CHAPTER 2

STIMULATED SCATTERING OF EM WAVES IN A MAGNETIZED PLASMA

2.1 Introduction

An important problem in parametric heating of plasmas by an intense EM wave is that of enhanced (or stimulated) scattering of incident EM waves by some ES waves. Physically we can understand the process as follows. The incident EM field induces electron oscillations through the Lorentz force. The electrons are initially driven along the electric field vector but then develop a longitudinal component through the \(\mathbf{v} \times \mathbf{B}\) force, where \(\mathbf{v}\) is the velocity of the electrons and \(\mathbf{B}\) is the magnetic field of the EM wave. The ions do not respond directly to the field due to their large mass. Thus local charge imbalances are generated which tend to be restored to neutrality by the opposing coulomb interaction. On a macroscopic scale, the density oscillations are coupled to the pump field by the ponderomotive force density which varies essentially as the gradient of the intensity. For suitable phase matching between the incident and the scattered
waves growth is induced in slowly moving density waves above a threshold intensity determined by the dissipation of the interacting waves.

In the absence of the pump wave, an unmagnetized plasma supports two natural ES modes of propagation. They are (1) the high frequency ES plasma wave and (2) the low frequency ES ion-acoustic wave. Accordingly in an unmagnetized plasma we can get two types of scattering processes. When EM pump wave is scattered by a plasma wave, the process is known as stimulated Raman scattering (SRS) and when it is scattered by an ion acoustic wave, the process is known as stimulated Brillouin scattering (SBS).

Historically study of scattering of EM waves by a plasma started from the work of W.B. Gordon\(^{(1)}\), in 1958, who predicted that if a powerful beam of radio waves with a frequency well above the penetration frequency were sent vertically through the ionosphere, an extremely small, but still measurable amount of power would be scattered back to the ground from the randomly distributed free electrons in the ionosphere. Later on Dougherty and Farley\(^{(2)}\), and Salpeter\(^{(3)}\) also studied the problem of incoherent scattering of radio waves by a plasma and Goldman and DuBois\(^{(4)}\) studied the incoherent scattering of light from plasmas.
SRS in plasmas was studied theoretically by Comisar\textsuperscript{(5)} in 1966. Gorbunov\textsuperscript{(6)}, using one dimensional fluid and kinetic descriptions, calculated threshold powers and growth rates for the backward SBS process. A unified formation for the stimulated scattering of an EM wave off RS waves, in a homogeneous unmagnetized plasma, was derived by Drake\textsuperscript{(7)} et al in 1974. These authors derived a general dispersion relation for such a scattering process and discussed various instabilities including SRS, SBS, Compton scattering, and filamentation and modulational instabilities. These instabilities have direct relevance to laser produced plasmas as these instabilities can be excited in the under dense region of the laser produced plasma resulting in a partially reflecting induced dielectric mirror which can lead to substantial reflection of the incident radiation.

Recently Stamper et al\textsuperscript{(8)} have observed intense spontaneously generated magnetic fields in the laser produced plasmas. Rough estimates of the field strength in the plasma, by numerical simulation, has shown that fields of the order of mega gauss are produced\textsuperscript{(9)}. These intense fields can completely modify the spectrum of electrostatic modes in the laser produced plasma. At the same time, if the laser frequency is much greater than the electron cyclotron frequency, these magnetic fields will not significantly influence the propagation characteristics of the incident and scattered EM waves. It is, therefore,
conceivable that in a magnetized plasma stimulated scattering can occur from a large number of ES modes and it is important to assess the relative potentialities of each one of them. In this chapter we shall discuss the scattering of the incident EM wave off various electrostatic modes occurring in magnetized plasma. The low frequency ES modes which we have considered are (1) the Bernstein modes (2) the lower hybrid modes and (3) the drift modes. We have calculated the threshold powers and the maximum growth rates for scattering off each of these modes.

These growth rates correspond to usual idealized infinite medium linear theory and for a realistic situation we must consider the stabilizing mechanisms present in the system. On the level of linear theory we may consider three stabilizing effects, viz. (1) damping of the product waves, (2) finite pump wave band width and (3) plasma inhomogeneity. The effect of damping of decay waves is to stabilize the decay if \( \nu_1 \nu_2 > \gamma_0^2 \) where \( \nu_1 \) and \( \nu_2 \) are the damping rates of the decay products (in the present case the low frequency ES mode and the scattered EM wave) and \( \gamma_0 \) is the growth rate obtained in the idealized case. If the damping is classical (i.e. collisional and Landau damping) then this does not appear to be an important mechanism of stabilization for laser fusion conditions. Also if \( \gamma_0 \) is greater than the band width \( \delta \omega \) then the finite band width effect is not important for stabilization.
Thus the principal linear stabilization mechanism appears to result from the inhomogeneity effect. If the plasma is infinite but inhomogeneous, then the matching conditions \( \omega_0 = \omega_1 + \omega_2 \) and \( k_0 = k_1 + k_2 \) can be satisfied only over a finite region, as the frequencies and the wave vectors of the interacting waves depend on the plasma density through the dispersion relation of the waves. As a wave travels away from the point of perfect matching, it grows until the phase mismatch is so large that the proper phasing for growth is lost. The wave equation has a turning point there and the wave propagates without growth from there on. A turning point occurs when the phase mismatch \( \int k dx \simeq K l_t \) is of order 1, where \( K = k_0 - k_1 - k_2 \) is the phase mismatch and \( l_t \) is the length of the interacting region. Thus the decay must occur before the product waves have time to convect out of the interaction region. A detailed consideration shows that this results in a growth of fluctuations by a factor \( \varepsilon^\alpha \) where \( \alpha = 2 \pi \sqrt{2}/V_1 V_2 K' \) is the amplification factor; \( V_1 \) and \( V_2 \) are the group velocities of the decay products and \( K' = dk/dx \).

