CHAPTER II

INTERACTION OF GRAVITY WAVES WITH THE IONOSPHERIC E REGION PLASMA AND GENERATION OF ELECTRIC FIELDS AND CURRENTS.

2.1 Introduction:

This chapter is devoted to the studies of electron density irregularities in the E region, with an emphasis on the equatorial E region irregularities. Experimental observations on the irregularities are discussed in the light of theoretical explanations put forward and their deficiencies are pointed out. The main emphasis in the chapter is on the three dimensional model, developed to study the effects of gravity wave winds on the E region plasma. The sectionwise breakup of this chapter is as follows.

Experimental and theoretical studies on the temperate zone sporadic E are discussed in section 2.2.

Section 2.3 is on the equatorial sporadic E in which, observations and theoretical explanation to these observations are discussed.

In section 2.4, earlier studies on the effects of the gravity wave winds on the equatorial E region are briefly discussed.

In section 2.5, a three dimensional model of the interaction of the gravity wave winds with the E region plasma is developed, detailing its basic concept and the theory.
Section 2.6 details the results of these studies in which electric field, currents and the plasma convergence rates are calculated. These results are discussed to bring out several new features of these studies.

In section 2.7, the conclusions of the studies carried out in this chapter are given.
The E region has been a subject of active study from the early days of ionospheric research. Ground based ionosonde were used to determine the electron density profiles in the E region. Appleton (1930) using the ground based ionosonde observed the abnormal layers in the electron density around 100 km in the nighttime. Later studies showed the presence of dense layers or patches of ionization in the E region, at heights of about 100-120 km, which did not relate to the normal daytime layers. Because of the irregularity of its behaviour, this phenomenon has been known as sporadic E or Es. Sometimes Es appears as sheets of ionization blanketing the overlying F layer on the ionograms. At other times it may be patchy and partially transparent. Much of statistical data has been accumulated over the past decades on various aspects of the sporadic E phenomena. These studies have helped in establishing that Es is a world wide phenomena. Smith (1957) has classified Es into equatorial, temperate and auroral zone type, although the boundaries are not so clearly defined. It is now believed that the causative mechanisms of the sporadic E, in the three different zones are different. A detailed review of the sporadic E phenomena has been given by Whitehead (1970).

2.2 Temperate Zone Sporadic E

Using the ground based ionosonde and radars the occurrence statistics of the sporadic E, that is, its variation with the time of the day, season and sunspot cycle; and various features
like the height, its variation with the strength of the magnetic field etc; have been studied extensively. The altitude region of their occurrence has been studied using the in-situ techniques (Pfister and Ulwick 1950; Aono et al 1961; Seddon 1962; Aurby et al 1966; Smith 1966; Bowhill 1966; Smith and Mechitly 1972).

The Wind Shear Theory

Several theories have been proposed to explain the formation of sporadic E layers. Dungey (1956, 1959) proposed that the north south shears in the north south wind can compress the ionization to give the appearance of a layer. This mechanism was found to be ineffective in the E region and hence given up.

Whitehead (1961) proposed that a height varying east west wind, that is, the vertical shears in the east west wind, can redistribute the E region ionization and produce the ionization layers.

In the E region, the neutral wind drags the ions along with it because the collision frequency of ions with the neutral \( \nu_{in} \) exceeds the ion gyrofrequency \( \Omega_i \). The ions experience a Lorentz force \( \mathbf{W} \times \mathbf{B} \), where \( \mathbf{W} \) is the neutral wind velocity and \( \mathbf{B} \) is the geomagnetic field, in the vertical direction. Under the combined action of these forces, the ions move in a vertical direction determined by \( \Theta = \tan^{-1}(\Omega_i/\nu_{in}) \). If the wind changes the direction with the altitude, then the ions
are compressed within the shear to give the appearance of a layer. Since the electrons are constrained to move only along \( \mathbf{B} \) in the E region (\( \eta_{\text{E}} \ll \eta_{\text{E}} \)), the neutralization of space charge takes place due to the flow of electrons along the geomagnetic field lines which are inclined in the mid latitudes.

It is this mechanism which is now believed to be responsible for the formation of ionization layers and has been known as the wind shear theory of sporadic E. However, this theory is also not without difficulties. Several objections and advancements have been made in the original wind shear theory of whitehead. Axford (1961, 1963) developed the theory along similar lines considering the effects of diffusion and recombination. He pointed out that the theory should not work at the geomagnetic equator. He also proposed the cork screw mechanism for the downward transport of the plasma. The theory was developed in detail by Chimonas and Axford (1968). Hines (1964) gave the physical picture of the mechanism as discussed above and pointed out that the ionization converges at the point where the wind shear has a maximum negative value. Layzer (1964) objected that the recombination coefficient of the major ions, as demanded by the wind shear theory was an order of magnitude smaller in value than the experimentally verified values. He also pointed out that, in contradiction to the theory, while the peak electron densities were several times the background, the minimum densities did not seem to be less than the background. Axford and Cunnold (1966) suggested
the meteoric ions of long life times to overcome the objection raised by Layzer. Mac Leod (1966) took into account, both the wind shears and the height variation of the ion neutral collision frequency. Kato et al (1966) considered the effect of a superimposed electric field on a gravity wave motion.

It is now believed that the metallic ions are very important to the formation of ionization layers.

2.3 Equatorial Sporadic E:

Early ionosonde studies of the equatorial E region revealed that the equatorial sporadic E occurs during the daytime almost every day (Matsushita 1951; Rawer 1953; Smith 1957; Ratcliff 1962). Rawer (1960) found that around the dip equator sporadic E becomes transparent over a wide range of frequencies. A decrease in occurrence of the blanketing $E_S$ from 6° dip down to the equator was reported by Kretch and Schlitt (1961) and Bandyopadhyay and Montes (1963). It is now known that the equatorial sporadic E consists of blanketing type, called $E_{sb}$, and transparent type, called $E_{sq}$. Matsushita (1951) showed that the intensity of the $E_{sq}$ is well correlated with the electrojet strength.

