Exam Performance = \( Mw_M + A (1 - w_M) \), \hspace{1cm} (2)

where \( M \) and \( A \) denote motivation and ability and \( w_M \) and \( (1 - w_M) \) represent the relative weights of the motivation and ability information.

Recent research by Singh and Bhargava (1982a) shows that prediction of exam performance is usually made by the averaging rule of Equation 2. Although they obtained evidence for an adding rule in a few cases, the most often used rule was indeed the averaging one. On the basis of their results and the present result, it can be claimed, therefore, that prediction of exam performance obeys an averaging rule.

Psychologically, an averaging rule is analogous to a causal conception that motivation and ability are qualitatively similar. One cause can thus be expected to compensate for the other. In attribution framework, this averaging rule can be considered as equivalent to the multiple sufficient causal schema (Gupta, 1978; Kun & Weiner, 1973). That is, even one of the two causes can produce the effect.

**Puzzle competition.** Prediction of performance in puzzle competition yielded evidence for two different data patterns. Children of Standard IV had a pattern of parallelism; children of Standard VIII had a pattern of convergence.
Children of these two age levels seem to have followed two different integrational rules. Whereas children of Standard IV followed an adding rule:

\[ \text{Puzzle Performance} = M + A, \]  

(3)

children of Standard VIII followed an inverted multiplying rule:

\[ \text{Puzzle Performance} = M + A - MA. \]  

(4)

Equation 4 is similar to the one proposed by Oden (1977) for fuzzy disjunctions and by Wyer (1976) for disjunctions of probabilistic events. Surber (1981b) has also suggested its plausibility to account for the converging pattern in Motivation x Ability effect of her studies with college students.

The two data patterns mentioned above reflect on two different causal schemata for prediction of performance in puzzle competition. The parallelism pattern with children of Standard IV implies that prior preparation for participation in competition is equally effective with children of low and high puzzle-solving ability. The converging pattern with children of Standard VIII implies that prior preparation for participation in competition is more effective with children of low than of high ability. These beliefs about prediction of perfor-
mance suggest that the children of two age groups have different causal schemata for prediction of performance in puzzle competition.

It deserves mention that Kun et al (1974) studied prediction of performance on puzzles. They found evidence for the multiplying rule beginning Standard II. But results of the present research are completely different from those of Kun et al. The cultural difference hypothesis (Singh, 1981) thus seems to hold true beyond the academic task also.

Music competition. Prediction of performance in singing competition yielded three different data patterns: Truly fan pattern, parallelism pattern, and approximate fan pattern. Children of Standard II and IV had truly fan pattern. Their judgments can perhaps be represented by a multiplying rule:

\[
\text{Music Performance } = M \times A. \quad (5)
\]

Psychologically, the fan pattern denotes an altogether different kind of causal schema. It implies that prior preparation for participation in music competition is more effective with children of high than low music ability. This result is identical to those of American studies (Kun et al, 1974; Surber, 1980).
Children of Standard VIII and XI had a nice pattern of parallelism. Their judgments may thus be represented by an adding rule of Equation 3. Its psychological interpretation is exactly the same as for the parallelism pattern obtained in puzzle task with children of Standard IV.

Judgments by children of Kindergarten and Standard VI did not conform to either the fan or the parallelism pattern. Both groups of children had one of the higher order trends as statistically significant. Accordingly, their judgments were considered to represent a transition stage. Predictions by Kindergarten children illustrate a transition from parallelism to fan pattern; predictions by children of Standard VI illustrate another transition from the fan pattern to parallelism. This shows that cognitive algebra underlying prediction of performance in singing competition undergoes several developmental changes perhaps due to different notions as to what makes one successful singer.

Hypothesis of age and task differences. Taken collectively, results from the present set of three experiments indicate that cognitive algebra of prediction of task performance is in fact linked with the nature of task and with the developmental level of the subjects. Some tasks have just one causal schema, and prediction of exam performance
appears to be one such task. On the contrary, there are some tasks which have multiple causal schemata. With these tasks, integration rules change as a function of the developmental level of the judges. Prediction of life performance (Bhargava, 1983; Singh & Bhargava, 1982b) and prediction of performance in singing competition appear to belong to the second category of tasks. These results confirm the hypothesis of interaction between nature of task and age of judges in determination of integration rule for prediction of performance.