Liu et al. studied SRS and SBS in an inhomogeneous plasma and calculated the amplification factors for these scattering processes. In this chapter we have calculated the amplification factors for the different scattering processes in a magnetized plasma mentioned above.
In section 2.2 we shall briefly discuss the general dispersion relation for scattering of an EM wave off a ES mode. In section 2.3 we shall outline the derivation of the amplification factor $\alpha$. In section 2.4 we shall present estimates of threshold powers, maximum growth rates and amplification factors for the instabilities under consideration. Section 2.5 gives a discussion of our results and compares them with those obtained by other authors for similar calculations.

2.2 Derivation of the General Dispersion Relation.

Let us consider a large amplitude, plane polarized EM pump wave

$$E_0 = 2E_0 \hat{E}_0 \cos(k_0 \cdot x - \omega_0 t)$$

(2.1)

propagating in a homogeneous plasma. We assume $(\omega_0, k_0)$ to satisfy the linear dispersion relation,

$$\omega_0^2 = \omega_{pe}^2 + \frac{c^2}{\epsilon_0} k_0^2$$

(2.2)

In equilibrium, electrons oscillate with high velocity in the incident electric field $E_0$ with the ions forming a stationary background. If there is a density perturbation $(\omega, \mathbf{k})$ associated with an electrostatic wave, then the electron density perturbation $\delta n_e$ will be driven by $E_0$ leading to currents at $\omega \pm \omega_0$ and $\mathbf{k} \pm \mathbf{l} k_0$ where $l$ is an integer. These currents will generate mixed ES-EM side band modes at $\omega \pm l \omega_0, \mathbf{k} \pm \mathbf{l} k_0$ which will in turn interact with the pump wave field producing a ponderomotive bunching force $\nabla E^2$ that amplifies the density perturbation. Thus
there is a positive feedback system that will lead to
instability of the original density perturbation and the
side band modes.

The Fourier transformed wave equation for the side
band modes $\omega_\pm = \omega \pm \omega_o$, $k_\pm = k \pm k_0$ can be written as

\[
\left( \frac{k_\pm^2 - \omega_\pm^2}{c^2} \right) \tilde{E} = - \frac{4\pi i \omega_\pm}{c^2} \tilde{J}_\pm
\]

where we have considered the lowest order coupling $\ell = 1$
which is justified when $e E_0 / m \omega_o c \ll 1$ (here $e$ is
the unit charge, $c$ is the velocity of light). In equation
(2.3) $\tilde{I}$ is the unit dyadic and $\tilde{E}_\pm = E(\omega_\pm, k_\pm)$. The
current density perturbation $\tilde{J}_\pm$ arises from the linear
response of electrons to $\tilde{E}_\pm$ and from the coupling between
the oscillating velocity produced by the pump and the electron
density perturbation produced by the electrostatic wave.

Substituting these terms for $\tilde{J}_\pm$ in equation (2.3) and re­
arranging we get,

\[
\left[ \left( k_\pm^2 - \omega_\pm^2 \right) \frac{c^2}{e} \right] \tilde{E}_\pm = - \frac{\omega_\pm^2}{c^2} \delta n e(\omega_\pm, k_\pm) \tilde{E}_0 \pm (2.4)
\]

where $\varepsilon_\pm = 1 - \omega_\pm^2 / \omega_p^2$ is the linear dielectric constant at
frequencies $\omega_\pm$, $\omega_p = (4\pi e^2 n_0 / m)^{1/2}$ is the plasma
frequency, $n_0$ is the equilibrium density and $\tilde{E}_0 \pm = \tilde{E}_c$
$E(\omega_\pm, k_\pm)$ refer to the components of the pump wave. We
have assumed here $\omega_o \gg \omega$ and $\omega_o / \omega_p \gg 1$ (or if $\omega_o / \omega_p$ is
arbitrary then $k \lambda_D \ll 1$ where $\lambda_D$ is the Debye wave­
length). Inverting equation (2.4) we get,

\[-\mathbf{E}_\pm = \omega \mathbf{p}_e \frac{\delta n_e(k, \omega)}{n_0} \left[ \left( \frac{1 - \frac{k^2}{k^2 \pm k} \omega_c^2}{k^2 \pm k} \right) \right] \omega \frac{1}{D_\pm} \frac{\mathbf{e}_0^\perp \mathbf{E}_0^\perp}{k^2 \pm k \omega_c^2 \epsilon_\pm} \mathbf{E}_0^\perp \]

where \( D_\pm = k^2 \omega_c^2 - \omega^2 \epsilon_\pm = c^2 k^2 + 2 k o_c^2 + 2 \omega \omega_0 - \omega^2 \).

(2.5)

In order to calculate \( \delta n_e(k, \omega) \) we first calculate the low frequency force \( F_\omega \) on the electrons, due to the presence of two high frequency waves, which is given by the gradient of the ponderomotive potential \( \psi_\omega \) i.e.

\[ F_\omega = -\nabla \psi_\omega = -\left( e^2 / m \omega_0^2 \right) \nabla \left( E_{0+} \cdot E_{-} + E_{0-} \cdot E_{+} \right) \]

(2.7)

Inserting this force term in the Vlasov equation and solving it for electron density perturbation we get

\[ \delta n_e(k, \omega) = \frac{\chi e(k, \omega)}{4 \pi e} \i k \mathbf{E}(k, \omega) + \frac{\omega}{e} \psi(k, \omega) \]

