CHAPTER V

Steady and random velocities of ionospheric irregularities
Auto - and Cross - Correlations

1. Introduction

Mitra's method (1949) of similar fades assumes that the diffraction pattern produced on the ground by ionospheric irregularities does not change in the form as it moves past the observing points on the ground. It also assumes that the diffraction pattern is isotropic which means that its properties do not change along any direction at a fixed distance from the origin.

It is important to consider fading of a radio wave which has been singly reflected from the ionosphere and devise mathematical methods to analyse such records. Briggs et al (1950) introduced the method of auto - and cross - correlograms and demonstrated the usefulness of such concepts in the interpretation of complex fading patterns. Let us briefly review their method.

It is clear that fading could be produced by a movement of the diffraction patterns past the observing points as would happen if a horizontal wind is blowing in the ionosphere. The fading recorded at two points in the direction of wind would be exactly similar but displaced in time. Another way in which fading could be produced would be by random changes
in ionospheric irregularities without any drift; such a mechanism would produce random changes in the diffraction pattern on the ground and the fading at two nearby receivers would be similar. If receivers were separated from each other in any direction the fading becomes more and more dissimilar.

The two mechanisms mentioned above may operate simultaneously and would introduce changes of form in a moving diffraction pattern. It is possible to reduce from fading records taken at three points situated at the corners of a right-angle triangle (1) the steady velocity of the diffraction pattern moving on the ground, (2) the rate at which the pattern alters as it moves, (3) the size of the irregularities in the pattern. It is also possible to make some rough estimate of the shape of the irregularities if they are not isometric. For all these, it is found useful to consider the auto-correlation and cross-correlation functions of the fading records at the three receiver-points. Once we deduce the nature of the diffracting pattern, it could be connected with large scale movements in the ionosphere and associated changes in the pattern due to random movements therein. A full analysis of the amplitudes of reflected signals has been made by Briggs et al (1950), and this has been extended by Phillips and Spencer (1955) to include anisometric patterns. A comprehensive review of aspects of diffraction theory and their application to the ionosphere is given by Ratcliffe (1956).
2. Measurement of correlation

(1) A single time-record

A fading record is found to have random character like the curves in Figure 5.1. To study a variable of this kind, use can be made of auto-correlation function, i.e., the amplitude or phase correlation coefficients of the received signal. It is clear that the correlation coefficient of a sequence of amplitudes \( R_1, R_2 \ldots \) of the signal between time \( t \) and \( t + \tau \), so that \( R_1 = R(t_1) \), and \( R_1 + 1 = R(t_1 + \tau) \), is by definition given by function:

\[
\rho(0,0,\tau) = \frac{\overline{R(0,0,t)} - \overline{R}}{\overline{R(0,0,t)} - \overline{R}}
\]

where \( \overline{R} \) is the mean value of \( R \) and \( \rho(0,0,\tau) \) is the correlation coefficient of the envelope of the random oscillations and measures the correlation between values of \( R \) separated by a time interval \( \tau \). It has the value unity for time displacement \( \tau = 0 \) and generally decreases smoothly at first as \( \tau \) increases from zero.

A typical auto-correlogram is shown in Figures 5.2(a) and 5.2(b). Clearly the value of \( \tau \) at which \( \rho(0,0,\tau) \) goes to zero is the time interval at which the two values of \( R \) have become completely independent.

In the analysis of fading of radio signals it is important to find a measure of the "speed of fading". It is
FIG. 51  FADING RECORDS AT TWO RECEIVERS
Figure 5.2(a) Fading curve on 2.6 Mc/s at 0830 hr on 22-6-60 and its auto-correlogram.
Figure 5.2(b) Fading curve on 2.6 Mc/s at 1730 hr on 13-7-60 and its auto-correlogram.
possible to derive it either from the function $\rho(0,0,r)$ as defined in equation (1) or with less labour in the following way:

$$S = \frac{\frac{\partial R}{\partial t}}{R}$$  \hspace{1cm} (2)

Here, the speed of fading $S$, is the mean rate of change of $R$, normalized to a constant mean value of $R$.

\begin{enumerate}
\item \textbf{Space variation and the two dimensional auto-correlation function}
\end{enumerate}

We may also measure the rate of variation $R$ over the ground at a given instant and call it $G$, where

$$G = \frac{\frac{\partial R}{\partial x}}{R}$$  \hspace{1cm} (3)

where $\Delta x$ is the distance between two points at which $R$ is measured at the same time. To start with, $G$ may be assumed to be independent of the direction on the ground in which the variable $x$ is measured.

