In Chapters 4, 5 and 6 the zonal wave number of the perturbation fields were taken to be multiples of zonal wavenumber of the basic Rossby wave by assuming \( \lambda_R = 0 \) in (4.13). In this chapter we intend to take a more general perturbation zonal wavenumber by allowing nonzero values of \( \lambda_R \). The stability analysis using a two-level primitive equation model in Chapter 6 does not give any additional information regarding the Rossby modes.
The characteristics of growing Rossby modes obtained from the quasi-geostrophic model in Chapter 4 are not changed considerably when we extend the study to primitive equation model. Hence in this chapter we will consider a two-level quasi-geostrophic model with a beta-plane centred at 18°N. The basic zonal flow and the stationary Rossby wave are the same as used in Chapter 4. The stationary Rossby wave has a wavelength of 30° longitude and its amplitude is predominantly large in the lower troposphere.

7.1 **Conditions for resonant interaction:**

Before conducting the stability analysis we will mention about the conditions for resonant interaction. It is known that the basic wave and perturbation can be regarded as weakly interacting waves and perturbation can grow when the condition for resonant interaction is satisfied or nearly satisfied. Essentially two waves combine to force a third wave to form a 'triad' for resonance. Longuet-Higgins and Gill (1967) have shown that all wave vectors can participate in a resonant triad with a family of wave vectors. In this case there is nonlinear coupling and strong energy transfer between the
waves. However, two waves of either the same wavelength or parallel wave vectors will not resonate as in that case the interaction co-efficient vanishes. In order for resonance to occur the triad must satisfy the condition

$$\Theta_j + \Theta_m + \Theta_n = 0 \quad (7.1)$$

where the $m^{th}$ and $n^{th}$ waves combine to produce a phase, $(\Theta_m + \Theta_n)$ which is equal to the phase angle of a third free mode $\Theta_j$. If

$$\Theta_j = k_j x + l_j y + \lambda_j t,$$

then the resonance conditions can be given by

$$k_j + k_m + k_n = 0$$
$$l_j + l_m + l_n = 0 \quad (7.2)$$

$$\lambda_j(k_j, l_j) + \lambda_m(k_m, l_m) + \lambda_n(k_n, l_n) = 0.$$

We are considering the basic Rossby wave to be directed eastward i.e. $k_j \equiv k_o$ and $l_j = 0$. In Chapters 4, 5 and 6 we had taken $k (\equiv k_m) = 0$. This implies that the basic Rossby wave and the perturbation wave vectors are parallel and hence there should not be any instability. However, for small $\lambda_j$, this perturbation can be as close as desired to satisfying a resonance condition (Gill, 1974).

Under present circumstances it can be shown (Pedlosky, 1979) that the two wavevectors $k_m$ and $k_n$ lie on the locus at opposite ends of the line passing
through \( k = -k_0/2 \) and intersecting the locus. Hence we assume the zonal wavenumber of a perturbation to be equal to \( n k_0 + k \) i.e. multiples of the basic wavenumber \( k_0 \) plus a constant wave number ' \( k \) ', which is constrained to satisfy a resonant condition is taken to be equal to \( -k_0/2 \).

7.2 Stability analysis:

The linearised potential vorticity equations at levels 1 and 3 are given by (4.9) and (4.10). The basic flow for the stability analysis is the same as in Chapter 4 given by (4.3). We assume solutions (4.13) for the perturbation geopotential fields. In previous three chapters we had limited the stability analysis to the case \( k = 0 \). But here we put \( k = -k_0/2 \). Using (4.11), (4.12) and (4.13) equations (4.9) and (4.10) can be simplified. Finally collecting the co-efficients of 
\[
\exp\left[\frac{i}{2}(nk_0x+kx+ly+\lambda t)\right]
\]
we get algebraic equations identical to (4.14) and (4.15), but here the parameters \( a_n \) and \( b_n \) are redefined as follows.

\[
a_n \equiv nk_0+k \quad \text{and} \quad b_n \equiv (nk_0+k)^2+l^2
\]

Truncating the series (4.13), at \( n = N \) and rearranging the
the terms we get eigenvalue equation (4.16) as in Chapter 4. Here also we truncate the series (4.13) at \( N = 10 \).

