3.1 Introduction

Electron density irregularities in the equatorial F region has been a topic of great interest ever since the discovery of Spread-F was made by Borkman and Wells (1934). Dagg (1957) proposed that these irregularities are first generated in the E-region and transmitted to the F-region via highly conducting geomagnetic field lines. This suggestion was intimately connected to that of Martyn (1955) who proposed that the large scale electric fields in the E-region are transmitted to the F-region via the highly conducting geomagnetic field lines. Farley (1959, 1960) studied the dynamical coupling between the E and F-regions for electric fields of small scale sizes. To account for the source of electric fields of small scale sizes, he used a system of horizontal winds which were derivable from a stream function. The source region was assumed to have a very small vertical extent such that the parallel conductivity within the source region was assumed to be constant. The electrostatic coupling between the equatorial F region and low latitude E region was provided through the geomagnetic field lines which were assumed to be
parabolic. His studies showed that

(i) Electrostatic coupling between the E and F region
may exist at all latitudes, the effect being weakest
at the equator.

(ii) The electrostatic source field strength will usually
be considerably less than the $W \times B$ field where $W$ is
the neutral wind and $B$ is the geomagnetic field. The
source field is maximum at 45° dip latitude and
decreases on either side of it.

(iii) No electron density variations could be produced at
the poles. At lower latitudes some fluctuations in
electron density could be produced, but they would
probably be too weak to be significant.

Hence he concluded that the Dagg's suggestion is not at
all satisfactory.

Farley assumed, in the determination of these solutions
that the distribution of the electrostatic potential is
symmetric about the equatorial plane. This condition
corresponds to an assumption that no current flows
across the equatorial plane, and requires that the field
generating mechanism operative in the dynamo region produces
identical electric fields at the conjugate points in the
two hemispheres of the earth. This condition was much too
unrealistic, as was realised by Spriter and Briggs (1961);
for their is no qualifying reason to believe that the
small scale electric fields produced by irregular winds be symmetrical with respect to the equator. Spieter and Briggs (1961) considered that the mechanisms that operate to generate electric fields at the conjugate points function independently and the linearity of the equations assures that the properties of the fields generated in each hemisphere can be treated separately. The appropriate boundary condition for the other hemisphere being that owing to a very small conductivity at the base of the ionosphere, current vanishes at the bottom of the ionosphere.

But they did not propose any mechanism for the generation of small scale electric fields in the source region. Their calculations were only for the attenuation of the electric fields during their transmission.

Reid (1968) proposed that the Dagg's suggestion may still work if the source region is considered to be "thick". He proposed that if the whole region from 100 km to 140 km altitude is considered as the source region, it could generate electric fields which are essentially in phase. So that the short circuiting effect, to be expected from the varying winds, does not arise. However, he did not examine the idea quantitatively.

Thus the earlier studies could not establish the efficiency of neutral winds in generating small scale
electric fields on a strong footing. In the previous chapter we have summarized the works of Kato (1973) and Anandarao et al (1977) in generation of electric fields due to the gravity wave winds. Also, pointing out the shortcomings of these model we developed a three dimensional model for the interaction of gravity wave winds with the ionosphere plasma. These studies showed that the electric fields can be generated by the gravity wave winds under certain conditions. In that, the importance of the wavelength parallel to the geomagnetic field lines was realised for the generation of the electric fields. In this chapter, we investigate rigorously, the problem of generation of electric fields due to the three dimensional gravity wave winds in the low latitude ionosphere, and the transmission of these fields to the other regions of the ionosphere. In the present investigations, the geomagnetic field line curvature has been taken into account. The following is the brief outline of this chapter.

In section 3.2 the geometry of the geomagnetic field line is described.

In section 3.3, the variation of wave numbers $k_x$ and $k_z$ with the dip angle is given. It is found that the wave number ($k_x$) parallel to the field line varies rapidly with the dip angle, going from positive to negative. The
variation in $k_z$ is, however, negligibly small over a large range of dip angles.

In section 3.4, the nature of the gravity wave winds has been studied. It is found that, on a given field line, the wind varies symmetrically around the point where $k_x = 0$. The wind remains in the same direction over a large distance along $B$ around the point where $k_x = 0$, and further away from it, it varies sinusoidally with decreasing wavelength on either side of this region.

In section 3.5, it is shown that the region where $k_x = 0$ defines the source region of the electric field. It is found that the vertical wave length of the gravity wave winds determines the extent of the source region.

In section 3.6, the theory of generation of electric fields due to the gravity wave winds is developed using the assumed form of the potential and the generalised Ohm's law. A set of second order partial differential equations in potential (corresponding to the real and imaginary parts) is thus obtained. On the basis of physical arguments, this set of equations is reduced to a single equation. Boundary conditions and the numerical method employed for solving these equations are also discussed in this section.

In section 3.7, the conductivity profiles (for different times of the day) used for the calculations are given.
In section 3.8, approximate solutions, giving information on the generation and attenuation of electric fields are obtained.

In section 3.9, by meeting the condition $k_x = 0$ in different regions of the ionosphere, the electric field in the source region is calculated for a variety of cases. Thus the condition for the most efficient generation of the electric fields is obtained.

In section 3.10, a comparison is made of the results obtained using the sinusoidal winds (which are present everywhere) of section 3.4 with the constant winds present only around the region where $k_x = 0$. The results of the two cases are found to be in good agreement.

In section 3.11, detailed calculations have been made on the generation and attenuation of electric fields due to the gravity wave winds of different vertical wave lengths under a variety of situations.

Fig. 3.1 defines the coordinate systems used in this investigation. The coordinate system $x' y' z'$ is such that $x'$ and $y'$ are horizontal pointing towards magnetic north and magnetic west respectively, and $z'$ is in the vertically upward direction. In this coordinate system, the gravity wave winds are normally defined. In another coordinate system, $x$ axis is along $\mathbf{B}$, $y$ axis is parallel to $y'$ axis and $z$ axis is in the plane perpendicular to the $xy$ plane and is upwards.
3.2 Geomagnetic Field Line Geometry

We use the geomagnetic field line geometry as envisaged by Farley (1960). From the dipole approximation of the field line we have

\[ \tan \theta = 2 \tan \phi \]  

where \( \theta \) is the dip angle and \( \phi \) is the latitude. The latitude \( \phi \) is given by \( \phi = x'/R \), where \( x' \) is the horizontal distance from the equator and \( R \) is the radius of the earth. Thus for small \( \phi \) we get,

\[ \tan \theta = 2x'/R \]  

If \( I \) is small then the horizontal distance \( x' \) is equal to the distance \( x \) along the field line. Using these facts we get

\[ dx = \frac{R}{2} d\theta \]  

The equation of the field line is given by

\[ Z_f - Z = x^2/R \]  

where \( Z_f \) is the altitude of the field line at the equator and \( Z \) is the altitude which it crosses at a distance of \( x'(x) \) from the equator.