VHF forward radar studies (Cohen and Bowles, 1963) over transequatorial path showed their generation in the altitude range of 95 to 100 km, a range agreeing well with the electrojet current system (Singer et al, 1951). With an oblique looking radar at Huancayo, Peru, it was found that the E region irregularities are strongly field aligned (Egan 1960), confirming
an earlier prediction. The equatorial electrojet irregularities are associated with the east-west electric field which drives the electrojet. The irregularities are present both during the day and night when the electron density is greatly reduced.

Significant contributions to the equatorial electrojet irregularities have been made by the VHF backscatter radar at Jicamarca, Peru. These studies have helped to determine the spatial distribution and drift velocity of the irregularities besides their various features. The Jicamarca backscatter radar operates at 50 MHz which corresponds to 3 m irregularity scale size. On the basis of the spectral characteristics, the irregularities have been classified as Type I and Type II (Bowles, 1967; Balsley 1967; Balsley 1969). Recently, the equatorial electrojet irregularities have been studied using the backscatter radars at Thumba, India (Prakash et al 1973; Muralikrishna 1975) and at central and east Africa also (Crochet et al 1976; Crochet 1977; Hamiise and Crochet 1977, 1978).

The type I spectra are characterised by a narrow returned power band of frequencies, Doppler shifted from the transmitted frequency by about 130 Hz. This Doppler shift corresponds to the drift velocity of the echoing region, close to the ion acoustic velocity in the medium. The Doppler shift was found to be independent of the antenna elevation angle. The type I spectra are observed only when the electrojet currents exceed a critical value.

Power spectrum of the type II irregularities is broader
than the type I and these irregularities were found to drift horizontally (Balsley 1967). The type II spectra dominates the type I when the electrojet currents are weak and also when the antenna is pointed in the vertical direction. A detailed study of the region of occurrence of the type II irregularities was carried out by Fejer et al (1975) using the range time intensity diagrams. These studies showed that while during the daytime the echoes came from more or less one continuous region; in the evening and nighttime they came from different altitude regions.

2.3.1 Rocket Results:

With the establishment of the rocket range at Thumba, India, near the dip equator, a new dimension to the study of the equatorial electrojet irregularities has been added. A comprehensive study of the ionization irregularities in the D and E regions has been carried out by Prakash et al (1970, 1971, 1977, 1979). These studies are basic to the understanding of various physical processes responsible for the generation of different ionization irregularities. The ground-based studies, using ionosonde and backscatter radar, are inadequate in this respect as these studies by themselves, do not give the localised parameters of the medium which have a direct bearing on the generation processes of various types of irregularities. Thus the ground based and insitu studies are complementary to each other.

The rocketborne studies of the irregularities using Langmuir probe, resonance frequency probe and magnetometers (Prakash et al loc.cit) have given the shape, size, spectrum of the irregularities and their relationship with the ambient parameters
like the electron density and gradients in it, electric fields and streaming velocity of charged particles. These studies have been very helpful in the development of various theories of the electrojet irregularities. However, there are many rocket observations which are not fully explained theoretically (Prakash et al 1979). A classification of the irregularities, based on their scale size and generation mechanisms, has been made by Prakash et al (1973) in which type M irregularities have a scale size range of 30-300 m and type S have the range 1-15 m. The classification is listed below.

(i) Large scale irregularities type L, (scale size>300 m).
(ii) Due to cross field instability mechanism (type Mc and Sc).
(iii) Due to streaming of electrons (type Ss).
(iv) Due to neutral turbulence (type Mn and Sn).
(v) Rocket induced irregularities.

2.3.2 Theories of the Equatorial Electrojet Irregularities:

To explain the type I radar echoes, a theory of two stream plasma instability was proposed (Farley 1963; Buneman 1963) which requires that the relative drift between the ions and electrons should exceed the ion acoustic velocity in the medium. However, the rocket observations (Prakash et al 1971) show that the threshold criterion for the excitation of two stream plasma instability was not satisfied, yet the irregularities were observed.

Several theories have been proposed to explain various features of these irregularities (Rogister 1971; Sato 1972; Wein Stock and Sleeper 1972; Kaw 1972; Lee 1972).
The type Mc (Rocket borne) irregularities are believed to be due to the gradient drift instability (or cross field instability). This instability mechanism was first proposed by Simon (1963) and Hoh (1963) in a different context. The gradient drift instability operates in a plasma, if the gradients in the electron density are in the direction of the field. Rocket observations by Prakash et al (1970) confirm this feature of the irregularities. The linear theories of gradient drift instability, in application to the equatorial E region irregularities, have been developed by many authors (Maeda et al 1963; Knox 1964; Tsuda et al 1965; Reid 1968; Register and D'Angelo 1970). These theories could explain the type Mc irregularities alone. To explain the type Sc and type II irregularities, Sudan et al (1973) developed a two step theory. But this theory suffers from the use of unrealistic parameters for the calculations as has been pointed out by Sinha (1976).

Mc Donald (1974, 1979) performed 2-dimensional computer simulation of the type II irregularities. Non linear effects have been considered by Sudan and Keskinen (1977).

2.3.3 Other features of equatorial E region irregularities:

Rocket experiments for the study of nighttime equatorial E region have revealed the existence of layered structures in the electron density profiles (Sinha 1976). The most illustrative example of such layers of ionization (Prakash et al 1970) is shown in figure 2.1. In some of these layers the electron density varied by over an magnitude. The vertical half width
NIKE APACHE 20.08
AUG. 29, 1968, 2300 HRS IST
THUMBA, INDIA

FIG. 2.1
of these layers was 3–4 km, with a horizontal extent of more than 50 km in the east-west direction. Such layered structures in the electron density profile at the equator were not reported earlier. Attempts to explain the formation of ionization layers in the equatorial E region were initiated by Kato (1973), followed by Anandarao et al (1977), and Prakash and Pandey (1979) using the neutral winds of the gravity wave origin.