For different values of \( \overline{U}_3 \) the corresponding values of \( \overline{U}_1 \) are calculated from (4.12). For each set of values of \( \overline{U}_3 \) and \( \overline{U}_1 \), doubling times, frequencies and amplitudes of disturbances are obtained by varying the meridional wavenumber of perturbation. As explained in Chapter 4, we write \( l = \frac{\pi}{12} k \) and vary \( J \) from 1 to 12 by steps of one to satisfy cyclic boundary conditions both in \( \chi \) and \( \psi \) directions. The characteristics of growing modes are reported in Tables 21 to 24. It is seen that the frequencies of growing modes are smaller than the coriolis parameter and hence they are Rossby modes. For each value of \( \overline{U}_3 \) there is a meridional scale length of the disturbance for which the growth rate is maximum (Fig. 34). In agreement with Gill (1974), and Duffy (1975) the most unstable mode has a growth rate less than that of the case \( k = 0 \). This is evident by comparing Figs. 17 and 34. Also, disturbances do not grow unless \( \overline{U}_3 = 15 \text{ m s}^{-1} \), as compared to \( \overline{U}_3 = 9 \text{ m s}^{-1} \) when \( k = 0 \).

The growing modes are almost stationary (Tables 21 to 24). But for each growing mode there are two frequencies.

The eigenfunctions corresponding to different
Figure 34: Meridional wavenumber \( l = J k_0 / 12 \) dependence of the growth of Rossby modes in quasi-geostrophic model when \( k = -k_0 / 2 \).
Table 21

Characteristics of growing modes in quasi-geostrophic model for $k = -k_0/2$ and $\Omega_0 = 15\,\text{m}\,\text{s}^{-1}$ (Meridional scale length of perturbation is $\frac{12}{J}$ times wavelength of the stationary Rossby wave).

<table>
<thead>
<tr>
<th>$J = \frac{12L}{k_0}$</th>
<th>Frequency $\lambda_\gamma,(\text{s}^{-1})$</th>
<th>Doubling time $\tau_d,(\text{days})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-1.4 \times 10^{-5}$</td>
<td>$6.9 \times 10^{15}$</td>
</tr>
<tr>
<td>2</td>
<td>$\pm1.3 \times 10^{-5}$</td>
<td>18.2</td>
</tr>
<tr>
<td>3</td>
<td>$\pm1.3 \times 10^{-5}$</td>
<td>7.4</td>
</tr>
<tr>
<td>4</td>
<td>$\pm1.2 \times 10^{-5}$</td>
<td>5.2</td>
</tr>
<tr>
<td>5</td>
<td>$\pm1.1 \times 10^{-5}$</td>
<td>4.6</td>
</tr>
<tr>
<td>6</td>
<td>$\pm9.4 \times 10^{-6}$</td>
<td>5.2</td>
</tr>
<tr>
<td>7</td>
<td>$7.8 \times 10^{-6}$</td>
<td>$1.7 \times 10^{13}$</td>
</tr>
</tbody>
</table>
Table 22

Characteristics of growing modes in quasi-geostrophic model for $k = -k_0/2$, and $\bar{u}_3 = 16 \text{ m s}^{-1}$ ($L_y = \frac{12}{J} L_z$).