3.3 Variation of Wave Numbers with Latitude

Variation of the wave vector components with the latitude can be studied using the expression (see chapter II),

\[ k_x = k'_{x}c - k'_{z}S \]
\[ k_z = k'_{z}c + k'_{x}S \]
\[ k_y = k'_{y} \]
where \( C = \cos I \) and \( S = \sin I \), \( I \) is the dip angle.

From the above expressions, it can be seen that for a gravity wave, with given wave vector components \( k_x \) and \( k_z \), there is always a dip angle where \( k_x = 0 \). Let this dip angle be \( I_0 \). At this point (hereafter referred to as point S) we have

\[
\tan I_0 = \frac{k_x}{k_z}
\]

As one moves away from this point \( k_x \) is finite. Since the ratio \( \frac{k_x}{k_z} \) can have practically any value for different gravity waves (Hines, 1960), it is possible to realise \( k_x = 0 \) at any desired dip latitude. For a fixed ratio of \( k_x \) and \( k_z \) equal to 10, variation of \( k_x \) and \( k_z \) with dip angle is shown in fig. 3.2. It can be seen from this figure that while variation of \( k_x \) from \(-30^\circ\) to \(+30^\circ\) dip angle is quite large, variation of \( k_z \) over the same range of dip angle is quite small and can be neglected.

### 3.4 The Nature of the Winds

Therefore the wind, say \( \mathbf{w}_y \), has the form

\[
\mathbf{w}_y = \mathbf{w}_0 \exp \left\{ i (k_x x + k_y y + k_z z - \Omega t) \right\}
\]

In the neighbourhood of point S, the concept of wavelength along \( B \) is lost. To study the wind pattern around the point S, it is quite instructive to study the phase of the wind \( \mathbf{w}_y \) at various points along a field line with respect to its phase at the point S. This can be done by evaluating
Fig. 3.3: Schematic variation of wind amplitude around $k_x = 0$ point on a given field line.
the expression \[ \int_{\chi_0}^{\chi} k_x \, d\chi \], where \( \chi_0 \) and \( \chi \) are the distance of the point S and the given point respectively, with respect to some reference point on a given field line. Using eqns. (3) and (5) we get,

\[ \int k_x \, d\chi = R \left( k_x \chi - k_x S \right) \, d\chi \]

(8)

\[ \int k_x \, d\chi = \frac{k_x R}{z} \, \sec \theta \left( I_0 - I \right)^2 \]  

(9)

It can be seen from eqn (9) that the phase of the wind varies symmetrically around the point S, or in other words around the dip angle \( \theta \), where \( k_x = 0 \). For an initial phase of zero at the point S, the wind pattern around the point S is shown in fig.3.3. It can be seen from this figure that the wind around the point S remains in the same direction over a large distance and then it varies sinusoidally with decreasing wavelength on either side of the point S.

The same situation is realised, more clearly, from fig.3.4. In this figure, different wave fronts at a given time with \( k_x = 0 \) and phase of \( (\eta \pi + \pi/2) \), where \( \eta \) is an integer are drawn intersecting a pair of field line at different points. The solid areas enclosed by the pair of the field lines denote that the wind is positive in these regions. Blank areas denote that the wind is in opposite direction, that is, negative.
3.5 The Source Region

The distance $2d$ (± $d$ on either side of the point $S$) over which the phase of the wind remains within $\pi/2$ with respect to its initial phase at the point $S$, can be calculated using eqn. (9) and is given by

$$2d = \left\{ \frac{R \lambda \zeta |C_{o3}I_{o}|}{\lambda_s} \right\}^{1/2}$$

For small $I_{o}$ eqn. (10) can well be approximated to

$$2d = (2R \lambda \zeta)^{1/2}$$

Thus for $R = 6400$ km and $\lambda \zeta = 16$ km we get

$$2d = 320 \ km$$

As has been discussed in chapter II, although the alternating winds give rise to alternating electric field along $E$, they get shorted due to the fact that the ratio $\sigma_{c}/\sigma_{p}$ is very large. $\sigma_{o}$ and $\sigma_{p}$ are the parallel and Pedersen conductivities, respectively.

Since the undirectional winds in the region $AA'$ (see fig. 3.3) do not have their counterpart anywhere on the given pair of field lines, they alone contribute to the production of an electric potential. Thus, the winds in the region $AA'$ serve as the source of an electric field, that is, the region $AA'$ is the dynamo region. The complete
region of the ionosphere connected with the field lines serves as a load to this potential. Hence, the distance 2d, over which the winds are unidirectional around the point S, defines the extent of the source region along B. From eqn. (12) it can be seen that the perpendicular wavelength \( \lambda_z \) is very important in determining the extent of the source region. Since the vertical wavelength of the gravity wave winds expected to be present in the E and F regions is, of the order of 10 to 100 km, the dynamo region can extend to hundreds of kms along the field line.

### 3.6 Theory

As has been pointed out earlier, that the charge excess along B due to the variational winds tends to short out, the electric potential produced due to such a wind will not have the same variational form along B as that of the wind. The electric potential due to such a wind can therefore be assumed to be of the form

\[
\phi = \phi_0(x) e^{k_x x} \{ i(k_y y + k_z z - \omega t) \} \tag{14}
\]

where \( \phi_0(x) \) varies monotonically with \( x \). If the polarization fields are assumed to be electrostatic then,

\[
E = - \nabla \phi \tag{15}
\]