The studies with the backscatter radar in the equatorial electrojet region (Reddy and Devasia 1976) have indicated the existence of fluctuating currents with periods similar to those of the gravity waves. The current in a given region may be considered to be comprised of two parts. One is due to the large scale global wind system and the other due to the local winds. The currents due to the large scale wind system may be inferred from the ground based and rocket borne magnetometers. However, due to inherent limitations of the techniques in use, it is difficult to determine experimentally, the currents produced due to the local winds. Therefore, the effectiveness of the local winds in driving current is not known experimentally and can only be estimated theoretically. The knowledge of the local currents is very crucial to the understanding of certain plasma instability processes responsible for the generation of the ionization irregularities. One such mechanism is the theory of two stream plasma instability (Farley, 1963; Bunemann, 1963) which has been considered to be the likely candidate for explaining type-I radar spectra. For this mechanism to be
operative, the relative velocity between ions and electrons should exceed \( v_3(1 + \Psi) \), where \( v_3 \) is the ion acoustic velocity in the medium and \( \Psi = R_i R_e \) (Register and D'Angelo, 1970).

The studies with the rocket borne probes have shown that the average velocity of the electrons relative to the ions was not sufficiently large as to meet the threshold requirement for the generation of two stream instability (Prakash et al., 1971). It is therefore necessary to look for the processes which could give rise to localised streaming of the charges (or currents) in addition to that due to the large scale electrojet currents.

### Gravity Wave-Ionosphere Interaction

The gravity waves play a very important role in the upper atmosphere and these have been used to explain many experimental observations of ionospheric D, E and F regions (Hines, 1960). In recent years, the problem of interaction/neutral winds with the ionospheric plasma has received increased attention and their effects on the equatorial electrojet have been evaluated using various models (Richmond 1973; Forbes & Lindzen 1976; Anandarao 1976; Reddy & Devasia 1977; Prakash & Pandey 1979).

The wind shear theory (Whitehead 1961; Axford 1963) which is usually invoked to explain the formation of ionization layers at the mid-latitudes has previously been considered to be not applicable at the equator because of the special configuration of the magnetic field lines there. It was believed that, at the equator, the shorting of the polarization
field $E$, does not take place and hence the field gets fully
developed. Such a polarization field would cancel out the wind
induced field $W \times B$ (in the frame of reference of the wind).
i.e., the total electric field $E^w = E^i + W \times B = 0$ where $E^w$
is the electric field in the frame of reference of the wind,
$W$ is the wind and $B$ is the geomagnetic field. From the
following equation for the ion velocity (Kato, 1965)

$$V_i = \frac{E^w R_i}{E(1 + R_i^2)} \left\{ R_i E^w + \frac{E^w B}{R_i^2} \right\} + W \quad (1)$$

where $V_i$ is the ion velocity and $R_i$ is the ratio of ion neutral
collision frequency and ion gyro-frequency (Equation (1) is
valid for electrons also when $R_i$ is replaced by $-R_e$), it can
be seen that for a constant plasma density, the ion convergence
rate, when $E^w = 0$ is given by $\nabla \cdot V_i = \nabla \cdot W \quad (2)$
since the winds of gravity wave origin are nearly non divergent
(Hines, 1960) hence

$$\nabla \cdot V_i = 0 \quad (3)$$

therefore, when $E^w = 0$, the winds will not give rise to the
accumulation of ionization. Later on Kato (1973), taking into
account the curvature of the geomagnetic field lines, pointed
out that the vertical polarization field $E_z$ would not be fully
developed even at the equator due to the partial shorting of
the polarization field by the electron currents along the
magnetic field lines. The partial shorting of the wind induced
polarization fields at the equator would mean that the gravity
wave winds would give rise to electric current, the importance
of which has been emphasized earlier. Using the winds of gravity wave origin, he calculated an efficiency factor $R$, given by

$$R = \left| \frac{(\mathbf{E} + \mathbf{w} \times \mathbf{B})_Z}{(\mathbf{w} \times \mathbf{B})_Z} \right|$$

(4)

for the production of the net vertical field in the frame of reference of the wind. He found that $R$ has a non zero value even at the equator. Its value is zero when $\lambda_z = 0$ and reaches an asymptotic value for $\lambda_z \gg 20$ kms. He, therefore suggested that the wind shear theory, in forming the ionization layers, is applicable even at the equator, though with much lesser efficiency as compared to that in the mid-latitudes.

Anandarao et al (1977) developed a two dimensional model of the interaction and calculated the parameter $R$ (as defined above) and the ion convergence rates at 100 km. They concluded that the results of Kato (1973) were an over estimate. Kato (1973) has neglected the horizontal wavelengths, both north-south and east-west, while Anandarao et al (1977) have not considered the north-south wavelength, Kato (1973) also neglected the north-south and vertical velocities. Inclusion of the wavelength along the geomagnetic field line is very important, as was demonstrated by Prakash and Pandey (1979) and is discussed in detail in the following sections.

The gravity wave induced electric fields, at a given place, can have their origin

(a) due to the local gravity wave winds

(b) due to the gravity wave winds present in some other region
of the ionosphere (c) due to a combination of situations (a) and (b).

We treat the situation (a) and (b) separately. In this chapter, effects of the local gravity wave winds, situation (a), alone are discussed. The problem of generation and transmission of electric fields, the situation (b), is discussed in chapter III, separately.

In what follows, the interaction of local winds (gravity wave winds) with the ionospheric plasma is studied using a three dimensional model. The electric field, current, streaming velocity of electrons relative to ions and the plasma convergence rates resulting from this interaction have been calculated using the winds of gravity wave origin. The results of these calculations which are applicable to the gravity waves of medium scale sizes are different, both qualitatively and quantitatively, from those obtained earlier (Kato 1973; Anandaraao et al 1977).

2.5 The Three Dimensional Model:

2.5.1 The field line geometry

The present model has been developed assuming the magnetic field to be homogeneous i.e. the magnetic field lines are straight everywhere. Although the geomagnetic field lines are curved, these can be assumed to be straight if the bending of the field lines (h) over half the wavelength (λx/2) along the magnetic field is much less, say an order of magnitude, than half the vertical wavelength λz/2.
Bending of field lines ($h$) over half the wavelength ($\lambda_z/2$) along the magnetic field. Vertical wavelengths for which the bending of the field lines can be ignored is also listed.