The present demonstration of developmental changes in integration rule underlying prediction of performance in singing competition is important in at least two ways. First, it shows that the multiplying rule is not necessarily restricted to American children (Kun et al., 1974; Surber, 1980) or to 20-year-olds in India (Bhargava, 1983; Singh & Bhargava, 1982b). It begins emerging by the fourth year and develops fully by the sixth year. The previous failures to yield evidence for a multiplying rule (Bhargava, 1983; Experiment 3; Gupta & Singh, 1981; Singh & Bhargava, 1982a; Singh et al., 1979) cannot, therefore, be ascribed to inability in Indians to employ a multiplying rule or to task-simplification (Singh, 1981). Perhaps nature of task and developmental level of judges could provide the best account of the existing literature.
on prediction of task performance. Thus, the general notion of multiplying rule, Performance = Motivation x Ability, proposed by Heider (1958) is not correct.

Second, it shows that the multiplying rule indeed evolves out of an adding rule (Kun et al, 1974; Surber, 1980; Wilkening, 1979, 1981). But linking of multiplying strategy with greater cognitive capacity of children does not seem to be tenable. Children of 6-9 years of age employed a multiplying rule but those of 12-17 years of age employed an adding rule in prediction of singing performance. Even in research by Singh and Bhargava (1982a), the same groups of college students who had adopted multiplying rule in prediction of life performance used an adding rule in prediction of exam performance. Accordingly, integration rules may better be treated as reflective of causal schemata and social philosophy than as measures of cognitive capacity (Gupta & Singh, 1981; Singh, 1982).

Alternative Interpretations

Can the age and task differences mentioned above be accounted for at levels other than integration rule? If we look at the functional measurement diagram (Anderson, 1981) of Figure 16, we notice that information processing involves three successive stages. At the
Figure 3.6, Functional measurement diagram.
first stage, physical stimuli, \( s_1, s_2, \) and \( s_3 \), available to the judges are converted into subjective values, \( s^1, s^2, \) and \( s^3 \). This is psychophysics in the traditional sense and valuation in the framework of information integration theory. At the second stage, various subjective values are combined together. This is the combination or integration stage. At the third and final stage of response reproduction, the implicit response (\( r \)) based on integration process is reproduced along the response scale (\( R \)) provided by the experimenter.

Within functional measurement (Anderson, 1976), integration rule serves as the base for scaling both stimuli and responses along equal interval scales. Furthermore, integration rules allow direct comparisons between age and cultural groups with respect to pattern in judgments (Gupta & Singh, 1981; Singh et al, 1979). The position taken here is thus that age and task differences occurred at the stage of information integration itself. However, these differences may occur at the levels of response reproduction/or valuation as well. These possibilities are examined below.

Response reproduction. Surber (in press) argues that the same scale values and an additive integration rule can result in convergence, parallelism, and diverging
patterns in factorial plots if subjects reproduce their implicit responses differently along the response scale. More specifically, convergence, parallelism, and divergence patterns can be yielded if subjects employ logarithmic, interval, and exponential functions while reproducing responses based on a simple additive integration rule. From this angle, age and task differences may be accounted for by differences in the process of response reproduction rather than integration rule per se.

Difference in integration function can be distinguished from that in response reproduction function by the "scale free method" (Birnbaum, 1974, 1982). Surber (1980) used this method and ruled out the possibility that response function could account for the differences in data patterns for prediction of performance. It can thus be stated that the present differences in data patterns may not be attributed to changing processes of response reproduction either.

Information valuation. Within averaging model (Anderson, 1981), each piece of information is described by two parameters - scale value, $s$, and weight, $w$. The former refers to the location of a particular piece of information along the dimension of judgment; the latter refers to the relative importance of various pieces of
information available for judgment. As already noted, when weights of the row and column factors remain constant over rows and across columns, the factorial plot of two-way data evinces parallelism. Under the condition of equal weighting, therefore, the averaging model makes the same prediction as does the adding model.