The electric current due to an electric field \( E \) and wind \( W \) is given by

\[
J = \left( \frac{\epsilon_0}{\mu_0} \right) E + \nabla \times \{ \left( \frac{\epsilon_0}{\mu_0} \right) W \times B \} \tag{16}
\]
where \( \parallel \) and \( \perp \) denote the components parallel and perpendicular to \( B \), and \( \sigma_c \), \( \sigma_p \), and \( \sigma_H \) are the parallel, Pedersen and Hall conductivities respectively. Using the above set of equations and imposing the condition

\[
\nabla \cdot j = 0
\]

the following second order partial differential equation is obtained

\[
\frac{C_p}{\partial x^2} + \frac{\partial \sigma_c}{\partial x} \frac{\partial \Phi}{\partial x} = \{ \sigma_p (k_x^2 + k_z^2) - i(k_x \frac{\partial \sigma_p}{\partial z} + k_y \frac{\partial \sigma_H}{\partial z}) \} \Phi
\]

\[
+ \int \left[ \left( \sigma_p (k_y \omega_{0y} - k_z \omega_{0z}) - i \sigma_H k_x \omega_{0x} - \omega_c \frac{\partial \Phi}{\partial z} \right) + \omega_{0z} \frac{\partial \sigma_H}{\partial z} \right] \exp(i k_x dx)
\]

wherein we have used the fact that \( \nabla \cdot \omega = 0 \)

Eqn.(18) can be further simplified if the terms with gradients in Pedersen and Hall conductivities are ignored in comparison to \( k_z \). This requires that

\[
k_z > \frac{1}{\Omega_p} \frac{\partial \sigma_p}{\partial z}
\]

Even if the \( \sigma_p \) scale length during the day time and the night time is taken to be 50 km and 6 km respectively, these are smaller than the wave number \( k_z \) corresponding to a wavelength of 25 km. If the wavelength is smaller,
then the neglect of these terms is more justified.

Neglecting these we get,

\[ \sigma_0 \frac{\partial^2 \phi_0}{\partial x^2} + \frac{\partial \sigma_0}{\partial x} \frac{\partial \phi_0}{\partial x} = \sigma_0 (k_I^2 + k_2^2) + B \int \sigma_0 (k_I w_0 - k_2 w_2) \exp(i k_2 x) \, \text{d}x \]

\[ - i \sigma_0 k_x w_0 \int \sigma(x) \exp(i k_2 x) \, \text{d}x \]

(21)

If \( \phi_0 = \phi_{01} + i \phi_{02} \)

(22)

From eqn. (21) we get following two equations

\[ \sigma_0 \frac{\partial^2 \phi_{01}}{\partial x^2} + \frac{\partial \sigma_0}{\partial x} \frac{\partial \phi_{01}}{\partial x} = \sigma_0 (k_I^2 + k_2^2) \phi_{01} \]

\[ - B \int \sigma_0 (k_I w_0 - k_2 w_2) - \sigma_0 k_x w_0 \sigma(x) \sin(S k_2 x) \, \text{d}x \]

(23)

\[ \sigma_0 \frac{\partial^2 \phi_{02}}{\partial x^2} + \frac{\partial \sigma_0}{\partial x} \frac{\partial \phi_{02}}{\partial x} = \sigma_0 (k_I^2 + k_2^2) \phi_{02} \]

\[ + B \int \sigma_0 (k_I w_0 - k_2 w_2) - \sigma_0 k_x w_0 \sigma(x) \cos(S k_2 x) \, \text{d}x \]

(24)

Since \( k_2 \gg k_I \) and \( w_0 \gg w_0 \) for the gravity wave winds, eqns. (23) and (24) can be further simplified and we get,

\[ \frac{\partial^2 \phi_{01}}{\partial x^2} + \frac{1}{\sigma_0} \frac{\partial \sigma_0}{\partial x} \frac{\partial \phi_{01}}{\partial x} = \sigma_0 k_2^2 \phi_{01} \]

\[ + B \int \frac{\sigma_0}{\sigma_0} k_2 w_0 \sigma(x) \sin(S k_2 x) \, \text{d}x \]

(25)

\[ \frac{\partial^2 \phi_{02}}{\partial x^2} + \frac{1}{\sigma_0} \frac{\partial \sigma_0}{\partial x} \frac{\partial \phi_{02}}{\partial x} = \sigma_0 k_2^2 \phi_{02} \]

\[ - B \int \frac{\sigma_0}{\sigma_0} k_2 w_0 \sigma(x) \cos(S k_2 x) \, \text{d}x \]

(26)
Representing the potential in terms of a dimensionless quantity \( \psi \), such that
\[
\psi = \psi_1 + i \psi_2 \tag{27}
\]
\[
|\psi| = (\psi_1^2 + \psi_2^2)^{1/2}
\]
where
\[
\psi_1 = k_z \phi_{01} / B_{woy} 
\]
\[
\psi_2 = k_z \phi_{02} / B_{woy}
\]
Equations (25) and (26) thus reduce to
\[
\frac{\partial^2 \psi_1}{\partial x^2} + \frac{1}{\sigma_0} \frac{\partial \sigma_0}{\partial x} \frac{\partial \psi_1}{\partial x} = \frac{\sigma_P}{\sigma_0} \left( k_j^2 + k_z^2 \right) \psi_1
\]
\[
+ k_z \left\{ \frac{\sigma_P}{\sigma_0} \left( k_z - k_y w_{Oz}/w_{Oy} \right) + \frac{\sigma_P}{\sigma_0} k_x w_{Ox}/w_{Oy} \right\} \sin \left( k_x x \right) \tag{29}
\]
\[
\frac{\partial^2 \psi_2}{\partial x^2} + \frac{1}{\sigma_0} \frac{\partial \sigma_0}{\partial x} \frac{\partial \psi_2}{\partial x} = \frac{\sigma_P}{\sigma_0} k_z^2 \psi_2
\]
\[
- k_z \left\{ \frac{\sigma_P}{\sigma_0} \left( k_z - k_y w_{Oz}/w_{Oy} \right) + \frac{\sigma_P}{\sigma_0} k_x w_{Ox}/w_{Oy} \right\} \cos \left( k_x x \right) \tag{30}
\]
The dimensionless quantity \( \psi \) represents the ratio of the vertical polarization field to the wind induced field in that direction. At the point \( S \), since \( k_x = 0 \) using eqn.(19) we get,
\[
k_z/w_{oy} = - k_y/w_{Oz} \tag{31}
\]
From the above expression it is clear that \( \psi_1 \) and \( \psi_2 \) can also be used to represent the electric field in the
Further simplification for the solution of eqns. (29) and (30) can be sought on the following physical reasoning. Since it is the wind around the point S which contributes predominantly to the generation of electric field, the wind effect being cancelled out in the other regions due to its variational form; it may as well be assumed that a constant wind is present only in the region around the point S which is as effective in generating electric field as is the variational wind of fig. 3.3. Therefore, if we consider the constant wind only around the point S where \( k_x \approx \omega \), we get only one equation, corresponding to eqn. (30) which contains the source term (wind term) and is given by

\[
\frac{\partial^2 \psi_2}{\partial x^2} + \frac{1}{\omega^2} \frac{\partial}{\partial x} \frac{\partial \psi_2}{\partial x} = \frac{\sigma_p}{\varepsilon_0} (k_x^2 + k_z^2) \psi_2 - k_z \left\{ \frac{\sigma_p}{\varepsilon_0} (k_z - k_y) \frac{\partial \psi_2}{\partial y} \right\}
\]