Thus in practice the signal amplitude over the ground and in time could be represented by a function $R(x,y,t)$ of the space coordinates $x,y$ and the time coordinate $t$. We can simplify first the analysis by considering $R$ as a function of $a$ and $t$ and obtain the two-dimensional auto-correlation function of $R(a,0,t)$ from equation (1). Thus
\[ \rho(a,0,\tau) = \frac{R(0,0,t) - \bar{R}}{\left[ R(0,0,t) - \bar{R} \right]^2} \frac{R(a,0,t + \tau) - \bar{R}}{\left[ R(a,0,t + \tau) - \bar{R} \right]^2} \]  \hspace{1cm} (4)  

where \( R(0,0,t) \) and \( R(a,0,t + \tau) \) are the amplitudes of the signal at the origin at time \( t \) and at distance \( a \) and \( t + \tau \) respectively, \( a \) being the separation between the pair of aerials. A bar drawn over a quantity signifies that the average of the quantity is taken over a time which is long compared to the time scale of the variations. The functional form of the correlation can be described mathematically as,

\[ \rho(0) = 1 \]
\[ \rho(\tau_1) \geq \rho(\tau_2) \text{ for } \tau_2 > \tau_1 \]
\[ \rho(-\tau) = \rho(\tau) \]

Equation (4) gives the cross-correlation between two records obtained at two aerials for different values of time difference \( \tau \). In this case \( \rho(a,0,\tau) \) will not necessarily be peaked at \( \tau = 0 \). This might be expected since the cross-correlation function uses two time series not necessarily identical.

Suppose there is a systematic time shift between the two series, the peak cross-correlation may occur with a time lag \( \tau = T_x \) in the fading records obtained with an E-W pair of aerials and a time lag \( \tau = T_y \) in that with N-S pair of aerials. Thus, in general, the lag correlograms \( \rho(a,0,T_x) \); \( \rho(0,a,T_y) \) would be less than unity.
It is important to bear in mind that the use of this type of correlation analysis requires that the amplitude variations with time in a fading be statistically steady. This means that the mean amplitude and its standard deviation should not vary within the sampling time.

Some typical fading records and their correlograms

We shall compare here the results obtained by the method of similar fades and by the method of correlation functions. A few tracings of some typical fading records are shown at the top of Figures 5.3(a), 5.3(b) and 5.3(c). The auto- and cross-correlation function were calculated from equation (1) and (4) and are shown below the fading records. Inset is a table showing the drift speed and direction calculated by the method of similar fades and by the method of correlation functions. The same table is also typed below the figure. It may be seen that the results obtained by the two methods do not differ seriously in some type of records.

Groupings of the fading records were made as described below:

1. Fading records showing a high degree of similarity in the amplitude of fading and the individual time shifts between corresponding maxima or minima such as shown in Figure 5.3(a). The auto- and cross-correlation curves in this case vary smoothly with time. The speed and direction obtained by the two methods agree fairly well.
Figure 5.3(a) Fading curves on 2.6 Mc/s at 0830 hr on 22-6-60 and their auto- and cross-correlograms.

C-W forms E-W pair
N-C forms N-S pair.

<table>
<thead>
<tr>
<th>Method</th>
<th>Speed</th>
<th>Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation</td>
<td>24 m/s</td>
<td>960° E of N</td>
</tr>
<tr>
<td>Similar Fades</td>
<td>30 m/s</td>
<td>920° E of N</td>
</tr>
</tbody>
</table>
Figure 5.3(b) Fading curves on 2.6 Mc/s at 1630 hr on 13-7-60 and their auto- and cross-correlograms.

C-W forms E-W pair
N-C forms N-S pair.

<table>
<thead>
<tr>
<th>Method</th>
<th>Speed</th>
<th>Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation</td>
<td>53 m/s</td>
<td>116° E of N</td>
</tr>
<tr>
<td>Similar Fades</td>
<td>66 m/s</td>
<td>114° E of N</td>
</tr>
</tbody>
</table>
Figure 5.3(c) Fading curves on 2.6 Mc/s at 1430 hr on 8-2-61 and their auto- and cross-correlograms.