<table>
<thead>
<tr>
<th>Sr.No.</th>
<th>$\lambda_z/2$ (km)</th>
<th>$h$ (km)</th>
<th>$\lambda_z/2$ (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>0.1</td>
<td>&gt;1.0</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>0.4</td>
<td>&gt;4.0</td>
</tr>
<tr>
<td>3</td>
<td>200</td>
<td>1.6</td>
<td>&gt;16.0</td>
</tr>
</tbody>
</table>
The bending of the geomagnetic field lines was calculated using the formula given by Alfvén and Falthammer (1963) and its value is listed in Table-I for various values of $\lambda x/2$. Correspondingly, the lower limit of $\lambda z/2$ for which the bending of the field lines can be ignored is also listed. Noting the wavelength ranges which can exist in the ionosphere (e.g. Beer, 1974), the restrictions put on the values of $\lambda z/2$ for various values of horizontal wavelengths are not at all serious.

According to the earlier theories, the homogeneous magnetic field will be most favourable for the full development of the polarization field, thereby making $R = 0$ at the equator. However, the following discussion shows that in the case of a 3-dimensional model, this is not true.

We choose a cartesian system of coordinates in which $x$-axis is along the magnetic field, $y$-axis is horizontal and is toward magnetic west and $z$-axis is perpendicular to both $x$ and $y$ axes. It may be pointed out that the $x$-axis will not be horizontal except at the equator. The gravity wave winds are generally defined in the cartesian coordinate system where two axes are horizontal and the third one is vertical. Let such a reference system be denoted by $x', y'$ and $z'$. Let $y'$-axis be same as the $y$-axis, $z'$-axis is in the vertical direction and $x'$ towards magnetic north. The gravity wave wind, in this coordinate system can be written as
where $\mathbf{W}_0$ is the wind vector having components $W_0^x, W_0^y, W_0^z$ and $k_x, k_y, k_z$ are the components of the wave vector in the respective directions.

2.5.2. Transformation of the gravity wave winds:

The wind given by eqn. (5) can be easily transformed into the $x$, $y$, $z$ coordinate system by rotating the $x'$, $z'$ axes in the plane perpendicular to $y$-axis and is given by

$$\mathbf{W} = \mathbf{W}_0 \exp \left\{ i ( \Omega t - k_x x' - k_y y' - k_z z') \right\}$$  \hspace{1cm} (6)

where $\mathbf{W}_0$ is the wind vector having components $W_0^x, W_0^y, W_0^z$ and $k_x, k_y, k_z$ are the components of the wave vector in the respective directions.

$I$ is the dip angle, positive in the northern hemisphere. In the ionosphere, the ratio of the collision frequency to the gyro-frequency for electrons is much smaller than that for the ions. Therefore, the neutral wind in the ionosphere gives rise to charge separation and the re-distribution of the charges. If the polarization fields thus produced are assumed to be electrostatic then

$$\mathbf{E} = - \nabla \phi$$  \hspace{1cm} (7)

As the plasma parameters and the magnetic field have been assumed to be uniform, the potential $\phi$ can be assumed to be
of the same form as the wind and can be written as
\[ \phi = (\Phi_0 + i \Phi_2) e^{i\left(\omega t - k_x x - k_y y - k_z z\right)} \]  
where \( \Phi_0 \) and \( \Phi_2 \) are real constants. From the eqns. (7) and (8) it can be seen that electric field will be present in all the three directions. The presence of an electric field along the magnetic field may look surprising. However, its presence can be visualised from the following discussion.

The gravity wave winds given by eqn (6) can be resolved in two directions, one parallel to \( B \), and the other perpendicular to it. This can be achieved by rotating the \( y \) and \( z \) axes in the plane perpendicular to \( x \) by an angle \( \phi \) such that \( \phi = \tan^{-1}\left(\frac{w_{0z}}{w_{0y}}\right) \). If \( x, y, z \) is the new set of axes, then the transformed components of the wind will be given by
\[ W_x = W_{0x} e^{i\left(\omega t - k_x x - k_y y - k_z z\right)} \]
\[ W_y = W_{0y} e^{i\left(\omega t - k_x x - k_y y - k_z z\right)} \]
\[ W_z = 0 \]  
Thus the wind velocity in the plane perpendicular to \( B \) is only along \( z \) axis, however its amplitude changes sinusoidally as one moves along any of the three axes.

2.5.3 The ion motion:

Let us consider the effect of a neutral wind on the motion of ions and electrons when the polarization fields are small. It can be seen from eqn. (1) that the wind along \( B \) does not result in any charge separation because both ions and electrons...
Fig. 2.2: Variation of \((R_i, e, \left[\frac{R_i}{e} / (1 + R_i^2_{e})\right], \left[\frac{R_i^2}{e} / (1 + R_i^2_{e})\right]\) with altitude. Values of \(R_i\) and \(R_e\) have been taken from Richmond (1972). The scale on top of the figure is for curves drawn with dashed lines. The lower scale is for the curves drawn with solid lines.
move with the same velocity in that direction.

As can be seen from fig. 2.2 that Re ≪ Ri. Hence from eqn. (1), the motion of electrons in the plane perpendicular to \( B \), is negligible compared to that of the ions. Also, due to the wind \( W_z \) the ions move with velocity \( W_z \frac{R_i^2}{(1+R_i^2)} \) and \( -W_z \frac{R_i}{(1+R_i^2)} \) along \( \xi \) and \( \eta \) directions, respectively. The resultant direction of the ions will therefore make an angle \( \Theta \) with the \( \xi \) axis where \( \Theta = \tan^{-1} \left(-1/R_i\right) \). Therefore, if the axes \( \xi \) and \( \eta \) are further rotated by an angle \( \Theta \) around the x-axis, then the ion velocity components in the new set of axes \( x', \xi', \eta' \) can be written as

\[
\begin{align*}
V_x &= V_{ox} e^{i\omega t} \left\{ \epsilon (\alpha t - k_x x - k_{\xi'} \xi') - k_{\eta'} \eta' \right\} \\
V_{\xi'} &= (V_0 \xi' \cos \Theta + V_0 \eta' \sin \Theta) e^{i\omega t} \left\{ \epsilon (\alpha t - k_x x - k_{\xi'} \xi' - k_{\eta'} \eta') \right\} \\
V_{\eta'} &= 0
\end{align*}
\]