The equation in \( \psi_1 \) corresponding to eqn. (29) is not significant as it does not contain the source term. This equation gives information only on the attenuation of the field. To justify this approach, results obtained with the set of eqns. (29) and (30) were compared with those obtained using eqn. (31) alone. It was found that the results obtained
with the latter case were closely representing the results obtained with the former set of equations, as will be discussed later. It may be remarked here that this approach is fully justified for gravity waves having large vertical wavelengths.

3.6.1 Boundary Conditions

Eqn. (32) can be solved using a step by step numerical method if the initial conditions viz \((\psi_2)_{x=0}\) and \((\partial\psi_2/\partial x)_{x=0}\) are known. Due to very low conductivity in the lower E region, say at 90 km, it is quite reasonable to assume that the parallel current vanishes, in both the hemispheres, at and below this altitude (Sprieter and Briggs, 1961). It implies that \(\partial\psi_2/\partial x = 0\) at the boundaries of the ionosphere. So the two boundary conditions are obtained with this physical argument. To make the problem an initial value problem, various values of \(\psi_2\) were tried at the initial point (with \(\partial\psi_2/\partial x = 0\)) until the correct solution giving \(\partial\psi_2/\partial x = 0\) at the other boundary was obtained.

3.6.2 Numerical Method

To minimise the number of steps to obtain the correct solution, a novel method was employed. It is based on the method of logarithmic fractionation. In this method the value of \(\psi_2\) is chosen to be 1 at the initial point which is evidently the largest possible value of \(\psi_2\). Using this as
the initial value, the complete solution is obtained. Then this value is decreased in accordance with the formula

$$\psi_2 = (\psi_2)_{\text{initial}} e^{(-k + 2/5 - m/2.5 + n/12.5)}$$

where $k, l, m$ and $n$ are integers with values ranging from 0 to 5. Keeping the values of $l, m$ and $n$ zero, the $k$ value is increased until the sign of $\frac{\partial \psi_2}{\partial x}$ changes at the other boundary. This value of $k$ is then frozen and the value of $l$ is increased, keeping $m$ and $n$ zero, until $\frac{\partial \psi_2}{\partial x}$ at the other boundary again changes sign. Similarly, $m$ and $n$ values are increased keeping other variables ($k, l, n$ or $k, l$ and $m$) constant. The correct solution is intermediate to the solutions in which, with the increase of $n$ value, the sign of $\frac{\partial \psi_2}{\partial x}$ changes at the other boundary. The uncertainty in the value of $\psi_2$ at the initial point is about 2%.

Fourth order Runge-Kutta method with Adam-Bashforth predictor - corrector for the successive values was employed to solve the differential equation. The accuracy demanded of the solution was up to the sixth decimal place. The step size, during the calculations, was automatically decreased to give the desired accuracy.
Fig. 3.5: Electron density profiles used for calculating the conductivity profiles for different times of the day.
3.7 The Conductivity Profiles

Calculations were carried out for the daytime, evening time and two night time conditions. As the electron density profile, on which the conductivity profiles are mainly dependent, is highly variable during the evening and night hours; no standard model of the electron density can be used. Based on various rocket launches from Thumba, India and elsewhere, and the ionosonde data available in the literature, four electron density profiles were built up representing the day, evening and two night time situations as given in fig. 3.5. The \( \phi_0 \) and \( \phi_\rho \) profiles for these different periods of the day are given in fig. 3.6.

3.8 Approximate Solutions

Before attempting to solve eqn. (32) rigorously for various cases, the general nature of the results can be studied using following approximations.

3.8.1 Attenuation

Outside the source region, dependence of \( \psi_z \) on \( k_z \) and the attenuation length can be estimated by noting the importance of various terms in eqn. (32). In the region where \( \partial \phi_0 / \partial x \) is small, i.e. around the apogee of the field line, the second term on the left hand side of eqn. (32)
can be neglected and we get
\[ \frac{\partial^2 \psi_2}{\partial x^2} = k_x^2 \frac{C}{C_0} (\psi_2 - 1) \]  
(34)
whose solution is (ignoring the variation \( \bar{\rho} \) and \( C_0 \) around the apogee of the field line)
\[ 1 - \psi_2 = (1 - \psi_0) e^{k_x (\pm k_x \sqrt{C/\bar{\rho}} / C_0, x)} \]  
(35)
Since away from the source region, the value of \( \psi_2 \) should decrease i.e., \((1 - \psi_0)\) should be large, exponent in eqn.(35) should have the positive sign. The attenuation length of \((1 - \psi)\) is, therefore
\[ \frac{1}{L_1} = \frac{1}{1 - \psi_2} \frac{\partial}{\partial x} (1 - \psi_2) = k_x \sqrt{C/\bar{\rho} / C_0} \]  
(36)
Hence, around the apogee of the field line, for a given value of \( \lambda_2 \), the attenuation of the field depends on the square root of the ratio of \( \bar{\rho} \) and \( C_0 \). If \( k_x \) is small, i.e. large \( \lambda_2 \), the attenuation of the field is small and vice versa.

If the first term on the left hand side of eqn.(32) is smaller compared to the second term, which is true in the \( E \) region, then the first term can be neglected and we get,
\[ \frac{\partial}{\partial x} \psi_2 = \frac{C}{C_0} k_x^2 (1 - \psi_2) \]  
(37)
where \( H \) is the scale height of \( C_0 \) along \( E \). Hence the
attenuation length, in this case, is given by

\[
\frac{1}{\lambda_z} = \frac{1}{(1 - \psi)} \frac{\partial}{\partial z} (1 - \psi) = H \frac{\sigma_r^2}{\sigma_o} k_z^2
\]  \tag{38}

Thus for a given value of \( \lambda_z \), the attenuation length of \( 1 - \psi \) in the E region depends on the ratio of \( \sigma_r \) and \( \sigma_o \) and also on the scale height of \( \frac{1}{B} \) along \( \varepsilon \).