C-W forms E-W pair
N-C forms N-S pair.
(2) Fading records which do not show high degree of similarity are presented in Figure 5.3(b), but some similarity can still be seen between the records. The correlation is found to be less in this type but the results by both the methods are in tolerable agreement.

(3) Fading records which show random fading with few recognizable common features as are shown in Figure 5.3(c). Detection of corresponding maxima or minima is extremely difficult in such cases. The correlograms show poor correlation and hence such records may be rejected for analysis.

Results

Some 78 fading records obtained on 2.6 Mc/s from the E region at Ahmedabad during the period March 1960 - May, 1961 were selected for analysis. Of these, 17 records fell in group (3) and were rejected. The results of the remaining 61 records are presented below.

Variation of drift speed

Figure 5.4(a) shows the histograms of different ranges of drift speed obtained by the two methods. The most probable range is 40-60 m/s in both the cases with a small difference in the average speed. The error is likely to be larger in the range 60-100 m/s in the method of similar fades since the time delays measured are smaller in magnitude than those in the correlation method.
Figure 5.4(a) Histograms of drift speed obtained by method of cross-correlation and similar fades.
Figure 5.4(b) is a plot of drift speeds by the method of similar fades versus that by correlation functions. There is a good correlation at lower speeds.

Variation of drift direction

Figure 5.5(a) and 5.5(b) show similar plots of drift direction as calculated by the two methods. The results show tolerable agreement and hence it may be concluded that for a general statistical study, the less laborious method of similar fades may be adopted.

3. Amplitude and time analysis of a single downcoming wave

(1) Amplitude analysis

Ratcliffe (1948) gave an explanation of the rapid fading of the amplitude of singly reflected downcoming radio waves. He considered the ionosphere as a reflecting layer in which a large number of irregularities were imbedded in a horizontal plane. Each of these irregularities scatters part of the incident radio waves and if the irregularities are assumed to have a movement with a random velocity \( v \) in the line of sight the problem becomes similar to the motion of gas molecules in thermal agitation. The probability distribution of \( v \) can be easily worked out from Maxwell's theory as

\[
P(v) = \frac{1}{\sqrt{2\pi} v_0} \exp\left(-\frac{v^2}{2v_0^2}\right)
\]

where \( v_0 \) is the r.m.s. value of \( v \).
Figure 5.4(b) Plot of drift speeds by the method of similar fades versus that by correlation functions.
Figure 5.5(a) Histograms of drift direction obtained by method of correlation function and method of similar fades.
Figure 5.5(b) Plot of drift directions by the method of similar fades versus that by correlation functions.
The frequency of the scattered wave will suffer a Doppler shift due to the motion of the irregularity and the change in frequency $f$ is given by the well-known Doppler formula

$$f = f_0 \left( 1 \pm \frac{2v}{c} \right)$$

(6)

where $f_0$ is the frequency of the incident wave, $v$ is the velocity of the irregularity and $c$ is the velocity of light. If $v$ is small compared with $c$, the irregular change in frequency can be considered to be an irregular change in phase of a fixed frequency $f_0$.

The phase of the scattered wave from the individual irregularities or blobs can thus be considered to be distributed randomly and the fading of the received signal would be the result of the interference of a large number of wave-vectors with random phases. This problem is therefore similar to the "random walk" phenomenon. Lord Rayleigh showed that the probability $P(R)$ of finding the amplitude of the resultant vector between $R$ and $R + dR$ in a random walk process is given by

$$P(R) = \frac{R}{\Psi} \exp \left( -\frac{R^2}{2\Psi} \right)$$

(7)

where $R$ is the instantaneous amplitude of the resultant vector, and $\Psi$ is related to the amplitude $\overline{R}$, $R_0$ and $R_m$ as shown below:

$$\overline{R} = \sqrt{\frac{R^2}{\Psi}}$$

(8)
where $\bar{R}$ is the average value, $R_0$ the r.m.s. value, and $R_m$ the most probable value of the amplitude.

We see from equation (3) that if we plot $\ln\left(\frac{P(R)}{R}\right)$ vs. $R^2$, we should get a straight line.