Therefore, in a plane perpendicular to \( B \), the ions move only in the \( \xi' \) direction. It can be seen that although the motion of ions is only along \( \xi' \), its amplitude varies sinusoidally as one moves along any of the three axes \( x', \xi', \eta' \). As is evident the ion convergence takes place at the nodal points of these sinusoidal variations. In fig. 2.3(a) the horizontal and vertical lines are drawn every \( \lambda_{\xi'}/2 \) and \( \lambda x/2 \) distance and the direction of motion of the ions is shown by arrows in the \( x \xi' \) plane. A similar picture is valid for the \( \xi' \eta' \) plane, though not shown in the diagram. It can be clearly seen that the regions with excess and deficiency of positive charges
Fig. 2.3: Schematic of ion motion in the $\zeta \xi'$ plane is shown in Fig. 2.3a. The dividing lines are at half the wavelength in the respective directions. The ion motion in two $\zeta \xi'$ planes separated by $\lambda \eta'/2$ is shown in Fig. 2.3b. The arrows show the direction of motion of ions.
occur every half wavelength as one moves along the magnetic field axis \( x \) and the axis \( \xi' \). The ion motion in two \( x-\xi' \) planes separated by \( \lambda \eta'/2 \) is also shown in fig.2.3b. Here also, the excess and deficiency of charges occurs every half the wavelength along the respective directions. This excess and deficiency of charges gives rise to an electric field not only along \( \xi' \) and \( \eta' \) but also along \( x \) which is the magnetic field direction. The electric field along \( B \) will result in an electron current along the magnetic field lines due to very large parallel conductivity for electrons. This current will thereby reduce the polarization fields. The electric fields in the perpendicular plane will give rise to a small electron and ion current in that plane. These currents will modify the direction of the conventional current in the plane perpendicular to \( B \) from direction \( \xi' \) to some other direction, say \( \xi_1' \). The argument as given above still remains valid if a proper set of orthogonal axes is chosen.

The current density \( J \), in terms of the total electric field \( \mathbf{E} \) is given by

\[
J = \sigma_0 \mathbf{E}_\parallel + \sigma_P \mathbf{E}_\perp + \sigma_H (\mathbf{E} \times \mathbf{B}) \tag{14}
\]

where \( \sigma_0 \), \( \sigma_P \) and \( \sigma_H \) are the parallel, Pedersen and Hall conductivities respectively. \( \parallel \) and \( \perp \) denote the components parallel and perpendicular to the magnetic field. Also

\[
\nabla \cdot J = 0 \tag{12}
\]
using equations (6, 7, 8, 11 and 12) we get

\[ \Phi_{02} = 0 \quad (13) \]

\[ \Phi_{02} = B \left\{ \frac{\sigma_p(k_y w_{0z} - k_z w_{0y}) + \sigma_h(k_x w_{0y} + k_z w_{0z})}{\sigma_0 k_x^2 + \sigma_p(k_y^2 + k_z^2)} \right\} \quad (14) \]

using the condition \( \nabla \cdot \mathbf{W} = C (Hines, 1960) \), \( \Phi_{02} \) can be rewritten as

\[ \Phi_{02} = - \frac{B w_{0y}}{k_z} \left\{ \frac{\sigma_p(k_y^2 + k_z^2 + k_x k_z \gamma) + \sigma_h k_x k_z \gamma}{\sigma_0 k_x^2 + \sigma_p(k_y^2 + k_z^2)} \right\} \quad (15) \]

where \( \gamma = w_{ox}/w_{0y} \)

In deriving the above expressions we have ignored the gradients in the electrical conductivities.

The electric field components can then be obtained using equation (7) which reduces to

\[ \begin{align*}
E_x &= -k_x \Phi_{02} \exp \left\{ i (-\omega t - k_x x - k_y y - k_z z) \right\} \\
E_y &= -k_y \Phi_{02} \exp \left\{ i (-\omega t - k_x x - k_y y - k_z z) \right\} \\
E_z &= -k_z \Phi_{02} \exp \left\{ i (-\omega t - k_x x - k_y y - k_z z) \right\} \quad (16)
\end{align*} \]

From eqn. (15) and (16) we get

\[ E_{0z} = B w_{0y} \left\{ \frac{\sigma_p(k_y^2 + k_z^2 + k_x k_y \gamma) + \sigma_h k_x k_z \gamma}{\sigma_0 k_x^2 + \sigma_p(k_y^2 + k_z^2)} \right\} \quad (17) \]

As Kato (1973) has retained \( E_z \) only, the factor \( R \) in his case can be used to determine the extent of shorting. In our model calculations, the factor \( R \) does not
have a unique value as the field is present in all the three directions and the factor R does not convey the same meaning as is conveyed in his case. Nevertheless, for comparison we have calculated the ratio of the Z-component of the field and it is given by

$$R_Z = \frac{E_Z}{(\omega \times B)_Z} = k_x \left( \frac{\sigma_0 k_x - \sigma_P k_y y - \sigma_H k_z z}{\sigma_0 k_x^2 + \sigma_P (k_y^2 + k_z^2)} \right)$$

Similarly,

$$R_Y = \frac{E_Y}{(\omega \times B)_Y} = -\frac{k_x}{k_z w_0} \left[ \frac{k_y w_0 y (\sigma_0 k_x - \sigma_P k_y y - \sigma_H k_z z)}{\sigma_0 k_x^2 + \sigma_P (k_y^2 + k_z^2) + w_0 k_x} \right]$$

It can be seen from eqn. (18) and (19), the factors $R_Y \neq R_Z$.

When $k_x = 0$, $R_z = 0$ and $R_y = 0$. Thus when $k_x = 0$, the polarization fields, $E_y$ and $E_z$, are fully developed. The electric currents due to the gravity wave winds can be obtained using eqn. (11) and are given by

$$\mathbf{J} = \sigma_0 \mathbf{E}$$

$$J_x = \sigma_0 (E_x + B w_x) - \sigma_H (E_z - B w_y)$$

$$J_y = \sigma_0 (E_y + B w_y) - \sigma_H (E_y + B w_z)$$

### Results and Discussion

The electric fields, currents and the drift velocity of electrons relative to ions were calculated at the equator, at each altitude separately, by assuming gravity waves with the given parameters. The envelopes of these parameters are shown
in fig. 2.4 through 2.6. In these figures, the solid and dotted portions of the curves have been used to represent the phase relationship of various parameters with the wind. These parameters are drawn with solid curves when they are in phase with the wind and with the dotted curves when they are 180° out of phase with the wind. 'r' denotes the region where a reversal of phase has taken place.