When weight of a factor changes as a function of its value, then systematic deviations from parallelism manifest on the two-way factorial plot. If the weight of a factor is inversely related to its own scale value, the two-way plot exhibits divergence pattern analogous to linear fan pattern predicted by multiplying rule. On the contrary, when weight of a factor is positively related to its own scale value, the two-way plot exhibits a nice pattern of convergence similar to the one caused by inverted multiplying rule. Because of its weight parameter, the averaging rule can account for all the three data patterns, namely, convergence, parallelism, and divergence found in the present research. This means that the age and task differences could be accounted for by differential valuation of information rather than by different integration rules.
It is possible to distinguish differences which occur at the level of information valuation with those which occur at the level of information integration. This has been done by employing distinguishing tests between rival rules, and judgments based on motivation only and ability only in Experiment 1 were taken to accomplish this very purpose.

Recent research by Singh (1984) and by Singh and Bhargava (1982a, 1982b), however, suggest that distinguishing tests based on just one of the two types of information do not necessarily yield definitive information on the rule underlying judgment. When subjects assume value for the missing information, such distinguishing tests become ambiguous. They have, therefore, developed distinguishing tests which avoid problem of missing information altogether. Results from these tests have clearly indicated that differences in patterns in judgments arise primarily due to differences in integration rules. So the position taken in the present dissertation that age and task differences occurred at the level of information processing seems to be reasonable.

Comments. While the two alternative interpretations mentioned above may account for the age and task differences, their plausibility has been ruled out on the basis
of the results obtained from other experiments mentioned above. In the present research, the main purpose was to demonstrate changes in data pattern as a function of age of subjects and nature of task. This has clearly been accomplished. To provide solid refutation for the hypotheses of differences in response reproduction and information processing, however, it will be necessary to employ both "scale free method" (Birnbaum, 1974, 1982) and distinguishing tests between rival rules of information integration (Singh, 1984).

Implications

Why do predictions of performance sometimes conform to the linear fan pattern and sometimes to the parallelism and converging patterns? Singh and his colleagues (Gupta & Singh, 1981; Singh, 1981; Singh et al, 1979) proposed one hypothesis for this difference. According to them, fan pattern is characteristic of the American culture, and the parallelism pattern is characteristic of the Indian culture. An alternative hypothesis based on difficulty of task has been proposed by Surber (1981a, 1981b). According to her, converging, parallelism, and diverging patterns are characteristic of easy, moderately difficult, and difficult tasks, respectively. The present finding of age and task differences in prediction of performance has implications for both hypotheses.
Cultural difference hypothesis. According to this hypothesis (Gupta & Singh, 1981; Singh et al, 1979), the Motivation x Ability effect yields a pattern of parallelism because school and college students in India believe that trying is equally effective with persons of low and high ability. This hypothesis was originally based on studies of prediction of exam performance (Gupta & Singh, 1981; Singh et al, 1979) and subsequent experiments (Bhargava & Singh, 1982a) basically supported this result.

In prediction of life performance, however, 20-year-olds followed a multiplying rule (Bhargava, 1983; Singh & Bhargava, 1982b). Bhargava, therefore, suggested that cultural difference hypothesis is true for undergraduate college students only. Results from the present research bring further qualification on the hypothesis of cultural difference. Because younger children up to Standard VI had the fan pattern, it can now be said that the hypothesis of cultural difference is restricted to only high school and undergraduate college students.

If we carefully consider results obtained from studies of exam performance (Gupta & Singh, 1981; Singh & Bhargava, 1982a; Singh et al, 1979), life performance (Bhargava, 1983; Singh & Bhargava, 1982), and competition performance of the present research, we now find that all
these three tasks were handled in much the same way by high school and undergraduate college students. All the three tasks yield a pattern of parallelism. This means that this is perhaps a period of optimism and idealism in India, for these students believe that effort or trying will be equally effective with persons of low and high ability. In other words, each person regardless of his or her ability has equal opportunity to improve upon his or her lot. This outlook certainly reflects on the egalitarian attitude of this young group of people in India.