(2.8)

where \( \chi e(k, \omega) \) and \( \mathbf{E}(k, \omega) \) denote the electron susceptibility and the self consistent electric field respectively. We can calculate the ion density perturbation in the same way but without the ponderomotive force since it is smaller than the electron term by the mass ratio. Inserting this expression for the ion perturbation into Poisson's equation, we can solve for the electron density fluctuation

\[ \delta n_e(k, \omega) = -\frac{1 + \chi i(k, \omega)}{4 \pi e} \i k \cdot \mathbf{E}(k, \omega) \]

(2.9)
where $\chi_i(k,\omega)$ denotes the ion susceptibility. One can eliminate $\tilde{E}(k,\omega)$ between equations (2.8) and (2.9) and use equation (2.7) to get
\[
\tilde{E}(k,\omega) = \frac{k^2}{4\pi m \omega^2} \frac{\chi_i(k,\omega)}{E(k,\omega)} (1 + \chi_i(k,\omega)) 
\times (E_{0+} E_- + E_{0-} E_+)
\] (2.10)
using equations (2.5) and (2.10) we get the general dispersion relation,
\[
\frac{1}{\chi_i(k,\omega)} + \frac{1}{1 + \chi_i(k,\omega)} = k^2 \left[ \frac{|k \cdot v_0|^2}{k_{+}^2 D_{-}} - \frac{|k \cdot v_0|^2}{k_{-}^2 \omega_{-}^2 E_-} \right. \\
\left. + \frac{|k \cdot v_0|^2}{k_{+}^2 D_{+}} - \frac{|k \cdot v_0|^2}{k_{+}^2 \omega_{+}^2 E_+} \right]
\] (2.11)
where $v_0 = eE_0/m\omega_0$.

This dispersion relation describes the parametric coupling of a low frequency electrostatic mode at $\omega$ and two high frequency mixed ES-EM side bands at $\omega \pm \omega_0$. The $k \cdot v_0$ terms in equation (2.11) arise from the EM components of the side band modes and the $k \cdot v_0$ terms from the ES components, when $\omega_0 \sim \omega \neq \omega_0$, $E_\pm \sim 0$ and the side bands are predominantly electrostatic. If $k \cdot v_0 \ll 1$ then this reduced form of equation (2.11) correctly describes the parametric excitation of two ES waves by an incident EM wave. On the other hand when $D_\pm \sim 0$, we find $E_\pm \neq 0$. In this case, the side bands are predominantly EM and this represents the scattering process where the EM pump wave...
excites an ES wave and new EM waves at shifted frequencies. Let us consider a case where $D_0 \approx 0$ and $D_+ \neq 0$, i.e., the Stokes' components are resonant and anti-Stokes' components are non-resonant. In such a situation equation (2.11) reduces to

$$\frac{1}{\chi_E(\omega, k)} + \frac{1}{1 + \chi_i(\omega, k)} = k^2 \frac{|k - \chi V_0|^2}{k^2 D_-}$$

(2.12)

Equation (2.12) is valid for $\omega \ll \frac{c^2 k^2 k_0}{\omega_0}$ and breaks down for small $k$ or if $k \perp k_0$. For $\omega_0 \gg \omega$ we may write

$$D_-(\omega, k) = c^2 k^2 - \omega^2 + \omega^2 \approx 2\omega_0 \left(\omega - \frac{c^2 k^2 k_0}{\omega_0} + \frac{c^2 k^2}{2\omega_0}\right)$$

(2.13)

The last two terms in the parenthesis are of order $\omega_0$ and therefore must roughly cancel for $D_-$ to be small. This condition implies that $k \approx 2k_0 \cos \theta$, where $\theta$ is the angle between $k$ and $k_0$. Figure 2.1 expresses this result in a geometrical form. Thus substituting $2k_0 \cos \theta$ for $k$ everywhere except $D_-$ in equation (2.12) we get

$$\frac{1}{\chi_E(k, \omega)} + \frac{1}{1 + \chi_i(k, \omega)} = \frac{2k_0^2 V_0^2}{\omega_0(\omega - \Delta \omega)} \Psi \phi(\theta, \phi)$$

(2.14a)

where

$$\Psi = |\sin \phi| \cos \theta$$

(2.14b)

$$\sin^2 \phi = \frac{|k - \chi V_0|^2}{k^2 V_0^2}$$

(2.14c)
Figure 2.1 - Wave vector diagram for non forward scattering. Here $\mathbf{k}_0$, $\mathbf{k}_1$ and $\mathbf{k}_2$ (=$\mathbf{k}_0 - \mathbf{k}_1$) are the EM pump wave vector, electrostatic wave vector and the scattered EM wave vector respectively.
The dispersion relation (2.14a) is quite general and the basic form of this equation will not change when a magnetic field or density, temperature gradients are introduced provided these additions only influence the low frequency modes at frequency $\omega$ and leave the higher frequency modes unaffected. The correct dispersion relation can be obtained by substituting the appropriate electron and ion susceptibility tensors $\chi_e$ and $\chi_i$ in equation (2.14a). Equation (2.14a) can be rewritten as

$$\frac{(\omega - \Delta \omega)}{\omega_0} \varepsilon_{l} = 2 \chi_e (1 + \chi_i) \frac{V_0^2}{c^2} \psi^2$$

(2.15)

The linear dielectric function $\varepsilon_{l}(\omega_0 + i \gamma)$ can be expanded near $\omega$ giving

$$\varepsilon_{l} = \gamma \frac{\partial \varepsilon_{l,r}}{\partial \omega} \bigg|_{\omega = \omega_{l}} \quad (2.16)$$