A typical fading record which approximately follows the Rayleigh law of amplitude distribution is shown in Figure 5.6.

The distribution is different when the received signal is composed of a steady component specularly reflected from the layer and a random component contributed by time changes in the irregularities. For this situation, Rice (1945) modified equation (3) to the following form:

$$P(Q) = \frac{Q}{\Psi} \exp \left[-\frac{(Q^2 + B^2)}{\Psi} \right] \frac{J_0 \left(\frac{QB}{\Psi}\right)}{J_0 (\frac{B}{\Psi})} \quad (11)$$

where $B$ is the amplitude of the steady signal and $\Psi$ is a function of the amplitude of random component as given by equations (4), (5) and (6) and $J_0 (x)$ is the Bessel function of zero order. In general

$$Q^2 = B^2 + 2\Psi = B^2 + R_0^2 \quad (12)$$

That is, the mean square amplitude of the resultant is the
Figure 5.6 Fading record following Rayleigh law of amplitude distribution.
sum of the mean square values of the steady and random components. The general behaviour of $P(Q)$ can be described by parameter $b$ which is defined as

$$b = \frac{B}{\sqrt{\Psi}} = \frac{B}{R_m}$$ (13)

Alpert (1963) has called this $\beta$, the turbidity factor. When $b < 1$, that is, when the steady signal is less than the random signal, equation (11) reduces approximately to Rayleigh type distribution.

If $b > 3$, McNicol (1949) has shown that equation (11) simplifies to

$$P(Q) \approx \frac{1}{\sqrt{2\pi \Psi}} \exp \left[ -\frac{(Q - Q_m)^2}{2\Psi} \right]$$ (14)

where $Q_m$ is the most probable value of $Q$ given by

$$Q_m = (b^2 + 1)^{\frac{1}{2}}$$ (15)

A plot of $\log P(Q)$ against $(Q - Q_m)^2$ will also be a straight line, if the assumptions are valid, and its slope gives the value of $\frac{1}{2\Psi}$. Thus knowing $\Psi$ from the slope of the line and $Q_m$ from experimental observations, $b$ can be evaluated from equation (15). Two examples are shown in Figures 5.7(a) and 5.7(b).

(2) Time analysis

An approximate measure of the rate of fading can be
Figure 5.7(a) Fading curve and a plot of $\ln[P(Q)]$ against $(Q - Q_m)^2$. 
Figure 5.7(b) Fading curve and a plot of $P(Q)$ against $(Q - Q_m)^2$. 
made by counting the number of maxima $N$ occurring in time $t$ in a fading record. A more rigorous method has been suggested by Ratcliffe (1948) according to which the mean speed of fading $S$ whose amplitudes follow the Rayleigh type distribution is given by

$$S = \frac{|\Delta \overline{R}|}{R \tau} = \frac{8v_o}{\lambda} \quad (16)$$

where $\overline{R} =$ mean amplitude of the fading curve

$$|\Delta \overline{R}| = \text{modulus of mean of the difference between successive amplitudes}$$

$\tau =$ time interval between successive measured amplitudes using auto-correlation function

and $v_o =$ r.m.s. value of random velocity in the line of sight.

Booker et al (1950) have derived a relation between power spectrum of the returned wave to the auto-correlation function $\rho(0,0,\tau)$ of the fading record. It is shown that in the presence of only random motions of the scattering center, the amplitude correlation coefficient is

$$\rho(0,0,\tau) = \zeta \exp \left[ - \frac{16\pi^2 v_o^2 \tau^2}{\lambda^2} \right] \quad (17)$$

It can be seen from equation (17) that $\rho(0,0,\tau)$ reduces to $\frac{1}{\zeta}$ at

$$\tau = \frac{\lambda}{4\pi v_o} \quad (18)$$
Knowing the time $\tau$ at which the auto-correlation $f(0,0,\tau)$ falls to a value $\frac{1}{e}$ and the wavelength $\lambda$ of the incident wave, $v_0$ can be readily calculated from equation (18).

4. Results of amplitude and time analysis

Some fading records on 2.6 Mc/s obtained during March 1960 - February 1961 were selected for analysis. The total number of records selected was 310, out of which 249 were for the daytime (07-18 hrs) and 61 for the nighttime (19-05 hrs).