Solution of eqn. (37) is

\[
1 - \psi = (1 - \psi_\infty) e x p \left( \int H \frac{\sigma_r^2}{\sigma_o} k_z^2 dx \right)
\]  \tag{39}

Sign of the exponent in eqn. (39) depends on the sign of \( H \), which on integration yields the logarithmic ratio of \( \sigma_o \) in two regions and hence, is always positive. Thus away from the source region, the value of \( \psi_\infty \) should decrease which is, indeed, true.

If \( k_z \) is small, i.e. large \( \lambda_z \), it is clear from eqn. (39) that \( \psi_\infty \) tends to \( \psi_o \). Hence, for large value of \( \lambda_z \), the attenuation of the field is small. If \( k_z \) is large, then the term on the right of eqn. (39) has a large value, implying that \( (1 - \psi) \) has a largest possible value. Since \( \psi_\infty \) is positive, it implies that the value of \( \psi_\infty \) is very small. Hence for small values of \( \lambda_z \), the attenuation of the field is large.
3.8.2 Generation of fields for large values of $\lambda z$

Since for large values of $\lambda z$, the attenuation of the field is very small, the electric field in this case can be assumed to be constant and its value can be estimated in the following way. The derivatives of the currents $J_y$ and $J_z$ are given by (see chapter II).

$$\frac{\partial J_y}{\partial y} = i \sigma_p k_y (E_y + B \omega_z) - i \sigma_p k_y (E_z - B \omega_y)$$  (40)

$$\frac{\partial J_z}{\partial z} = i \sigma_p k_z (E_z - B \omega_y) + i \sigma_p k_z (E_y + B \omega_z)$$  (41)

Adding eqns. (40) and (41), and using the facts that $\nabla \cdot \mathbf{J} = 0$ and $k_z = 0$ we get

$$\frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} = i \sigma_p \left\{ k_y E_y + k_z E_z + B (k_y \omega_z - k_z \omega_y) \right\}$$

$$- i \sigma_p (k_y E_z - k_z E_y)$$  (42)

The form of the potential ensures that the last term on the right hand side of eqn. (42) is zero. Also, $k_z \gg k_y$ and $\omega_0 y > \omega_0 z$ for the gravity wave winds, we get

$$\frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} \equiv i \sigma_p (k_z E_z - B k_z \omega_y)$$  (43)

since $\nabla \cdot \mathbf{J} = 0$,

$$J_x = - i \int \sigma_p k_z (E_z - B \omega_y) \, d\chi$$  (44)

As shown in fig. 3.7, if the region bounded between $x_2$ and $x_3$ is the source region (wind region), the current flowing outside the wind region depends on $E_z$ only and
FIG. 3.7. SCHEMATIC DIAGRAM SHOWING THE WIND REGION AND THE LOAD REGION
The current in the wind region is given by
\[ J_x = -i k_z \int_{x_2}^{x_3} \sigma_p E_z \, dx \]  
(4.6)

Since the current flowing outside the wind region should be equal to the current generated in the wind region, we get
\[ \int_{x_2}^{x_3} \sigma_p E_z \, dx + \int_{x_2}^{x_3} \sigma_p E_z \, dx = -B w_y \int_{x_2}^{x_3} \sigma_p \, dx \]  
(4.7)

Hence,
\[ E_z = \frac{-B w_y \int_{x_2}^{x_3} \sigma_p \, dx}{\int_{x_2}^{x_3} \sigma_p \, dx} \]  
(4.8)

It is clear from eqn.(4.8) that, for large value of \( \lambda \), the generated electric field depends on the ratio of integrated Pedersen conductivity in the source region to the total integrated Pedersen conductivity. If the wind region overlaps the region of maximum \( \sigma_p \), larger field is generated. Also, since the extent of the wind region increases with \( \lambda \), for large value of \( \lambda \), the field is more.

3.9 Generation of Electric fields with \( \mathbf{R}_x = 0 \) in Different Regions.

From the foregoing discussions it is clear that for large values of \( \lambda \), \( E_z \) remains constant in the region bounded by a pair of field lines. This field is maximum when the wind region overlaps the region of maximum \( \sigma_p \). An estimate of the generated field can also be made in this case. However,
for small value of \( \lambda_z \), the field can not be estimated and only qualitative information on the attenuation of the field can be obtained.

To ascertain the efficiency of generation of fields by the winds (of a given \( \lambda_z \)), calculations were carried out whence the centre of the wind region was shifted from the equator down to the region where a given field line crosses the 90 km altitude. The centre of the wind region was shifted by 100 km in successive calculations. It is worth while to point out here that the wind region, as defined in this analysis, implies the region over which \( k_x \cdot C \). Two values of \( \lambda_z \), namely 25 km and 4 km, were used for such calculations.

Figs. 3.8 and 3.9 give the results of these calculations when the field line apogee is taken to be at 300 km and 200 km, respectively. The solid curves of these figures represent the results of the case when \( \lambda_z = 25 \text{ km} \) while the dashed curves represent for the case when \( \lambda_z = 4 \text{ km} \). The curves marked 1, 2 and 3 represent the results for the day time, evening time and night time situations, as represented by the conductivity profiles of fig. 3.6. Lower scale of these figures gives the distance along field line as reckoned from the equator. For successive calculations, the winds were assumed to be present in different regions along such a field line. For a given location of the wind
region, $v_{\text{max}}$ denotes the maximum value of $v$ for such a case. Vertical scale of these figures gives the value of $v_{\text{max}}$. Thus a point on these curves represents the maximum value of $v$ (i.e. $v_{\text{max}}$) when the winds are centred around the point as obtained from the corresponding point on the lower scale. Along with the results of these calculations, a field line which drops from a given altitude (200 km or 300 km in these cases) at the equator is also plotted. Top scale of these figures gives the corresponding altitude which is crossed by this field line.

The altitude range covered by the winds when assumed to be present in different regions on a field line can be obtained using the field line geometry as given in these figures. The extent of the wind region, for a given $\lambda_z$, is given by eqn. (12). For example, if $\lambda_z = 4$ km, the wind region extends 80 km on either side of the centre of the wind region. If the apogee of the field line is at 300 km and the centre of the wind region is at 500 km, the wind region extends from 420 km to 580 km along the field line. The altitude range corresponding to this extent of the wind region is approximately 248 km to 272 km, as can be seen from fig. 3.8.