Here $\omega_{l}$ is the real part of the frequency and $\gamma$ is the growth rate. $\varepsilon_{l,r}$ is the real part of the dielectric function. Substituting equation (2.16) into (2.15) we get

$$\gamma^2 + \Gamma_{l} \gamma = - 2 \chi_e (1 + \chi_i) \frac{\omega_0 V_0^2}{c^2} \left[ \frac{\partial \varepsilon_{l,r}}{\partial \omega} \right]_{\omega = \omega_{l}}$$

(2.17)

where we have introduced $\Gamma_{l}$ and $\Gamma_{l}$ to denote the phenomenological damping rates of the free ES and the free EM waves respectively. We have also taken $\omega = \omega_{l} + i \gamma$ and $\Delta \omega = \omega_{l}$.
The maximum growth rate is obtained from equation (2.17) for \( \gamma \gg \Gamma_\perp, \Gamma_\parallel \), and is given by:

\[
\gamma^2 = -\frac{2 \chi\epsilon(\omega)(1 + \chi_i(\omega))\omega_0}{(\partial \epsilon/\partial \omega)_{\omega = \omega_0}} \frac{V_0^2 \psi^2}{c^2} \tag{2.18}
\]

The threshold power is readily obtained by setting \( \gamma = 0 \) in equation (2.17).

\[
\frac{V_{\text{OT}}^2}{c^2} = -\frac{1}{2} \frac{\Gamma_\perp \Gamma_\parallel}{\omega_c \omega_0} \frac{1}{(\partial \omega / \partial \omega)_{\omega = \omega_0}} \frac{\chi\epsilon(1 + \chi_i)}{\chi(\omega, \chi)} \tag{2.19}
\]

In section (2.4) we shall present estimates of maximum growth rates and threshold powers for different ES modes, in a magnetized plasma, using equations (2.18) and (2.19) respectively.

2.3 Derivation of the amplification factor \( \alpha \).

In an inhomogeneous plasma, the matching condition can hold only locally and the mismatch \( K = k_{0x}(\chi) - k_{1x}(\chi) - k_{2x}(\chi) \) develops due to the spatially dependent quantities e.g. density, temperature which occur in the dispersion relations determining \( K(\omega, \chi) \). This mismatch then localizes the region of resonant interaction.

For simplicity, we consider a plasma slab with density gradient in the \( \chi \)-direction. In the weak pump
case, the equations for the amplitudes $a_1, a_2$ of the decay waves in an inhomogeneous medium are,

$$\frac{\partial a_1}{\partial t} + v_1 \frac{\partial a_1}{\partial x} = \gamma_0 a_2 \exp (i \int_0^x k \, dx) \tag{2.20}$$

and

$$\frac{\partial a_2}{\partial t} + v_2 \frac{\partial a_2}{\partial x} = \gamma_0 a_1 \exp (-i \int_0^x k \, dx) \tag{2.21}$$

The coupling factor $\gamma_0$ is taken to be the growth rate in the absence of damping for the homogeneous medium.

Laplace transforming in time and neglecting initial values, eliminating $a_2^x$ and putting

$$a_1 = \xi \exp \left[ i \int_0^x \frac{1}{2} k \, dx - \frac{1}{2} \left( \frac{1}{v_1} + \frac{1}{v_2} \right) x \right] \tag{2.22}$$

we easily find that

$$\frac{\partial^2 \xi}{\partial x^2} + \left[ \frac{1}{4} \left( k - i \rho \left( \frac{1}{v_1} - \frac{1}{v_2} \right) \right) + \frac{i}{2} \frac{d k}{d x} - \frac{\gamma_0^2}{\rho v_1 v_2} \right] \xi = 0 \tag{2.23}$$

where $\rho$ is the Laplace transformed variable.

Equation (2.23) is an eigenvalue equation for $\xi$ with $\rho$ its eigenvalue. If there is a well behaved solution with $\text{Re} \rho > 0$ then $\rho$ will correspond to the eigenvalue for a temporally growing mode with growth rate given by $\text{Re} \rho$. This mode is generally localized within certain regions and grows exponentially in time until some nonlinear mechanism limits the growth. If no such $\rho$ exists, then only spatial amplification over a given source at the thermal level, is possible. By spatial
amplification we mean that a thermal source, due to spontaneous emission of waves, can grow for a limited period of time and stops growing once a maximum level is reached. This level of growth eventually reached is limited principally by the duration of interaction, which in turn, is determined by the wave propagation out of the interaction region where the phase matching conditions are approximately satisfied. Since \( k \) increases with \( \kappa \) the possible behaviour at infinity is

\[
\alpha_1 = \exp \left( -\frac{p}{\nu_1} \right) \kappa
\]

and

\[
\alpha_II = \exp \left[ i \int_0^\infty k \, d\kappa - \left( \frac{p}{\nu_2} \right) \kappa \right]
\]  

(2.24) for both solutions are badly behaved at either plus or minus infinity for \( \text{Re}p > 0 \) and no temporally growing modes are possible. With \( \nu_1 \nu_2 < 0 \) (decay modes propagating oppositely) normal modes with \( \text{Re}p > 0 \) are possible, provided the solutions well behaved at \( \pm \infty \) can be joined at \( \kappa = 0 \).

For \( K = K(\kappa) \kappa \) we make a transformation

\[
k(\kappa) \kappa - i p \left( \frac{1}{\nu_1} - \frac{1}{\nu_2} \right) = K(\kappa) \kappa
\]

reducing equation (2.23) to the parabolic cylinder equation. Since this equation has no well behaved solution for \( \text{Re}p > 0 \) only spatial amplification is possible. We choose \( p = \epsilon \) where \( \epsilon \) is a small positive number to give the proper behaviour at infinity. Considering \( |\gamma_0^2/\nu_1 \nu_2| > K' \) for sizable
amplification equation (2.23) can be written as

\[ \frac{d^2 \xi}{dx'^2} + \left( \frac{1}{4} K' x'^2 - \gamma_0^2 / V_1 V_2 \right) \xi = \delta(x') \]  

(2.25)

where we have put the source at \( x' = 0 \).