<table>
<thead>
<tr>
<th>$j = \frac{12J}{k_0}$</th>
<th>Frequency $\lambda_r$ ($s^{-1}$)</th>
<th>Doubling time $\tau_d$ (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-1.4 \times 10^{-5}$</td>
<td>$7.6 \times 10^{15}$</td>
</tr>
<tr>
<td>2</td>
<td>$\pm 1.3 \times 10^{-5}$</td>
<td>17.1</td>
</tr>
<tr>
<td>3</td>
<td>$\pm 1.3 \times 10^{-5}$</td>
<td>6.6</td>
</tr>
<tr>
<td>4</td>
<td>$\pm 1.2 \times 10^{-5}$</td>
<td>4.4</td>
</tr>
<tr>
<td>5</td>
<td>$\pm 1.1 \times 10^{-5}$</td>
<td>3.7</td>
</tr>
<tr>
<td>6</td>
<td>$\pm 9.4 \times 10^{-6}$</td>
<td>3.6</td>
</tr>
<tr>
<td>7</td>
<td>$\pm 8.2 \times 10^{-6}$</td>
<td>4.5</td>
</tr>
<tr>
<td>8</td>
<td>$6.1 \times 10^{-6}$</td>
<td>$6.2 \times 10^{14}$</td>
</tr>
</tbody>
</table>
Table 23

Characteristics of growing modes in quasi-geostrophic model for $k = -k_0/2$ and $\bar{v}_3 = 18 \text{ m s}^{-1}$ ($L_y = \frac{12}{J} L_s$).

<table>
<thead>
<tr>
<th>$J = \frac{12}{L_s}$</th>
<th>Frequency $\lambda_y$ ($\text{s}^{-1}$)</th>
<th>Doubling time $\tau_d$ (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-1.4 \times 10^{-5}$</td>
<td>$1.3 \times 10^{16}$</td>
</tr>
<tr>
<td>2</td>
<td>$+1.3 \times 10^{-5}$</td>
<td>15.7</td>
</tr>
<tr>
<td>3</td>
<td>$+1.3 \times 10^{-5}$</td>
<td>5.5</td>
</tr>
<tr>
<td>4</td>
<td>$+1.2 \times 10^{-5}$</td>
<td>3.5</td>
</tr>
<tr>
<td>5</td>
<td>$+1.1 \times 10^{-5}$</td>
<td>2.7</td>
</tr>
<tr>
<td>6</td>
<td>$+9.4 \times 10^{-6}$</td>
<td>2.5</td>
</tr>
<tr>
<td>7</td>
<td>$+8.1 \times 10^{-6}$</td>
<td>2.5</td>
</tr>
<tr>
<td>8</td>
<td>$+6.9 \times 10^{-6}$</td>
<td>3.1</td>
</tr>
<tr>
<td>9</td>
<td>$5.7 \times 10^{-6}$</td>
<td>31.0</td>
</tr>
<tr>
<td>10</td>
<td>$7.9 \times 10^{-6}$</td>
<td>$2.2 \times 10^{16}$</td>
</tr>
</tbody>
</table>
Table 24

Characteristics of growing modes in quasi-geostrophic model for \( k = -k_0/2 \) and \( \bar{\vartheta}_z = 20 \text{ m s}^{-1} \) \( (L_y = \frac{12}{J} L_s) \).

<table>
<thead>
<tr>
<th>( J = \frac{12 J}{k_0} )</th>
<th>Frequency ( \lambda_y ) ( (\text{s}^{-1}) )</th>
<th>Doubling time ( \tau_d ) ( (\text{days}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( -1.4 \times 10^{-5} )</td>
<td>( 5.7 \times 10^{16} )</td>
</tr>
<tr>
<td>2</td>
<td>( \pm 1.3 \times 10^{-5} )</td>
<td>14.9</td>
</tr>
<tr>
<td>3</td>
<td>( \pm 1.3 \times 10^{-5} )</td>
<td>4.8</td>
</tr>
<tr>
<td>4</td>
<td>( \pm 1.2 \times 10^{-5} )</td>
<td>3.0</td>
</tr>
<tr>
<td>5</td>
<td>( \pm 1.1 \times 10^{-5} )</td>
<td>2.3</td>
</tr>
<tr>
<td>6</td>
<td>( \pm 9.3 \times 10^{-6} )</td>
<td>2.0</td>
</tr>
<tr>
<td>7</td>
<td>( \pm 7.9 \times 10^{-6} )</td>
<td>1.9</td>
</tr>
<tr>
<td>8</td>
<td>( \pm 6.6 \times 10^{-6} )</td>
<td>2.1</td>
</tr>
<tr>
<td>9</td>
<td>( \pm 5.3 \times 10^{-6} )</td>
<td>3.0</td>
</tr>
<tr>
<td>10</td>
<td>( -2.2 \times 10^{-6} )</td>
<td>( 2.8 \times 10^{16} )</td>
</tr>
</tbody>
</table>
harmonics are shown in Figures 35 and 36. Comparing these two figures it is evident that the disturbances are mainly confined to the lower troposphere. Also the amplitude has considerable magnitude for the first few lower harmonics only. Thus it is reasonable to truncate the series (4.13) at $N = 10$.