3.9.1 Field line apogee at 300 km

We first discuss the results of the case when the field line apogee is taken to be at 300 km. Results of this case are shown in fig. 3.8. For the case when $\lambda_z = 25$ km, the
Fig. 3.8: Variation of \( \Psi_{\text{max}} \) with distance along a field line. The curve marked B is the geomagnetic field line dropping from the equator at an altitude of 300 km (see top scale).
wind region has an extent of 400 km.

It can be seen from fig. 3.8 that for $\lambda z = 25$ km case, $\psi_{max}$ has the largest value at a distance of about 900 km from the equator (i.e. in the E region) for the daytime case whereas it is maximum around the equator (i.e. in the F region) for the night time case. The evening time case has two maxima, in the F region and in the E region, respectively. These results are in accordance with the $J_r^p$ profiles (fig. 3.6) in which, for the daytime case $J_r^p$ is maximum in the E region while it is maximum in the F region for the night time case. For the evening time case, $J_r^p$ values in the 140 km and 300 km regions are almost equal.

For a source region with an extent of 400 km, it was found that the integrated $J_r^p$ along a field line, $\int J_r^p \, dx$ is maximum in the region around 900 km for the daytime case, around the equator and 900 km for the evening time case. The ratio of $\int J_r^p \, dx$ to the total integrated $J_r^p$, $\int J_r^p \, dx$ (when the source is assumed to be situated around the regions mentioned above) comes out to be about 0.45, 0.40, and 0.25 for the day time, night time and evening time situations respectively. These values are close to the maximum value of $\psi_{max}$ for these situations.

Results of the case when $\lambda z = 4$ km are also given in fig. 3.8. These results are essentially similar to the ones obtained in $\lambda z = 25$ km case, except for the night time situation. In the night time case, $\psi_{max}$ has a maxima in the
Fig. 3.9: Variation of $\psi_{\text{max}}$ with distance along a field line. The field line apogee is at 200 km in this case.
E region also. This is owing to the shape of the $\sigma_p$ profile in the night time, which has a peak in the E region. Effect of this shape of $\sigma_p$ is suppressed (but is still visible) in $\lambda z = 25$ case. This suppression is owing to larger length of the source region.

For $\lambda z = 4$ km case, the maximum value of $\psi_{nal}$ for different times of the day, is more than that obtained for respective times in $\lambda z = 25$ km case. It implies that for smaller $\lambda z$, the field in the source region is larger.

### 3.9.2 Field line apogee at 200 km

Results of the case when the field line apogee is at 200 km are presented in fig. 3.9. It can be seen from fig.3.6 that $\sigma_p$ has the largest value in the E region. Hence the value of $\psi_{nal}$ is expected to be largest only in the E region. It is indeed so, as can be seen in fig.3.9. Rest of the features of the results for this case are qualitatively similar to the ones obtained in section 3.9.1.

### 3.10 Comparison of Results - Constant Wind Case and Sinusoidal Wind Case

We now compare the results obtained using the set of eqns.(29) and (30) with those obtained using eqn.(32) alone. This comparison is necessary to show that the constant winds around the region where $\lambda k = 0$ are as effective in generating electric fields as are the sinusoidal winds of section 3.4 (Fig.3.3).
Fig. 3.10: Comparison of results obtained using the constant winds in region $AA'$ (Fig. 2.3) with those obtained using the variational winds of Fig. 2.3. $\lambda = 15$ km for both the cases. $\psi$ is the representative of the total field.
We consider a daytime case for gravity wave winds with $\lambda z = 16$ km. We assume that the condition $k x=0$ is obtained at a distance of 940 km along a field line which has the apogee at 300 km. For $\lambda z = 16$ km, the distance $2d$, over which the winds are in the same direction around $k x=0$ point is 320 km.

Results of these calculations are presented in fig.3.10 in which curves marked $\Psi_1$ and $\Psi_2$ correspond to the solution of eqn.(29) and (30) respectively. $|\Psi|$ denotes the total field, as defined in eqn.(27) and is also plotted as solid curve.

Results of the case when constant winds are in the region from 780 km to 1100 km along the field line (centre at 940 km; $2d = 320$ km) are also given in fig. 3.10 with dashed curve. This curve is also labelled as $|\Psi|$.

A comparison of the curves marked $|\Psi|$ shows that both the curves have near identical characteristics with nearly same value. Hence the constant winds around $S$, over the distance $2d$ as determined by $\lambda z$, are as efficient in generating electric fields as are the sinusoidal winds of fig. 3.3. Hence in the following section, calculations are made with constant winds present in a limited region.

3.11 Calculations with Constant Winds in a Limited Region-Results and Discussion:

We now solve eqn.(32) for different values of $\lambda z$, for various times of the day. A schematic of the calculations carried out is given in fig. 3.11.
FIG. 3.11 SCHEMATIC OF CALCULATIONS
Calculations were carried out for four different values of \( \lambda z \); 25 km, 16 km, 9 km and 4 km. For each value of \( \lambda z \), the cases with the field line apogee at 300 km and 200 km were considered. With a given apogee of the field line, two positionings of the wind regions, in the E region (Case A or Case C) and in the F region (case B or Case D), were considered. Each of these cases (Case A etc) was investigated for the daytime (A1, B1 etc), eveningtime (A2, B2 etc), and nighttime (A3, B3 etc) situations separately.

3.11.1 \( \lambda z = 25 \text{ km} \):

The distance \( 2d^* \) the extent of the source region, can be calculated using eqn. (12) and is 400 km in this case.

3.11.1.1 Case A - Field line apogee at 300 km and winds in the E region:

The gravity wave winds of same velocity amplitude were assumed to be present, with centre at 940 km reckoned from the equator, extending from 740 km to 1140 km along the field line. The altitude range thus covered by the wind region is from 96 km to 214 km in one of the hemispheres. The solid curves of fig. 3.12 give the results of these calculations, in which curves marked (1), (2) and (3) represent the daytime (A1) eveningtime (A2) and nighttime (A3) cases respectively. The electric field represented by parameter \( \Psi \) is plotted against the distance along the field line, reckoned from the equator.
Fig. 3.12: Variation of $\psi$ with distance along B. The field line apogee is at 300 km in this case.
Generation of field in the source region:

It can be seen from fig. 3.12 that the maximum field in the source region is generated during the daytime and that the eveningtime field is larger than the nighttime field. This is because, for the daytime case, the wind region overlaps the region of maximum $\sigma_p$ and outside the wind region, the conductivity $\sigma_p$ is very small. For the eveningtime case, although the wind region overlaps one of the regions of maximum $\sigma_p$, its value outside the wind region is nearly the same. For the nighttime case, the wind region does not overlap the region of maximum $\sigma_p$.