When we look at the results obtained from high school and college students of the United States, we find that they usually follow the multiplying rule. This has happened in case of prediction of performance in puzzle (Kun et al, 1974), weight-lifting contest (Surber, 1981), and exam performance (Anderson, 1981; Anderson & Butzin, 1974; Surber, 1981b). Predictions by these groups of subjects are drastically different from those by their Indian counterparts. Accordingly, cultural difference hypothesis appears to offer a reasonable explanation for the difference in pattern in Motivation x Ability effect.
**Hypothesis of task difficulty.** According to Surber (1981a, 1981b), parallelism and converging patterns appear with less difficult tasks. Difficult tasks produce the fan pattern, and this is consistent with Kelley's (1972) multiple necessary causal schema. Can the fan pattern obtained with music task, then, be explained by its perceived difficulty?

The task difficulty explanation would have provided a reasonable account for the fan pattern, had subjects of all groups yielded the very linear fan pattern. Prediction of performance in singing competition obtained fan pattern with younger children but parallelism with older children. On intuitive basis, singing should be considered more difficult by older than by younger children. So fan pattern should be characteristics of older and not the younger children. This did not happen in the present research. It seems reasonable to state, therefore, that task difficulty cannot account for the results obtained from the present set of three experiments.

It should also be added that Singh and Bhargava (1982a, 1982b) and Bhargava (1983) provided direct tests of the task difficulty explanation suggested by Surber (1981b). For exam performance, the tests were described as varying in difficulty. For life performance, opportu-
nity available for growth varied from low to high. Bhargava had manipulated probability of success in department in his study of prediction of managerial performance. None of these studies yielded any evidence for task difficulty as being a moderator variable of Motivation x Ability effect. Effectiveness of task difficulty factor itself thus seems to be a culture-specific phenomenon. Accordingly, the task difficulty explanation cannot account for the data obtained from Indian subjects.

Conclusions

From the foregoing discussion, it appears that integration rules underlying prediction of task performance depends on nature of task as well as age and culture of subjects. While the hypothesis of nature of task has been able to reduce the confusion present in the literature considerably, the three factors, namely, nature of task, age of subjects, and culture of subjects bring a good deal of orderliness in literature. Nevertheless, it is obvious that Heider's proposal that Performance = Motivation x Ability is much more restricted than what has been believed to be true so far.

In developmental literature, integration rules have been considered to be reflective of cognitive capacity. The finding of development of parallelism pattern from the fan pattern around the age of 13 years indicates that integration rules
bear more on causal conceptions of the subjects than on their cognitive capacity. This reinforces the suggestion by Gupta and Singh (1981).

**Hypothesis of Limited Cognitive Capacity**

The second main finding of present research is that children are not as limited in cognitive capacity as they are generally believed to be. Kindergarten children utilized up to four pieces of information without any difficulty. Also, they paid equal attention to motivation and ability information irrespective of the orders in which the two types of information appeared. These results provide no support for the Piagetian notion that preoperational children employ centration as their attention style (Flavell, 1970; Ginsberg & Opper, 1969).

Results related to information utilization and order effects are important in two ways. First, they show that the number of significant main effects in the individual child analysis do not bear upon the cognitive capacity of the children. Perhaps utilization of a cue depends upon the relevance or weight of information (Singh, 1982), statistical sensitivity of the design (Gupta & Singh, 1981), and nature of the task. Second, they indicate that recency effects, which may be treated as an index of limited cognitive ability, do not show any
definite age trend. In the present research, recency effects occurred with older children. In the research by Kun et al (1974), they were present with first graders but not with third and fifth graders. In studies of moral judgments (Surber, 1982) and personal happiness (Singh, 1982), recency effects were present with children of 4-10 years of age. Accordingly, recency effects may not be a true index of limited cognitive capacity.

One implication of the result vis-a-vis information utilization and order effects is that these two measures also reflect upon properties of the task. Anderson (1981, pp. 179-191) reports that order effects do change as a function of nature of stimuli and response requirements even at the level of college adults. Thus, information processing strategies may also be linked with nature of the task.