Let us consider the case \( V_1 V_2 > 0 \) then from the boundary conditions we know that \( \xi = 0 \) for \( x' < 0 \) (since the amplification takes place from \( x' = 0 \)).

Beyond the turning point \( x_t = 2 \gamma_0^2 / K' (V_1 V_2)^{1/2} \) the solution is oscillatory while between \( 0 \) and \( x_t \) it has the approximate form

\[ \sinh \int_0^{x_t} \frac{d_0}{V_1 V_2 - K' x'^2/4} \frac{1}{2} d_0 \]

Thus there is a net \( e \)-folding given by

\[ \int_0^{x_t} \left[ \gamma_0^2 / V_1 V_2 - \frac{1}{4} K' x'^2/2 \right] d_0 = \pi \gamma_0^2 / x_0 V_1 V_2 K' \]  

(2.26)

Putting the source term at \( -x_t \) we get for the \( e \)-folding of intensity as

\[ I = I_0 \exp \left( 2 \pi \gamma_0^2 / V_1 V_2 K' \right) \]

(2.27)

where \( I_0 \) is the non-driven, thermal source intensity.

For effective growth \( \lambda = 2 \pi \gamma_0^2 / V_1 V_2 K' \) which means that the wave must grow substantially during the time it propagates to the point where the phase mismatch is substantial. The same result can be obtained for \( V_1 V_2 < 0 \) case also.
2.4 Calculation of growth rates, thresholds and amplification factors.

In this section we shall calculate the growth rates, threshold powers and amplification factors for different electrostatic modes using equations (2.18), (2.19) and (2.27). We take the geometry as follows: the magnetic field $B_0$ is in the $\hat{z}$-direction; there is a density gradient along the $x$-direction; the pump wave is applied along the $\chi$-direction.

2.4.1 Bernstein modes.

The Bernstein modes are polarized with their electric vector nearly parallel to wave vector $k$ and they are almost pure longitudinal waves. These waves are the counterpart of the field free plasma waves $\omega = \omega_{pe}$ and $\omega = k c_s$ and in the limit of the magnetic field $B_0$ going to zero, they reduce to the Langmuir Oscillations at high frequency and to ion sound waves at low frequency. The Bernstein modes propagate in frequency ranges that lie between harmonics of the cyclotron frequency. The lowest frequency for which propagation occurs lies above the electron cyclotron frequency $\Omega_e (\approx eB_0/m_e)$ i.e. $\omega > \Omega_e$. The exact location of the modes is a function of density, temperature and field strength.

The general dispersion relation for the Bernstein modes is given by

$$K^2 = \sum_n \sum_\alpha \left[ \frac{2n^2 \omega_{pe}^2 \Omega_s^2}{\omega^2 n^2 \Omega_s^2} \frac{m_\alpha}{T_\alpha} \ln \left( \frac{K^2 T_\alpha}{\Omega_s^2 m_\alpha} \right) \exp \left( -\frac{K^2}{\Omega_s^2 m_\alpha} \right) \right]$$

(2.28)
where \( \lambda \) denotes electrons or ions, \( n \) is an integer, \( T_\lambda \) is the temperature measured in energy units, \( I_n \) is the modified Bessel's function of the first kind, \( I_n(z) = \exp(-i\pi n/2)J_n(e^{i\pi/2}z) \). When the plasma temperature is low or when long waves are being studied, the Bessel function \( I_n \) can be expanded in powers of \( k a_\lambda \), \( a_\lambda \) being the gyroradius of species \( \lambda \). We shall consider here two special cases.

(1) Low density plasma

When the plasma density is low, all the Bernstein modes occur very close to cyclotron harmonics and the explicit solutions of equation (2.28) are

\[
\omega^2 = n^2 \Omega_e^2 (1 + \beta_n)
\]

(2.29)

where

\[
\beta_n = \frac{2\omega_p^2}{\Omega_e^2} \frac{1}{k^2 a_e^2} I_n(k^2 a_e^2) \exp(-k^2 a_e^2)
\]

and

\[
a_e^2 = \frac{T_e}{m_e} \Omega_e^2 \quad \text{and} \quad \omega_p^2 \ll \Omega_e^2
\]

For this mode, the ion and the electron susceptibilities are given by,

\[
\chi_i(\omega) = 0
\]

\[
\chi_e(\omega) = -n^2 \Omega_e^2 (1 + \beta_n)/\omega^2
\]

(2.30)

Therefore

\[
\chi_e(\omega_e) = -1
\]

(2.31)
The dielectric function \( \varepsilon \) is given by

\[
\varepsilon(\omega) = 1 - \frac{n^2 \Omega_e^2 (1 + \beta_n)}{\omega^2}
\]  

(2.32)

Therefore

\[
\left( \frac{\partial \varepsilon}{\partial \omega} \right)_{\omega = \omega_L} \approx \frac{2}{n \Omega_e}
\]

(2.33)

Substituting equations (2.30) to (2.33) in equations (2.18) and (2.19) we get the expressions for growth rate and the threshold power for the scattering of an incident pump wave off a Bernstein mode at low density as

\[
\gamma_0^2 = V_0^2 \omega_0 \Omega_e \psi^2 / c^2
\]

(2.34)

and

\[
\frac{V_0}{c^2} = \frac{1}{\psi^2} \frac{\Gamma}{\omega_0} \frac{\Gamma e}{\Omega_e}
\]

(2.35)

where we have taken \( n = 1 \).