In figs. 2.7 and 2.8, the results of the present studies are compared with those of Kato (1973) and Anandarao et al (1977). The variation of ion convergence rates per ion with altitude at varies latitudes is given in fig. 2.9.

2.6.1 The electric fields.

The electric fields can be calculated using eqns. (15) and (16). Fig. 2.4 gives the envelope of the electric field components. For these calculations, we have assumed \( \lambda_x = \lambda_y = 100 \text{ kms}, \lambda_z = 5 \text{ kms} \) and \( W_{ox} = 100 \text{ m/sec} \). Using \( \nabla \cdot \mathbf{W} = \mathbf{C} \), one can calculate \( W_{ox} \) and \( W_{oz} \) which in this case would be 100 m/sec and 10 m/sec respectively. Since we have chosen \( k_x = k_y \), the corresponding fields \( E_{ox} \) and \( E_{oy} \) are also equal. The field \( E_{oz} \) is greater than the field \( E_{ox} \) or \( E_{oy} \) by a constant factor \( k_z/k_y \).

It can be seen from fig. 2.4 that the electric field components do not maintain a constant phase relationship with the wind. Above 90 km the polarization fields \( E_{oy} \) and \( E_{oz} \) are seen to be very poorly developed in comparison to the wind.
Fig. 2.4: Envelopes of electric field components with the altitude at the equator. The solid and dotted portions of the curves have been used to represent whether the field is in phase or $180^\circ$ out of phase with the wind. $r$ denotes the region where such a reversal of phase occurs. The wave parameters are $\lambda_x = \lambda_y = 100$ km, $\lambda_z = 5$ km and $W_{0y} = 100$ m/sec.
induced fields \((\mathbf{W} \times \mathbf{B})_y\) and \((\mathbf{W} \times \mathbf{B})_z\) respectively, which are approximately \(3 \times 10^{-4}\) \(v/m\) and \(3 \times 10^{-3}\) \(v/m\) in the \(y\) and \(z\) directions respectively.

2.6.2 The electric currents

The electric fields obtained above were used to calculate the currents due to the gravity wave winds. Fig. 2.5 gives the envelope of the amplitudes of the currents with altitude. Corresponding to these currents, the streaming velocities of electrons relative to ions are shown in fig. 2.6.

Since the electric fields and winds have an exponential term, it can be seen that the gravity wave driven currents reverse their direction of flow at every half the wavelength as one moves in any of the three directions. From fig. 2.5 it can be seen that \(J_{oy}\) remains in phase with the wind and has a broad maximum centred around 106 km with a peak value of approximately 3 Amp/km.2.

The vertical profile of the electrojet current \((J_E)\) obtained in American zone (Maynard, 1967; Devis et al 1967) and in Indian zone (Maynard et al 1965; Sastry 1970) peaks around this altitude only. The peak value, was found to be 10-12 Amp/km² (Sampath, 1976). Therefore, the contribution of gravity wave driven current (for \(W_{oy} = 100\) m/sec) is substantial even near the peak of electrojet current. As \(J_E\) decreases more rapidly with altitude than \(J_{oy}\), the ratio \(J_{oy}/J_E\) will increase with the altitude. The current \(J_{oy}\) when superimposed upon the current \(J_E\), would give rise to regions of enhanced and reduced
Fig. 2.5: Envelopes of current components with altitude corresponding to the fields of Fig. 2.4. Solid and dotted portions of the curves have the same significance as in Fig. 2.4.
currents. In the regions where the current is enhanced, the streaming velocity of electrons relative to ions would be larger than what is calculated using average electrojet currents.

The variation of the current components as given in fig. 2.5 can be qualitatively explained when the polarization fields \( E_y \) and \( E_z \) are negligible in comparison to the wind induced fields \( (\mathbf{W} \times \mathbf{B})_y \) and \( (\mathbf{W} \times \mathbf{B})_z \), respectively. From fig. 2.4 it can be seen that above 96 km, the polarization fields are smaller by over an order of magnitude than the wind induced fields. Therefore, above 96 km the equation (20), can be approximated to

\[
\begin{align*}
J_x &= \sigma_H B W_y + \sigma_P B W_y (W_z/k_x) \\
J_y &= \sigma_H B W_y (1 + \sigma_P W_z) \\
J_z &= \sigma_H B W_y (W_z/W_y - \sigma_P/\sigma_H)
\end{align*}
\]

In the present case, both \( W_{0y} \) and \( W_{0z} \) are positive and \( W_{0z} = 0.1 W_{0y} \); hence the current \( J_{0y} \) will remain positive in the region of interest. Since \( \sigma_H \) is fairly constant and \( \sigma_P < \sigma_H \) between 105-125 km, \( J_{0y} \) has a broad maximum centred around 106 km.

The current \( J_{0z} \) will change its direction when \( W_{0z}/W_{0y} < \sigma_P/\sigma_H \). This condition is satisfied above 110 km. Above 126 km the ratio \( \sigma_P/\sigma_H > 1 \). Therefore, the current \( |J_{0z}| \) exceeds \( |J_{0y}| \).

The ratio of the two current components is given by

\[
|J_{0z}/J_{0y}| = \left(\frac{W_{0z}/W_{0y} - \sigma_P/\sigma_H}{1 + \sigma_P W_{0z}/\sigma_H W_{0y}}\right)
\]
As $\sigma_p/\sigma_H$ increases with altitude, the limiting value will be $\omega_{0y}/\omega_{0z}$ (10, in our case) which is achieved much above 190 kms. At an altitude of 190 kms $|J_{oz}/J_{oy}| = 7.5$.