In his review of literature on cognitive development, Gelman (1978) notes that traditional approach to cognitive development addressed itself to what children cannot do. This led to the notion that children are cognitively incompetent. The modern approach investigates what the children can do as well as what they cannot do. The present research may be regarded as belonging to this modern approach, for it shows that children are reasonably
good at information integration task. This success enables us to at least challenge the hypothesis that children are of limited cognitive capacity.

In developmental research, it would perhaps be better to talk of cognitive capacity for a task than the cognitive development in general. This would allow more critical evaluation of what children can do and what they cannot do. Results of this research show that methods of information integration theory may offer penetrating approach to such developmental analyses.

Further Work

Task Differences

Findings of the present research show that some tasks have just one causal schema or integration rule and some tasks have multiple causal schemata. If this hypothesis of nature of task as a determinant of causal schemata is correct, then developmental changes in integration rules for a particular task should hold true with other tasks also. Further developmental study of prediction of performance in puzzle competition considered in Experiment 2 and in athletic competition (Anderson & Butzin, 1974; Surber, 1980) is thus required. Studies of these nonacademic tasks would yield direct evidence on the tenability of the cultural difference hypothesis (Singh, 1981) as well.
The position taken in this dissertation is that integration rules reflect causal conceptions of judges and not necessarily on their cognitive capacity. If one is interested in cognitive capacity per se, then he or she should require young children to perform several information integration tasks and should compare these judgments with those by adults. This will allow him or her to find out what young children can do and what they cannot do like adults. This will directly be in line with the suggestion of Gelman (1978).

**Distinguishing Tests**

In the present research, the main focus was on patterns in factorial plot of Motivation x Ability effect. Since two rules can result in the same pattern, it is now necessary to perform distinguishing tests between rival rules. Experiment 1 of the present research had used motivation alone or ability alone descriptions just as in the experiments by Gupta and Singh (1981) and by Surber (1980) to make distinction between rules. Research by Singh (1984) and by Singh and Bhargava (1982a, 1982b) show that tests based on just one type of information do not yield clear evidence on the underlying rule. This happens because people assume some value for the missing information.
To distinguish adding from constant-weight averaging and multiplying from differential-weight averaging, judgmental tasks have to involve integrations at two stages. This can be accomplished by having more pieces of information about motivation and just one piece of information about ability. A simple example of such task would be a three-factor, Motivation 1 x Motivation 2 x Ability design and two two-factor, Motivation 1 x Ability and Motivation 2 x Ability designs. If subjects employ a three-term averaging rule, then prediction of performance would obey the following model:

\[ \text{Performance} = \frac{M_1 W_{M1}}{A} + \frac{M_2 W_{M2}}{A} + \frac{W_A}{A}, \]  

where three weights sum to one. Under the condition of equal weighting, Motivation 1 x Motivation 2, Motivation 1 x Ability, and Motivation 2 x Ability would all evince parallelism. On the contrary, the differential weighting for information would yield a different pattern of data. Whereas Motivation 1 x Motivation 2 effect would show parallelism, factorial plots of Motivation 1 x Ability and Motivation 2 x Ability would show the fan pattern. Furthermore, a combined factorial plot of the data from Motivation 1 x Ability and Motivation 2 x Ability from the three- and two-cue designs would not conform to the very parallelism or fan pattern. The two-factor curves
would cross over the corresponding curves from the three-factor design.

If the two-stage averaging-adding model is used, then prediction of performance will follow a different model:

\[
\text{Performance} = \sqrt{M_{1w_{M_1}}} + M_2 (1-w_{M_1}) + A. \tag{7}
\]

The two pieces of motivation information will be first averaged and then the averaged motivation will be added to ability information. The combined factorial plot of data from the three- and two-cue designs will, therefore, exhibit the very pattern of parallelism.