In order to calculate the amplification factor \( \alpha \) we need to calculate the group velocities \( V_1x \) and \( V_2x \) of the electrostatic wave and scattered EM wave respectively and \( K' \) the derivative of the phase mismatch. From equation (2.29) we have

\[
V_{1x} = \partial \omega / \partial k_{1x} = -\omega_p^2 a_e^2 k_{1x} / \Omega_e
\]

(2.36)

From the dispersion relation of the scattered EM wave viz.

\[
\omega_x^2 = \omega_p^2 + c^2 k_x^2 \approx c^2 k_x^2
\]

(2.37)

we have

\[
V_{2x} = \partial \omega_2 / \partial k_{2x} = c^2 k_{2x} / \omega_0
\]

(2.38)
Also equation (2.29) can be written as
\[ \omega^2 = \Omega^2 + \omega^2 \left( 1 - k_1^2 a_e^2 \right) , \]
from which
\[ k_1 x = \left[ (\omega^2 + \Omega^2 - \omega^2 a_e^2 k_{1 y}^2) / \omega^2 a_e^2 \right]^{1/2} \] (2.39)
using equation (2.37) we have for the scattered wave vector
\[ k_{2 x} = \left[ (\omega_0^2 - \omega_0^2) / c^2 \right]^{1/2} \] (2.40)
And for the pump wave we get
\[ k_{0 x} = \left[ (\omega_0^2 - \omega_0^2) / c^2 \right]^{1/2} \] (2.41)
Substituting equations (2.39) to (2.41) in the expression
\[ K' = \frac{d}{dx} (k_{0 x} - k_{1 x} - k_{2 x}) \] (2.42)
we have
\[ K = \frac{\omega_0^2 \sin^2 \theta/2}{L \sin^2 k_{0 x} \cos \theta} \left\{ 1 + \frac{\omega_0^2 \cos \theta}{\omega_0^2} - \frac{c^2 (1 - a_e^2 k_{1 y}^2) \cos \theta'}{4 \omega_0^2 a_e^2 \sin^4 \theta/2} \right\} \] (2.43)
where \( \theta' \) is the angle between \( k_0 \) and \( k_2 \) and we have written \( L_n = (1/\omega_0^2) (d\omega_0^2/dx) \). Substituting equations (2.34), (2.36), (2.38) and (2.43) in the expression
\[ \alpha = 2 \pi \gamma_0^2 / V_1 x V_2 x K' \] (2.44)
we get for the spatial amplification factor
\[ \alpha = \frac{\pi V_0^2 \sin^2 \phi \Omega^2}{V_e^2 \sin^2 \theta/2} \frac{\Omega_e^4}{\omega_0^2 \sin^2 \theta/2 \omega_0^4} \left\{ 1 + \frac{\omega_0^2 \cos \theta}{\omega_0^2} - \frac{c^2 (1 - a_e^2 k_{1 y}^2) \cos \theta'}{4 \omega_0^2 a_e^2 \sin^4 \theta/2} \right\} \] (2.45)
where \( v_e = (e/m_e)^{1/2} \) is the electron thermal speed.

We shall now consider two special cases:

Case (a): When \( \theta = 90^\circ \) i.e. for side scattering equation (2.45) becomes

\[
\chi_{q_0} = 2\pi \left( \frac{v_0}{v_e} \right)^2 \left( k_0 l_n \right) \left( \frac{\Omega_e}{\omega_{pe}} \right)^4 \sin^2 \phi
\] (2.46)

Case (b): When \( \theta = 180^\circ \) i.e. for back scattering equation (2.45) becomes

\[
\chi_{180^\circ} = 4\pi \left( \frac{v_0}{v_e} \right)^2 \left( k_0 l_n \right) \left( \frac{\Omega_e}{\omega_{pe}} \right)^2 \sin^2 \phi
\] (2.47)

(ii) High density plasma

When the plasma density is large and magnetic field strength is small, the lowest frequency mode is given by,

\[
\omega_1 = \frac{2 \Omega_e}{\omega_{pe}} \left( 1 - \frac{3 \omega_{pe}^2}{\omega_e^2} \frac{k^2_0 l_n}{m} \left( \frac{\omega_{pe}^2 - 3 \omega_e^2}{\omega_{pe}^2} \right) \right)
\] (2.48)

for \( \omega_{pe}^2 > 3 \omega_e^2 \) and \( k^2_0 l_n << 1 \).

For this mode the ion and the electron susceptibilities are given by,

\[
\chi_i(\omega) = 0; \quad \chi_e(\omega) = -\frac{2 \Omega_e}{\omega} \left( 1 - \frac{\omega_{pe}^2}{\omega_e^2} \frac{3}{(\omega_{pe}^2 - 3 \omega_e^2)16} k^2 l_n^2 \right)
\] (2.49)

Therefore

\[
\chi_e(\omega_e) = -1
\] (2.50)

The dielectric function \( \varepsilon \) is given by

\[
\varepsilon(\omega) = 1 - \frac{2 \Omega_e}{\omega} \left( 1 - \frac{\omega_{pe}^2}{(\omega_{pe}^2 - 3 \omega_e^2)16} k^2 l_n^2 \right)
\] (2.51)
Therefore \( \left( \frac{\partial E}{\partial \omega} \right)_{\omega = \omega_c} = \frac{1}{2 \Omega_e} \) \hspace{1cm} (2.52)

using equations (2.49) to (2.52) we get the following growth rate and the threshold power for the scattering of an incident EM wave off a Bernstein mode at high density,

\[
\gamma_0^2 = 2 \nu_0^2 \Omega_e \omega_e \Psi^2 / c^2
\] \hspace{1cm} (2.53)

and

\[
\frac{V_{GT}}{c^2} = \frac{1}{2 \Psi^2} \frac{\Gamma_f}{\omega_0 \omega_c}
\] \hspace{1cm} (2.54)