The most striking feature of fig. 2.5 is that above 110 kms the current $|J_{ox}|$ becomes large compared to $J_{oy}$ and $J_{oz}$. This can be explained from eqn. (12) which can be written as

$$J_{ox} = -k_y (J_{oy} + k_z J_{oz}/k_y) / k_x$$

In our case $k_z/k_y = -20$ and $k_x = k_y$. The contribution of $J_{oz}$ to $J_{ox}$ becomes larger than that of $J_{oy}$ above 115 km where $|J_{oz}| > |J_{oy}|$. Above this altitude the contribution of $J_{oy}$ is negligible and hence $|J_{ox}| < 20 |J_{oz}|$

2.6.3 The streaming velocities:

Fig. 2.6 gives the streaming velocity of electrons relative to ions calculated using currents given in fig. 2.5. It can be seen that the maximum parallel streaming velocity of 900 m/sec occurs around 130 km. This velocity is sufficiently large to excite the ion cyclotron instability. This instability mechanism has been invoked (D'Angelo, 1973) to explain the type III radar echoes as observed at the Auroral latitudes (Balsley and Ecklund, 1972). It was, however, pointed out (D'Angelo, 1973) that these echoes will not be observed at the equator because such large relative velocity between electrons and ions are not available. The present calculations show that this need not always be true.
Fig. 2.6: Envelopes of streaming velocities of electrons relative to ions with altitude. The scale on the top of the figure is for $V_{ox}$ only. The lower scale is for $V_{oy}$ and $V_{oz}$. Solid and dotted portions of the curves have the same significance as in Fig. 2.5.
As has been pointed out earlier, the threshold requirement for the excitation of the two stream plasma instability is that 
\[ \left(\frac{V_d}{1 + \Psi}\right) > \sqrt{\Psi} \]
where \( V_d \) is the drift velocity of electrons relative to ions. In the electrojet region \( \frac{V_d}{1 + \Psi} \) is maximum at an altitude where \( \Psi = \frac{1}{3} \), while at the peak of the electrojet \( \Psi = 1 \) (Fejer et al. 1975). In the E-region \( \sqrt{\Psi} \approx 360 \text{ m/sec} \) and hence, to satisfy the threshold criterion, \( V_d \) should be greater than 480 m/sec at an altitude few km above the peak of the electrojet. At the peak of the electrojet \( V_d \) should be even larger than 480 m/sec.

Rocket borne proton procession magnetometers, Langmuir probes and resonance probes were used to determine the average streaming velocity of electrons relative to the ions (Prakash et al., 1971 and Nike-Apache flight 10.44 and 10.45 from Thumba). A maximum velocity of 340 m/sec was obtained on flight 10.44 and 380 m/sec on flight 10.45, showing that the drift velocity of electrons relative to ions never exceeded the threshold value for the excitation of the two stream instability.

It can be seen from fig.2.6 that, for the gravity waves with the assumed parameters, the additional streaming velocity of electrons relative to ions in the E-W direction is quite large, its maximum value being around 100 m/sec at 106 km. If these velocities are added to those corresponding to the electrojet currents, it is quite possible that the threshold criterion may be satisfied, at least, in some of the regions.
Fig 2.7 Variation of $R$ (or $R_z$) with $\lambda_z$ (km).

The "Present Results" are for the cases when $\lambda_x = \lambda_y = 50$ km and $200$ km respectively.
In such regions the plasma would become unstable and the ionization irregularities would be generated.

2.6.4 The parameter $R$ 

The values of $R_z$ were calculated at the equator for parameters corresponding to 100 km altitude using various values of $\lambda_z$ and keeping $\lambda_x = \lambda_y = 50$ km. The results are given in fig.2.7 by a solid curve. The results of Kato (1973) and Anandarao et al (1977) have also been reproduced for comparison purposes. It can be seen from the fig.2.7 that the value of $R_z$, in our case is much larger than those of Kato (1972) and Anandarao et al (1977). The values of $R_z$ for various values of $\lambda_z$ keeping $\lambda_x = \lambda_y = 200$ km were also calculated for 100 km altitude and are shown in fig.2.7 with dotted curve. For the case when $\lambda_x = \lambda_y = 50$ km the value of $R_z$ in our case remains within $1.0 \pm 0.1$ for $\lambda_z \geq 0.75$ km. The value of $R_z$ lies in the same range for $\lambda_z \geq 3.0$ kms when $\lambda_x = \lambda_y = 200$ kms.

2.6.5 Ion convergence rate:

Assuming electron density to be constant with altitude the ion convergence rate per ion can be obtained using eqns.(1) and (7). Ignoring the variation in the ion gyro-frequency and ion neutral collision frequency with altitude, the convergence rate is given by

$$\nabla \cdot \mathbf{V}_i = \frac{\phi_z}{B(1 + \frac{R_i^z}{k_x^2})} \left( R_i \left( \frac{k_x^2}{k_z^2} + \frac{k_z^2}{R_i^x} \right) + \frac{R_i k_z w_y - k_x w_x}{(1 + \frac{R_i^x}{k_x^2})} \right)$$

(24)
Fig. 2.8. Variation of ion convergence rate with $\lambda_z$ at 100 km altitude at the equator. The value of $R_i$ has been taken from Fig. 2.2 and is approximately equal to 50.
For the sake of comparing our results with those of Kato (1973) and Anandarao et al (1977), we have calculated the convergence rates at 100 km at the equator, for various values of $\lambda_Z$ keeping $\lambda_x = \lambda_y = 50$ kms. The results of these calculations are given in fig. 2.8. While Kato (1973) has neglected the horizontal wavenumber in comparison to the vertical, Anandarao et al (1977) have assumed the horizontal wavelength to be wholly in the E-W direction. It should be noted here that Kato (1973) and Anandarao et al (1977) have used $R_i \sim 10$ for their calculations. We have taken $R_i \sim 50$ (following Richmond, 1972) and have re-calculated the convergence rates using appropriate equations for their cases also.

It can be seen from fig. 2.8 that the convergence rates obtained with our model are much larger compared to those of Kato (1973) and Anandarao et al (1977). For $\lambda_Z = 1$ km, the convergence rate in our case is approximately 5 times larger than that of Kato (1973) and 60 times than that of Anandarao et al (1977). With increasing $\lambda_Z$ this ratio decreases. The present calculations show that even the gravity waves with small vertical wavelengths can give rise to appreciable convergence rates.