It can be seen from fig. 3.12 that in all the cases, the polarization field in the source region is never fully developed. The field is always less than 0.5 times the wind induced field $(W \times B)$. For the daytime case, maximum field in the source region is about 0.45 times the $(W \times B)$ field, while it is about 0.24 and 0.09 times the $(W \times B)$ field for the evening and nighttime cases respectively.

Attenuation of the field:

Since the ionospheric regions are connected through the conducting geomagnetic field lines, the electric field generated in one region is likely to be transmitted to the other regions. It was shown earlier in section 3.8 that for large $\lambda z$, the attenuation of the fields will be small. The results as given in fig. 3.12 confirm this feature. For instance, in all the
three cases, the attenuation of the field from the point S to the equator is less than 10%. That is, an appreciable fraction of the source field is transmitted to the equator.

3.11.1.2 Case-B. Field line apogee at 300 km and winds in the F region.

In this case the winds are assumed to be present in the F region, symmetrically with respect to the equator around the apogee of the field line. The extent of the wind region being ± 200 km around the equator, along the field line. The altitude range thus covered is about 6 km. Results of these calculations are presented in fig.3.12 with dashed curves (the curve for the case B1 is not plotted as its value was less than 0.05).

It can be seen from the dashed curves of fig.3.12 that while there is little difference between the results of two eveningtime cases (cases A2 and B2), the daytime (B1) and nighttime (B3) cases differ significantly from the corresponding cases of case A.

Results of the two eveningtime cases are similar because, in both the cases the wind region overlaps one of the regions of maximum $\omega_p$. This is owing to the shape of the $\omega_p$ profile for the eveningtime case (Fig.3.6). Also, the value of $\omega_p$ in the source region for the two cases in question, is almost equal. For the daytime case (B1), the wind region does not overlap the region of maximum $\omega_p$. Thus the generated electric field is very small. For the nighttime case (B3), the winds are situated in the region of maximum $\omega_p$. Also, due to
very small Pedersen conductivity in the E region during the nighttime, most of the E region acts as an open circuit to the source field. Thus, the field is transmitted, as such, to the E region.

The attenuation of the field, for all the cases, is again less than 10%.

The results of the nighttime case (B3) bring out an important point. That is, in the nighttime, the dynamo (source of an electric field) is situated in the F region. The field generated in the F region is transmitted to the lower latitude E region in appreciable magnitude. This conclusion, on the location of the nighttime dynamo, is qualitatively similar to the one drawn by Rishbeth (1971) where he observed that the large scale east west winds in the equatorial F region during the nighttime are capable of generating electric fields in the F region. These fields can be transmitted to the E region in appreciable magnitude.

3.11.1.3 Case C - Field line apogee at 200 km and winds in the E region:

In this case, the winds were assumed to be present in the region extending from 400 km to 800 km along B, with their centre at 600 km. The altitude range thus covered by the wind region is from 100 km to 175 km. For this altitude range (by considering conductivity profiles upto 200 km altitude only), it can be seen from fig.3.6 that the region of maximum $C_P$ is covered in all the three cases. Hence,
Fig. 3.13: Variation of $\Psi$ with distance along B. The field line apogee is at 200 km.

$\lambda z' = 25$ kms

DISTANCE ALONG B FROM THE EQUATOR (km)
large electric fields are expected to be generated in all the cases.

Solid curves of fig. 3.13 give the results of the calculations for this case. Once again, the curves marked 1, 2 and 3 represent the results of the daytime (C1), eveningtime (C2) and nighttime (C3) situations respectively. A curve marked 4 represents the results of the second nighttime case corresponding to the electron density profile (numbered 4) of fig. 3.5.

It can be seen from fig. 3.13 that the maximum field in the source region is developed during the daytime. Its value being 0.43 times the wind induced ($W \times B$) field. The field which is transmitted to the equator is about 0.39 times the ($W \times B$) field. It implies that about 85% of the source region field is transmitted to the equator.

Comparison of the eveningtime (C2) and nighttime (C3) results shows that, in general, the evening time field is more than the nighttime field. In the source region, the fields are generated in almost equal amount in both the cases. This is because of the particular shape of the $\Omega_p$ profile for the nighttime case which has almost constant $\Omega_p$ in the region where the winds are assumed to be present. The amount of the field which is transmitted to the equator is approximately 90% and 82% (of the maximum field in the source region) for the evening and night time cases respectively. The maximum field in the source region for both the cases (C2 and C3) is
about 0.35 times the \((W \times B)\) field.

The curve for the second nighttime case (C4) has the lowest value of \(\Psi\) in comparison to the other cases. This is in accordance with the electron density profile (fig. 3.5) for this case.

A comparison of the results obtained in the cases A and C reveals that the attenuation of the field is less in case A. For the daytime situation, the maximum field in the source region is almost equal in the cases A1 and C1. This is because in both the cases, the wind region covers the region of maximum \(C_P\), within which the integrated conductivity is essentially equal. Results of the evening and nighttime cases are, however, quite different in the two cases A and C. For Case A2, it can be seen from fig. 3.13 that the value of \(C_P\) outside the wind region is comparable to that in the wind region. While in case C2, \(C_P\) outside the wind region is quite small. Hence, the field in case C2 is larger than in case A2. In case A3, the value of \(C_P\) outside the wind region is much larger than inside the wind region, while it is smaller in case C3. Hence the electric field in case A3 is much smaller than in case C3.

3.11.1.4 Case D - Field line apogee at 200 km and winds around the apogee of the field line:

The winds were assumed to be present in the region ±200 km (along the field line) around the equator. The altitude range covered by the wind region is very narrow and
\( \lambda Z' = 16 \text{ km FIELD LINE APOGEE AT 300 kms} \)

- Variation of \( \psi \) with distance along \( B \).

DISTANCE ALONG A FIELD LINE FROM THE EQUATOR (km)
Fig. 3.15: Variation of $\psi$ with distance along B. $\lambda_B = 16$ km. The field line apogee is at 200 km.
for the practical purposes it can be assumed that the conductivities $\sigma_0$ and $\sigma_p$ are constant in the wind region.