Next we shall calculate the spatial amplification factor \( \alpha \) for this case. The group velocity \( V_{\nu\nu} \) is given by

\[
V_{\nu\nu} = -3 \Omega_e \omega_{pe} k_{\nu\nu} a_e^2 / 4 (\omega_{pe}^2 - 3 \Omega_e^2)
\] \hspace{1cm} (2.55)

The wave vector \( k_{\nu\nu} \) is calculated to be

\[
k_{\nu\nu} = \sqrt{\frac{8}{3 \Omega_e \omega_{pe}}} \left[ 2 \Omega_e \left( 1 - \frac{3 \Omega_e^2}{\omega_{pe}^2} \right) - \omega_s \left( 1 - \frac{3 \Omega_e^2}{\omega_{pe}^2} \right) - \frac{3 \Omega_e k_{\nu\nu} a_e^2}{8} \right]^{1/2}
\] \hspace{1cm} (2.56)

Substituting equations (2.40), (2.41) and (2.56) in equation (2.42) we get,

\[
k' = \frac{\omega_{pe} \sin^2 \theta / 2}{\ln c^2 k_{\nu\nu} \cos^2 \theta} \left\{ 1 - \frac{3 \Omega_e \omega_{pe}^2 \cos^2 \theta}{\omega_{pe}^2 \sin^2 \theta / 2 (\omega_{pe}^2 - 3 \Omega_e^2)} \right\}
\] \hspace{1cm} (2.57)

Substituting equations (2.38), (2.53), (2.55) and (2.57) in equation (2.44) we get the following expression for the spatial amplification factor,

\[
\alpha = \frac{16 \pi V_0^2 \omega_0 \sin^2 \phi (\omega_{pe}^2 - 3 \Omega_e^2) \cos^2 \theta}{3 c^2 2 \omega_{pe}^2 a_e^2 \sin^2 \theta / 2 \left( 1 - \frac{3 \Omega_e^2 \omega_{pe}^2}{\omega_{pe}^2 \sin^2 \theta / 2 (\omega_{pe}^2 - 3 \Omega_e^2)} \right)}
\] \hspace{1cm} (2.58)
Thus for side scattering we get,

\[ \alpha_{90} = \frac{16\pi}{3} \left( \frac{V_0}{V_e} \right)^2 k_0 L_n \left( \frac{\Omega_e}{\omega_p e} \right)^2 \left( \frac{\omega_p^2 - 3 \Omega_e^2}{\omega_p^2} \right) \sin^2 \phi \]  

(2.59)

And for back scattering we get

\[ \alpha_{180} = \frac{8\pi}{9} \left( \frac{V_0}{V_e} \right)^2 k_0 L_n \left( \frac{\omega_p^2 - 3 \Omega_e^2}{\omega_p^2 \omega_0^2} \right) \sin^2 \phi \]  

(2.60)

2.4.2 Lower hybrid waves:

These electrostatic waves propagate almost perpendicular to the magnetic field \( B_0 \) having frequency lying between electron cyclotron and ion cyclotron frequencies. They are known as hybrid waves because the frequency of these waves depends both on the plasma density and the magnetic field. When the phase velocity of lower hybrid wave along the magnetic field is greater than the electron thermal velocity, i.e. for cold plasma approximation, the dispersion relation for the lower hybrid mode is given by,

\[ \omega^2 = \frac{\omega_p^2}{1 + \omega_p^2 / \Omega_e^2} \left( 1 + \frac{k_0^2 \frac{m_i}{m_e}}{k^2} \right) \]  

(2.61)

where \( \omega_p^i = (4\pi e^2 n_0 / M)^{1/2} \) is the ion plasma frequency and \( k_z \) refers to the wave vector parallel to the magnetic field such that \( k_z \ll k \). Ion and electron susceptibilities are given by

\[ \chi_i(\omega) = -\frac{\omega_p^i}{\omega^2} \quad \chi_e(\omega) = \frac{\omega_p^2}{\Omega_e^2} - \frac{\omega_p^2 k_z^2}{\omega^2 k^2} \]  

(2.62), (2.63)
The dielectric function $\varepsilon$ is given by
\[
\varepsilon(\omega) = 1 - \frac{\omega_{pe}^2}{\omega^2} \beta + \frac{\omega_{pe}^2}{\Omega_e^2},
\tag{2.64}
\]
where $\beta = \frac{m}{M} + \frac{k_z^2}{k^2}$, and we have assumed,
\[
\Omega_e^2 \gg \omega^2 > \Omega_i^2.
\]
From equation (2.64) we have
\[
\left(\frac{\partial \varepsilon}{\partial \omega}\right)_{\omega = \omega_e} = \frac{2}{\omega_{pe} \beta^{1/2}} \left(1 + \omega_{pe}^2 / \Omega_e^2\right)^{3/2},
\tag{2.65}
\]
Substituting equations (2.62), (2.63) and (2.65) into equations (2.18) and (2.19) we get, after some algebra, the following expressions for growth rate and the threshold power for the scattering of an incident EM wave off a lower hybrid wave as,
\[
\gamma_0^2 = \frac{\nu_0^2}{c^2} \frac{e^2}{\omega_0 \omega_{pe}} \left(\frac{k_z^2}{k^2} - \frac{\omega_{pe}^2}{\Omega_e^2}\right)^2 \left(1 + \frac{\omega_{pe}^2}{\Omega_e^2}\right)^{3/2} \left(1 + \frac{k_z^2}{k^2} \frac{m}{M}\right)^{3/2}
\tag{2.66}
\]
\[
\nu_{0T}^2 = \frac{1}{\psi^2 \omega_0 \omega_{pe}} \left(1 + \frac{\omega_{pe}^2}{\Omega_e^2}\right)^{3/2} \left(\frac{k_z^2}{k^2} + \frac{m}{M}\right)^{3/2} \left(\frac{k_z^2}{k^2} - \frac{\omega_{pe}^2}{\Omega_e^2}\right)^{2}
\tag{2.67}
\]
For $\Omega_e^2 \gg \omega_{pe}^2$ and $\frac{k_z^2}{k^2} \frac{m}{M} \gg 1$, equations (2.66) and (2.67) become
\[
\gamma_0^2 = \frac{\nu_0^2}{c^2} \frac{e^2}{\omega_0 \omega_{pe}} \frac{k_z^2}{k^2} \psi^2
\tag{2.66a}
\]
\[
\nu_{0T}^2 = \frac{1}{\psi^2 \omega_0 \omega_{pe}} \frac{k}{k_z}
\tag{2.67a}
\]
Next we shall calculate the amplification factor for the lower hybrid mode. The group velocity $v_{lx}$ is given by