Fig. 2.9 shows the altitude variation of the ion convergence rates at various dip angles, calculated using eqn. (24). The gravity wave parameters used for this purpose were $\lambda_x = \lambda_y = 100$ km, $\lambda_Z = 5$ kms and $W_{0y} = 100$ m/sec. It can be seen from these curves that the ion convergence rate below 120 km is maximum at the equator and minimum at the poles.
Fig. 2.9: Variation of ion convergence rate with altitude at various latitudes. The gravity wave parameters used for these calculations are $\lambda_x' = \lambda_y' = 100$ km, $\lambda_z' = 5$ km and $W_{oy} = 100$ m/sec.
This is contrary to the results of the earlier workers (i.e. Axford 1963) which predicted zero or smaller convergence rates at the equator compared to that in the mid-latitudes. Above 130 km however, there is a marked difference in the convergence rates. The convergence rates increases from the equator to 40° dip latitude and thereafter it decreases and becomes minimum at 80° dip latitude.

Hence when a three dimensional model of gravity wave winds is used, the ion convergence rates below 130 kms at the equator are quite large even when the geomagnetic field lines are assumed not to be curved as is essential for the models of Kato (1973) and Anandarao et al (1977).

The formation of the ionization layers in the E-region would be possible if and only if the ion convergence rate is much larger than the rate of recombination. While the ion convergence depends upon the gravity wave parameters, the geometry of the geomagnetic field lines and \( R_i \), the recombination rate depends on the recombination constant \( \xi_R \) and the electron density, \( n_e \).

The normally assumed value of \( \xi_R \) in the E region is \( 3 \times 10^{-7} \text{ cm}^{-3} \text{ sec}^{-1} \). In the 100 km region, \( n_e \approx 10^5 \text{ cm}^{-3} \) during day time and \( 5 \times 10^3 \text{ cm}^{-3} \) during nighttime. It can therefore be seen that during the nighttime the recombination rate \( \approx 1.5 \times 10^{-3} \text{ sec}^{-1} \). This is less than the calculated value of the convergence rate which is \( 6 \times 10^{-3} \text{ sec}^{-1} \). (It should be again pointed out here that Kato (1973) and...
Anandarao et al (1977) have used $R_i \approx 10$ at 100 km whereas we have used $R_i \approx 50$; hence their values of the convergence rates should be divided by a factor of 5 for comparison with our results. Hence during nighttime the formation of the ionization layers through gravity wave winds is feasible. During daytime, the recombination rate is $\approx 3 \times 10^{-2}$ sec$^{-1}$ which is larger than the calculated convergence rate and hence, during the day time, the formation of the ionization layer is not feasible with the normally assumed value of $\lambda_R$. It should be noted here that, to explain the formation of the ionization layers during daytime in the midlatitudes, the presence of metallic ions having a much smaller recombination constant than what is assumed here has been resorted to.

We will now discuss the convergence rates in the following two limiting cases:

1. When $k_x = 0$, i.e., the wavelength along B is infinitely large. As $\nabla \cdot \mathbf{W} = 0$, we get from eqn. (15)

$$\Phi_{O_2} = -B W_0 y / k_z = B W_{O_2} / k_y$$

Substituting these values in eqn. (24) we get $\nabla \cdot \mathbf{V} \geq C$. Therefore when wavelength along B is very large the convergence rate is very small or zero.

2. In the lower E region $R_i >> 1$. As $k_z \gg k_x, k_y$ and $W_{O_2} < \delta_{O_2} W_{O_2}$, eqn. (24) can be greatly simplified and we get

$$\nabla \cdot \mathbf{V} = |k_z (B W_0 y - E_{O_2}) / B R_i|$$

The field $E_{O_2}$ when calculated for various combinations of wave parameters was found to be much smaller than the wind.
induced field \((W \times B)_z\). Above 87 km for the wave parameters assumed here it was found to be smaller than the wind induced field, at least by a factor of eight. Therefore, in such cases the polarization field can be neglected in comparison to the wind induced field and we get

\[ \nabla \cdot \mathbf{V} = \left| k_z w_0 y / R_i \right| \]

Using the transformations given by eqn. (16) required to go from the coordinate system \(x', y', z'\) to \(x, y, z\); the eqn. (27) becomes

\[ \nabla \cdot \mathbf{V}' = \left| \left( k_z' c + k_x' s \right) w_0 y / R_i \right| \]

At equator this expression is similar to the one obtained in the wind shear theory and we recover the result that in the lower E region, the ion convergence is due to the vertical shears in the E-W wind. However, the ion convergence in our case is maximum near the equator as pointed out earlier also.

2.7 Conclusions:

A three dimensional model, for the interaction of small and medium scale size gravity waves with the ionospheric E region, was developed. In the model calculations, the east west and north south winds were assumed to be equal, with their velocity amplitude equal to 100 m/sec. The highlights of the present studies are as follows.

(i) For the medium and small scale size gravity waves with the wavelength \(\lambda \approx \kappa\), along the geomagnetic field lines less than about 200 km, the curvature of the geomagnetic field lines can be ignored and hence, the geomagnetic field can be assumed to be
homogenous.

(ii) The wavelength, $\lambda_{\chi}$ of the gravity waves, parallel to the geomagnetic field, is very important to the generation of net electric field $E^\omega$ in the wind frame of reference. The polarization fields $E$, above 90 km are substantially shortened, even at the equator, if $\lambda_{\chi}$ is finite, implying $E^\omega$ to be large.

(iii) The efficiency factor $R$, for the generation of net electric field, was found to be much larger than that obtained earlier.

(iv) The gravity wave driven currents in the east west direction were found to be a significant fraction of the main electrojet currents, even at the peak of the electrojet. These currents, together with the electrojet current, may satisfy the threshold requirement for the excitation of the two stream plasma instability in localised regions.

(v) The gravity waves give rise to strong field aligned currents which were found to peak around 130 km altitude. The streaming velocity of the electrons was found to be sufficiently large as to excite the ion cyclotron instability in the 130 km altitude region.

(vi) The gravity waves can converge the ionization at the equator also, giving rise to the ionization layers in the nighttime. The ion convergence rates below 120 km at the equator, were found to be larger than those in the mid latitudes. Therefore, the gravity wave winds would be, at least, as effective at the equator as they are believed to be in the
mid latitudes in forming the ionization layers.

(vii) The ion convergence in the lower E region was found to be due to the vertical shears in the east west wind. The ion convergence rates at 100 km at the equator due to the gravity wave winds were found to be much larger than those given earlier.