7.3 Energy conversions:

The rates of conversion from kinetic energy of the basic wave, available potential energies of the basic wave and zonal flow are same as given by (4.28), (4.29) and (4.30) respectively. However, here the perturbation fields are redefined as

$$
\phi'_n = \sum_{n=-\infty}^{\infty} \left\{ \phi^{(1)}_n e^{i(nk_0x + kx + ly + \lambda t)} + \phi^{(1)}_{-n} e^{-i(nk_0x + kx + ly + \lambda t)} \right\}
$$

$$
\phi'_3 = \sum_{n=-\infty}^{\infty} \left\{ \phi^{(3)}_n e^{i(nk_0x + kx + ly + \lambda t)} + \phi^{(3)}_{-n} e^{-i(nk_0x + kx + ly + \lambda t)} \right\}
$$

where $k = -k_0/2$.

Hence the rates of conversion from different energies
Figure 35: Fourier components of lower tropospheric geopotential perturbation of the fastest growing Rossby mode in Fig. 34.
Figure 36: Fourier components of upper tropospheric geopotential perturbation of the fastest growing Rossby mode in Fig. 34.
may be different from those obtained in Chapter 4 where $k = 0$.

7.3.1 **Conversion from kinetic energy of the basic wave**

From (4.32) we can calculate the rate of conversion from kinetic energy of the basic wave to perturbation kinetic energy. We have

$$C(K_w, K') = \int \frac{k_0 \Delta k}{2g f_0} \left\{ \overline{u_1} \frac{\partial \phi_1'}{\partial y} \frac{\partial \phi_1'}{\partial x} + \overline{u_3} \frac{\partial \phi_3'}{\partial y} \frac{\partial \phi_3'}{\partial x} \right\} (e^{ik_0x} - e^{-ik_0x}) dx dy$$

(7.4)

Let us consider the first integral on the right hand side of (7.4). Using (7.3) we can write

$$\int \frac{\partial \phi_1'}{\partial y} \frac{\partial \phi_1'}{\partial x} (e^{ik_0x} - e^{-ik_0x}) dx dy$$

$$= -1 \int \sum' \left\{ \phi_n^{(1)} e^{i(nk_0x+kx+ly+\lambda t)} - \phi_n^{(1)} e^{-i(nk_0x+kx+ly+\lambda t)} \right\} \sum (mk_0+k)$$

$$\times \left\{ \phi_m^{(1)} e^{i(mk_0x-kx+ly+\lambda t)} - \phi_m^{(1)} e^{-i(mk_0x-kx+ly+\lambda t)} \right\}$$

$$\times (e^{ik_0x} - e^{-ik_0x}) dx dy$$
Integrating over \( y \) we get

\[
\int \! \int \frac{\partial \Phi_i'}{\partial y} \frac{\partial \Phi_i'}{\partial x} (e^{ik_0 x} - e^{-ik_0 x}) \, dx \, dy
\]

\[
= 2\pi e^{-2\lambda^2} \int_0^{2\pi/k_0} \sum_{n,m} (m k_0 + k) \left[ \Phi_n \overline{\Phi_m} e^{i(n-m+1)k_x} - e^{i(n-m-1)k_x} \right] \, dx
\]