Results of the calculations for this case are presented in fig. 3.13 as the dashed curves. The general conclusions drawn from the results of case B are valid for this case also. It can be seen that the maximum field is generated in the nighttime and the least during the daytime. The field in the nighttime (D3) and eveningtime (D2) is about 0.21 and 0.15 times the wind induced field, $(\mathbf{W} \times \mathbf{B})$, respectively.

A comparison of results of cases B and D reveals that the generation of the field is more in case B. This is because of larger value of integrated $\sigma_p$ in the wind region in case B as compared to that in case D.

The attenuation of the fields during the transmission is again quite small.

3.11.2 $\lambda_Z = 16$ km

The extent of the source region in this case is 320 km along the geomagnetic field line.

Calculations carried out under the conditions similar to those for $\lambda_Z = 25$ km case were repeated. The results of the calculations are presented in figs. 3.14 and 3.15. It can be seen from these diagrams that the results of this case are qualitatively similar to those obtained in section 3.11.1 for $\lambda_Z = 25$ case.
Fig. 3.16: Variation of $\psi$ with distance along B. The field line apogee is at 300 km.
$\lambda_{z'} = 9 \text{ kms}, \text{ APOGEE} = 200 \text{ kms}$

DISTANCE ALONG B FROM THE EQUATOR (km)

Fig. 3.17: Variation of $\psi$ with distance along B.
Since the results of this section are not different from those of section 3.11.1, no further discussion of the results is necessary.

3.11.3 $\lambda z = 9 \text{ km}$

In this case the extent of the wind region is 240 km along $B$ and the wind region was so situated that it does not, necessarily, overlap the region of maximum $C_p$ in all the cases. For example, in case A, the wind region was from 820 km to 1060 km (along $B$) which corresponds to an altitude of range of about 125 km to 195 km in one of the hemispheres. Similarly, in case C, in which the field line apogee is at 200 km altitude, the wind region was from about 130 km to 175 km altitude.

Results of this case are presented in figs. 3.16 and 3.17 which correspond to the case with the field line apogee at 300 km and 200 km respectively. It can be seen from these figures that in this case, the attenuation of the field during transmission is larger than in the cases considered earlier in sections 3.11.1 and 3.11.2. In this case, the attenuation of the field in case C is more than in case A, and in case D is more than in case B. A good fraction of the $(W \times B)$ field is, however, generated in the source region in cases A1, A2 and B3 and is transmitted to the other regions also.
3.11.4 \( \lambda_Z = 4 \, \text{km} \)

This is the last of the cases considered for these studies. For \( \lambda_Z = 4 \, \text{km} \), wind region is 160 km along a given field line. Again, the winds were assumed to be centred around the apogee of the field line in cases B and D, and at a distance of 940 km (Case A) and 600 km (case C) from the equator along B. In cases A and C, the wind region does not overlap the region of maximum \( \sigma_p \).

Results of this case are presented in figs. 3.18 (for cases A and B) and 3.19 (cases C and D). It can be seen from these figures that for all the cases, the attenuation of the field is larger in the E region.

During transmission of the source region field to the equator, the field suffers an attenuation of about 25% in case A, while the attenuation is more than 50% in case C. These results, on the attenuation of the field, are in conformity to the conclusions drawn in section 3.8 for small value of \( \lambda_Z \). Although the generation of the field in the source region, in this case, is more than in the cases considered earlier in section 3.11.1 through 3.11.3, these fields suffer strong attenuations and become very small within a short distance. Hence for small \( \lambda_Z \), appreciable amount of the electric field can not be transmitted to the other regions.
Fig. 3.18: Variation of $\Psi$ with distance along B. Apogee of the field line is at 300 km.
\[ \lambda_{z^{'}} = 4 \text{ kms, APOGEE}= 200 \text{ kms} \]

Fig. 3.19: Variation of \( \psi \) with distance along B.

DISTANCE ALONG B FROM THE EQUATOR (km)
3.12 Summary:

The generation of electric fields due to the three-dimensional gravity wave winds and their transmission to the other regions of the ionosphere was investigated in this chapter. The gravity wave winds in different regions of the ionosphere with different vertical wavelengths were used to calculate the electric fields at different times of the day. The following are the salient features of this study.

(i) While \( k_z \) varies slowly with the dip angle, \( k_\chi \) varies rapidly and undergoes a change of sign (fig. 3.2).

(ii) On a given geomagnetic field line, the wind pattern around the point where \( k_\chi = 0 \) is symmetrical (figs. 3.3 and 3.4). This point has been referred to as point S. Around the point S, the winds remain in the same direction over a large distance (\( \zeta \eta = \sqrt{\lambda_z k} \)). For example, for \( \lambda_z = 16 \) km, this distance is 320 km. Away from the point S, the wind varies sinusoidally with wavelength decreasing with increasing distance.

(iii) The electric fields are mainly generated due to the winds around S and hence this region has been referred to as the source region. The electric fields due to the sinusoidal winds get shorted, approximately, within a wavelength.

(iv) The geomagnetic field lines act as conductors carrying current. The rest of the region connected by these field lines to the source region acts as load.
(v) In the load region, the attenuation of the electric field increases with the decrease of $\lambda_z$. For example, for $\lambda_z = 25$ km (fig. 3.12, case A1), the attenuation of the field from the point $S$ to the equator is less than 10%. Similarly for $\lambda_z = 16, 9$ and 4 km (Case A1 in figs. 3.14, 3.16 and 3.18 respectively) the attenuation of the field is less than 10%, about 15% and 25% respectively.

For a given $\lambda_z$, attenuation length of the field in the E and F regions can be calculated using the expressions as given in section 3.8. For $\lambda_z = 25$ km, the attenuation length in the E and F regions is of the order of $10^4$ and $10^5$ km respectively. These values are in close agreement with the one obtained from fig. 3.12 (Case A1).

(vi) The generation of the electric field is most efficient when the source region overlaps the region of maximum $\psi_p$. For example, for the daytime case when $\psi_p$ is maximum in the E region, the field is maximum when the source region is around 140 km. Similarly for the nighttime case when $\psi_p$ is maximum in the F region, the field is maximum when the source is in the F region (fig. 3.14, $\lambda_z = 16$ km case).

The electric field in the source region is always less than the ($\vec{w} \times \vec{B}$) field. However, it is a substantial fraction of ($\vec{w} \times \vec{B}$) field. For example, for $\lambda_z = 25$ km, 16 km, 9 km and 4 km; the field in the source region is 0.45, 0.43, 0.41, 0.33 times ($\vec{w} \times \vec{B}$) field, respectively.