Table 5.1 shows the frequency of occurrence of different ranges of $b$ for daytime and nighttime. The most frequently occurring value lies between 1.0-2.0 for day and 1.6-2.6 for night which means that specular reflection and scattered reflections are equally important during most of the time.

There is a small frequency of occurrences for $b \leq 1$, namely those with dominant random component, and quite a significant number for $b > 3$ indicating the dominance of a steady component.

There were about 51 day time cases and 14 nighttime cases where neither a Rayleigh nor a normal distribution curve could be fitted. These showed some mixed type of amplitude distribution, a few examples of which are shown in Figure 5.8.
Table 5.1

Frequency table of distribution of $b$ during day and night

\[ b = \text{Steady signal/Most probable random signal} \]

<table>
<thead>
<tr>
<th>Range of $b$</th>
<th>Frequency of occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Daytime (07-18 hr)</td>
</tr>
<tr>
<td>0.00-0.59</td>
<td>10</td>
</tr>
<tr>
<td>0.60-1.09</td>
<td>24</td>
</tr>
<tr>
<td>1.10-1.59</td>
<td>53</td>
</tr>
<tr>
<td>1.60-2.09</td>
<td>47</td>
</tr>
<tr>
<td>2.10-2.59</td>
<td>35</td>
</tr>
<tr>
<td>2.60-3.09</td>
<td>10</td>
</tr>
<tr>
<td>3.10-3.59</td>
<td>5</td>
</tr>
<tr>
<td>3.60-4.09</td>
<td>4</td>
</tr>
<tr>
<td>4.10-4.59</td>
<td>2</td>
</tr>
<tr>
<td>$\geq 4.6$</td>
<td>8</td>
</tr>
</tbody>
</table>
Figure 5.3 Examples of mixed type of amplitude distributions.
Table 5.2 gives the mean values of $v_0$ for different ranges of $b \leq 2.09$ obtained by Ratcliffe's method and by the auto-correlation method. It can be seen that there is fair agreement between the two methods. It is interesting to note that $v_0$ at night is less than that during day for $b \leq 1.09$ but is greater during night than during day when $b$ is greater than 1.09.

5. Results obtained by other workers

Mitra and Srivastava (1957) reported that the amplitude distribution showed that the value of $b$ ranged between 0.6 and 2.6. Dasgupta and Vij (1960) found that the distribution was of the Rayleigh type in the case of rapid fading and was of a double-hump type in the case of quasiperiodic fading.

The magnitude of $v_0$ as reported by Mitra (1949) and Mitra and Srivastava (1957) ranges from 2 to 25 m/s, whereas Dasgupta and Vij (1960) and Khastgir and Singh (1960) obtained values of $v_0$ to be less than 10 m/s. These may be compared to the values of $v_0$ in Table 5.2 which is 7 to 8 m/s.
Table 5.2

Distribution of average value of $v_o$ obtained by Ratcliffe's method and by the method of auto-correlation function against different values of $b$.

<table>
<thead>
<tr>
<th>Range of $b$</th>
<th>DAYTIME (07-18 hr)</th>
<th>NIGHTTIME (19-05 hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean $v_o$ by Ratcliffe's method m/s</td>
<td>Mean $v_o$ by the method of auto-correlation function m/s</td>
</tr>
<tr>
<td>0.00-0.59</td>
<td>7.9</td>
<td>7.9</td>
</tr>
<tr>
<td>0.60-1.09</td>
<td>7.2</td>
<td>6.1</td>
</tr>
<tr>
<td>1.10-1.59</td>
<td>7.0</td>
<td>6.2</td>
</tr>
<tr>
<td>1.60-2.09</td>
<td>6.5</td>
<td>6.6</td>
</tr>
<tr>
<td>2.10-2.59</td>
<td>-</td>
<td>7.3</td>
</tr>
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<td>2.60-3.09</td>
<td>-</td>
<td>6.8</td>
</tr>
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<td>3.10-3.59</td>
<td>-</td>
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</tr>
<tr>
<td>3.60-4.09</td>
<td>-</td>
<td>3.9</td>
</tr>
<tr>
<td>4.10-4.59</td>
<td>-</td>
<td>3.0</td>
</tr>
<tr>
<td>&gt;4.6</td>
<td>-</td>
<td>0.4</td>
</tr>
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</table>

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