While integrating over \( x \), terms with \( m = n + 1 \) from the first series, with \( m = n - 1 \) from the second, with \( m = n - 1 \) from the third and terms with \( m = n + 1 \) from the last series contribute. Other terms vanish.
Hence
\[ \iint \frac{\partial \Phi'_1}{\partial y} \frac{\partial \Phi'_1}{\partial x} \left( e^{i k_0 x} - e^{-i k_0 x} \right) \, dx \, dy \]
\[ = \frac{4 \pi^2}{k_0} e^{-2 \lambda_1 t} \sum_{n=-\infty}^{\infty} \left[ \{k+(n+1)k_0\} \left( \Phi_n^{(1)} \Phi_n^{(1)} - \{k+(n-1)k_0\} \left( \Phi_n^{(1)} \Phi_n^{(1)} - \Phi_n^{(1)} \Phi_{n-1}^{(1)} \right) \right] \]

Similarly the second integral on the right hand side of (7.4) can be evaluated.
\[ \iint \frac{\partial \Phi'_3}{\partial y} \frac{\partial \Phi'_3}{\partial x} \left( e^{i k_0 x} - e^{-i k_0 x} \right) \, dx \, dy \]
\[ = \frac{4 \pi^2}{k_0} e^{-2 \lambda_1 t} \sum_{n=-\infty}^{\infty} \left[ \{k+(n+1)k_0\} \left( \Phi_n^{(3)} \Phi_n^{(3)} - \Phi_n^{(3)} \Phi_{n+1}^{(3)} \right) \right. \]
\[ \left. - \{k+(n-1)k_0\} \left( \Phi_n^{(3)} \Phi_n^{(3)} - \Phi_n^{(3)} \Phi_{n-1}^{(3)} \right) \right] \]
Expressing $\phi_n$ and $\phi_\tilde{n}$ in terms of $\phi^r_n$ and $\phi_i^n$ and substituting for the above integrals in (7.4) we get

\[
C(K_W, K') = \frac{4\pi^2 A b}{g f_0^2} e^{-2 \lambda_i t} \sum_{n=-\infty}^{\infty} \overline{\nu}_1 \left\{ k + (n+1)k_0 \right\} 
\times \left( \phi^r_n \phi_{n+1}^{\text{i}} - \phi^r_n \phi_{n+1}^{\text{i}} \right) - \left\{ k + (n-1)k_0 \right\} \left( \phi^r_n \phi_{n-1}^{\text{i}} - \phi^r_n \phi_{n-1}^{\text{i}} \right)
\]

\[
- \phi^r_n \phi_{n-1}^{\text{i}} + \sum_{n=-\infty}^{\infty} \overline{\nu}_3 \left\{ k + (n+1)k_0 \right\} \left( \phi^r_n \phi^{3\text{i}}_{n+1} - \phi^r_n \phi^{3\text{i}}_{n+1} \right)
\]

For $k = -k_0/2$

\[
C(K_W, K') = \frac{4\pi^2 A b k_0}{g f_0^2} e^{-2 \lambda_i t} \sum_{n=-\infty}^{\infty} \overline{\nu}_1 \left\{ \begin{array}{l}
(n + \frac{1}{2}) \left( \phi^r_n \phi_{n+1}^{\text{i}} - \phi^r_n \phi_{n+1}^{\text{i}} \right) \\
(n - \frac{3}{2}) \left( \phi^r_n \phi_{n-1}^{\text{i}} - \phi^r_n \phi_{n-1}^{\text{i}} \right) + \overline{\nu}_3 \left( (n + \frac{1}{2}) \left( \phi^r_n \phi^{3\text{i}}_{n+1} - \phi^r_n \phi^{3\text{i}}_{n+1} \right) \\
- \phi^r_n \phi_{n+1}^{\text{i}} - (n - \frac{3}{2}) \left( \phi^r_n \phi_{n-1}^{\text{i}} - \phi^r_n \phi_{n-1}^{\text{i}} \right) \end{array} \right. 
\]