When the two-stage integration will involve averaging-multiplying, the prediction of performance would be represented by the following model:

\[
\text{Performance} = \sqrt{M_{1w_{M_1}}} + M_2 (1-w_{M_1}) \times A. \tag{8}
\]

Like Equation 7, this model also assumes that two pieces of motivation information will be averaged. But it assumes that ability information will multiply the averaged motivation information. If this model is correct, then combined factorial plot of data for Motivation 1 x Ability and Motivation 2 x Ability from three-cue and corresponding two-cue designs will all yield a linear fan pattern.
The distinguishing tests described above have two distinct advantages. The first is that it avoids the problem of imputations about the missing information completely. The second is that it rules out the possibility of nonlinear response scale as explanation of the fan pattern in Motivation x Ability effect. Because this test presents evidence for both parallelism and fan patterns, scale nonlinearity cannot account for the fan or convergence pattern (Lampel & Anderson, 1968). Thus, distinguishing tests based on two-stage operation logic deserve more use in future developmental research.

Group Differences in Causal Schemata

The finding that high school and undergraduate college students have just one causal schema for all tasks but young children and postgraduate students vary in their causal schemata for tasks suggests that cognitive algebra underlying prediction of task performance reflects on the learning and experience of the subjects. This suggestion is consistent with the hypothesis of cultural difference in prediction of performance. If causal conceptions are experience- and role-based, then people from different groups/roles in any society may be expected to differ in their integration rules for prediction of task performance (Gupta & Singh, 1981; Singh et al, 1979).
Group differences in cognitive algebra can directly be examined by giving one task to several groups of subjects just as the present set of three experiments did with age groups. If age is held constant but roles differ such as with parents, teachers, and managers, any difference in rule will be attributed to the group differences in experience and role.

Group differences in causal conceptions based on experience can also be studied by giving several tasks to the same group of subjects. With a uniform experience, they should be able to handle all tasks according to one integration rule just as high school and undergraduate college students did in the previous research. Any evidence for task difference would not question the hypothesis of experience. It would simply reflect on different experiences with different tasks. In this way, it may be possible to account for differences in cognitive algebra for task performance by a general hypothesis of differences in experiences of the subjects. Accordingly, it is necessary now to study prediction of performance by parents, teachers, and managers who deal with the development of human resources in society.
Concluding Comments

In the first review of the literature on developmental psychology in India, Parameswaran (1972) lamented that most of the studies in developmental psychology were confined to survey methods, and experimental studies were "conspicuous by their absence". Not only were these studies vague in "discussion of data collected with loosely designed methods on badly selected samples" (p.60) but also were devoid of any developmental orientation. He was surprised to note that not a single study was based on any theoretical model. The second review of literature on developmental processes by Anandalakshmy (1980) hardly shows any improvement in the gloomy picture.

In the last five years, however, there have been experimental studies of children's social cognition. These experiments have tested applicability of information integration theory (Anderson, 1981) to children's judgments of personal happiness (Singh, 1982; Singh, Sidana & Saluja, 1978a), attractiveness of playgroups (Singh, Sidana & Saluja, 1978b) and family groups (Singh, Sidana, & Srivastava, 1978) and prediction of exam (Gupta & Singh, 1981) and life (Bhargava, 1983) performances. Because of the encouraging results from these studies, the present research also applied information integration theory to
the study of developmental trends in prediction of task performance.

The present information integration analysis of predictions by children challenges the commonly held belief of limited cognitive capacity in preoperational children. Kindergarten children utilized up to four pieces of information efficiently in their judgments. Also, they paid equal attention to motivation and ability information irrespective of the order in which they were presented. More importantly, their judgments conformed to the precise requirements of the adding, averaging, and multiplying rules. These results portray young children as a "highly adaptive information processor" (Leon, 1980, p. 94) contrary to what they are commonly believed to be.

This conception of children as highly adaptive information processors emerged primarily because of the penetrating approach provided by information integration theory. The rating scale method, individual child analysis, and diagnosis of integration rules offer much more powerful analytic tools for the study of children's cognition than the traditional Piagetian choice tasks (Anderson, 1980; Anderson & Cuneo, 1978; Leon, 1980). The author thus hopes that the present work will draw
attention of developmental psychologists in India to the potential power of information integration theory for cross-age and cross-cultural comparisons in social cognition.