(7.5)
7.3.2 Conversion from available potential energy of the basic wave:

Using (4.36) we can calculate the rate of conversion from available potential energy of the basic wave to perturbation available potential energy. We have

\[ C(A_\omega, A') = \frac{k_0 (\overline{\nu}_3 - \overline{\nu}_1)}{4\pi g (\Delta p)^2} \int \left( \frac{\partial \phi'_3}{\partial y} \phi'_3 - \frac{\partial \phi'_3}{\partial y} \phi'_1 \right) \left( e^{i k_0 x} \overline{\nu}_3 - e^{i k_0 x} \overline{\nu}_1 \right) \, dx \, dy \]  

(7.6)

As in Section 4.3.2 the integrals on the right hand side of (7.6) can be evaluated to get

\[ C(A_\omega, A') = -\frac{2 \pi^2 k_0 (\overline{\nu}_3 - \overline{\nu}_1)}{\sigma g k_0 (\Delta p)^2} e^{-2 \lambda t} \sum_{n=-\infty}^{\infty} \left[ (\phi_n^{1i} - \phi_n^{3i}) \left( \phi_{n+1}^{1i} + \phi_{n+1}^{3i} + \phi_{n-1}^{1i} + \phi_{n-1}^{3i} \right) \right. \]

\[ - \left. \left( \phi_n^{1r} - \phi_n^{3r} \right) \left( \phi_{n+1}^{1r} + \phi_{n+1}^{3r} + \phi_{n-1}^{1r} + \phi_{n-1}^{3r} \right) \right] \]

(7.7)

This is identical to (4.38).
7.3.3 Conversion from available potential energy of the 
zonal flow:

Using (4.39) the rate of change of available 
potential energy of zonal flow can be calculated. We have

\[ C(A_z, A') = \frac{k_0 (U_3 - U_1)}{2 \pi^2 g (\Delta \rho)^2} \iiint \left( \frac{\partial \phi_3'}{\partial x} \phi_3' + \frac{\partial \phi_1'}{\partial x} \phi_3' - \frac{\partial \phi_1'}{\partial x} \phi_1' \right) dx \, dy \]

(7.8)

Let us consider the second integral on the right hand side of (7.8). Substituting for \( \phi_1' \) and \( \phi_3' \) from (7.3) first 
we integrate over \( y \). Then integrating over \( x \), terms 
with \( m = n \) contribute.

Thus

\[ -\iiint \frac{\partial \phi_3'}{\partial x} \phi_1' \, dx \, dy \]

\[ = - \frac{i 4 \pi^2}{k_0} e^{-2 \lambda i t} \sum_{n=-\infty}^{\infty} (k + nk_0) \left( \phi_n^{(3)} (\phi_n^{(1)}) - \phi_n^{(3)} \phi_n^{(1)} \right) \]
For $k = -k_0/2$,

$$-\iint \frac{\partial \Phi^3}{\partial x} \phi_1' \, dx \, dy = -i \frac{4\pi^2}{l} e^{-2\lambda_1 t} \sum_{n=\infty}^{\infty} (n-\frac{1}{2})(\phi_n^{(3)} \phi_n^{(1)} - \phi_n^{(3)} \phi_n^{(1)})$$

Similarly,

$$\iint \frac{\partial \phi_1'}{\partial x} \phi_3' \, dx \, dy = i \frac{4\pi^2}{l} e^{-2\lambda_1 t} \sum_{n=\infty}^{\infty} (n-\frac{1}{2})(\phi_n^{(1)} \phi_n^{(3)} - \phi_n^{(1)} \phi_n^{(3)})$$

It can be shown that first and last integrals on the right hand side of (7.8) vanish.

Hence,

$$C(A_2, A_1') = \frac{9\pi^2 \rho_0 (U_3 - J_1)}{l \sigma g (\Delta b)^2} e^{-2\lambda_1 t}$$

$$\times \sum_{n=\infty}^{\infty} (n-\frac{1}{2})(\phi_n^{1\tau} \phi_n^{3\tau} - \phi_n^{1\tau} \phi_n^{3\tau})$$

$$= \frac{9\pi^2 \rho_0 (U_3 - J_1)}{l \sigma g (\Delta b)^2} e^{-2\lambda_1 t}$$

$$\times \sum_{n=\infty}^{\infty} (n-\frac{1}{2})(\phi_n^{1\tau} \phi_n^{3\tau} - \phi_n^{1\tau} \phi_n^{3\tau})$$

(7.9)

The rates of conversion from the kinetic energy and available potential energy of the basic wave and rate of change of available potential energy of the zonal
flow are calculated from (7.5), (7.7) and (7.9) respectively, for a typical case \( \overline{U}_3 = 20 \text{ m s}^{-1} \). The rate of conversion from kinetic energy of the basic wave to perturbation kinetic energy is the maximum as in Chapter 4 where \( k = 0 \). Following ratios give some idea about the magnitudes of different energy conversion rates.

\[
\frac{C(K_w, K')}{C(A_z, A')} = 2 \times 10^2
\]
\[
\frac{C(K_w, K')}{C(A_w, A')} = 7 \times 10^2
\]
\[
\frac{C(A_w, A')}{C(A_z, A')} = 0.3
\]

Comparing these results with those of the case \( k = 0 \) in Chapter 4, it is seen that the rate of change of available potential energy of the zonal flow is more in case of \( k = -k_0/2 \) than in the case \( k = 0 \).

7.4 Summary and Conclusions:

In this chapter the stability analysis is extended to the case where the perturbation has a zonal wavenumber equal to \( \pi |k_0 + k \) \) and \( k = -k_0/2 \) satisfying conditions for resonant interaction. In other words we have
studied the stability of a stationary Rossby wave of zonal wavenumber $k_0$ (corresponding to wavelength of 30° longitude) superposed on the monsoon zonal flow to a perturbation having a zonal wavenumber equal to the sum of the multiples of the basic wavenumber and a constant wavenumber $i.e.$ $n k_0 + k$ where $k = -k_0/2$.

The stability analysis is conducted for different values of meridional velocity at lower and upper troposphere satisfying the relation $\overline{U}_3 = 22 \overline{U}_4$. For each pair of values of $\overline{U}_3$ and $\overline{U}_4$ the meridional wavenumber of the perturbation is varied using the equality $l = \frac{J}{12} k_0$ and varying $J$ from 1 to 12 by steps of one.

It is found that as in the previous case of $k = 0$, here for each value of Rossby amplitude there is a meridional wavenumber for which the growth rate of perturbation is maximum. It is also seen that the maximum shifts towards higher value of $l$. The maxima lie in the range $\frac{5}{12} k_0 < l < \frac{7}{12} k_0$.

There is a threshold value of Rossby amplitude for which perturbations grow. Increasing $\overline{U}_3$ from 5 m s$^{-1}$ to 20 m s$^{-1}$ it is seen that perturbations do not grow unless $\overline{U}_3 = 15$ m s$^{-1}$. In comparison with $k = 0$ case, it is found that the minimum value of $\overline{U}_3$ for the growth of perturbations is more in case of $k = -k_0/2$.
The magnitudes of disturbances are more in the lower troposphere than in the upper troposphere.

For the same value of Rossby wave amplitude the growth rate is found to decrease because of the introduction of nonzero value of $k$. Thus the fastest growth rate occurs for $k = 0$ case. This is in conformity with the results of Gill (1974) and Duffy (1975).

Energy calculations show that the perturbations grow mainly by drawing on kinetic energy of the basic wave.