$$v_{lx} = \frac{\partial \omega}{\partial k_{lx}} = - \frac{k_x^2}{k_y} k_{lx} \frac{\omega_p e}{\Omega_e^2} \frac{1}{1 + \omega_p e / \Omega_e^2} \frac{1}{\omega_L^2}$$  \hspace{1cm} (2.68)

where

$$\omega_L^2 = \frac{\omega_p e \beta}{(1 + \omega_p e / \Omega_e^2)}$$  \hspace{1cm} (2.69)

Again from equation (2.61) we have,

$$k_{lx} = \left( \frac{k_x^2 \omega_p e}{\omega^2 (1 + \omega_p e / \Omega_e^2) - \omega_{pi}^2} - k_y^2 \right)^{1/2}$$  \hspace{1cm} (2.70)

Substituting equation (2.40), (2.41) and (2.70) in (2.42) we get,

$$K = \frac{\omega_p e \sin^2 \theta / 2}{\ln c^2 R_{ox}} \left\{ \frac{1}{\cos \theta} - \frac{\omega_n^2}{\omega_p e \sin^2 \theta / 2} \right\}$$  \hspace{1cm} (2.71)

where

$$\eta = 1 - \frac{k_x^2}{k_y^2} \left( \frac{\omega_L^2}{\Omega_e^2} - \frac{m}{M} \right)$$

Now substituting equations (2.38), (2.66), (2.68) and (2.71) into equation (2.44) we get for the amplification factor for the lower hybrid mode as,

$$\alpha = \frac{4\pi v_0^2}{c^2} \left( \frac{k_x^2}{k_y^2} - \frac{\omega_p^2}{\Omega_e^2} \right)^2 \frac{c^2}{C_k} L \omega_p e \sin^2 \phi \frac{k_x^2}{k_y^2} \frac{2}{\omega_p e (1 + \omega_p e / \Omega_e^2) \left( \frac{k_x^2}{k_y^2} + \frac{m}{M} \right)} \left( 1 - \frac{\omega_n^2 \cos \theta \eta}{\omega_p e \sin^2 \theta / 2} \right)$$  \hspace{1cm} (2.72)
For side scattering equation (2.72) becomes,
\[
\alpha_{90^\circ} = \frac{4\pi V_0}{c^2} \frac{2 \left( k_z^2 / k^2 - \omega_p^2 / \Omega_e^2 \right) k_0 L_n}{\left( 1 + \omega_p^2 / \Omega_e^2 \right) \left( k_z^2 / k^2 + m_i M \right)} \frac{\omega_o^2}{\omega_p^2} \frac{k_i^2}{k_z^2} \sin^2 \phi. \tag{2.73}
\]

And for back scattering equation (2.72) becomes,
\[
\alpha_{180^\circ} = \frac{4\pi V_0}{c^2} \frac{2 \left( k_z^2 / k^2 - \omega_p^2 / \Omega_e^2 \right) k_0 L_n}{\left( 1 + k^2 m / k_z^2 M \right)} \frac{\omega_o^4}{\omega_p^4} \frac{k_i^4}{k_z^4} \sin^2 \phi. \tag{2.74}
\]

For \( \Omega_e^2 \gg \omega_p^2 \) and \( k_z^2 M / k^2 m \gg 1 \), we get the following expressions for \( \alpha_{90^\circ} \) and \( \alpha_{180^\circ} \) as
\[
\alpha_{90^\circ} = \frac{4\pi V_0}{c^2} \left( k_0 L_n \right) \frac{\omega_o^2}{\omega_p^2} \sin^2 \phi. \tag{2.75}
\]
and
\[
\alpha_{180^\circ} = \frac{4\pi V_0}{c^2} \left( k_0 L_n \right) \sin^2 \phi. \tag{2.76}
\]

### 2.4.3 Drift waves

When the density inhomogeneity is perpendicular to the magnetic field, then drift waves propagate almost perpendicular to the magnetic field. The density inhomogeneity scale length \( L_n \) is assumed to be greater than the wavelength of perturbation. The phase parallel to the magnetic field is assumed to be greater than the ion thermal velocity \( \Upsilon_i \) but smaller than the electron thermal
velocity $v_e$. The frequency of the drift mode is given by,

$$\omega = \frac{\omega_x \beta'}{2 - \beta'}$$  \hspace{1cm} (2.77)

where

$$\omega_x = -\frac{k_D y T_e}{m \Omega_e L n}$$

$$\beta' = 1 - b i \ ; \ b i = \frac{k^2 v_e^2}{\Omega_e^2} \ll 1.$$  

For this mode the ion and the electron susceptibilities are

$$\chi_i(\omega) = \frac{k^2}{k_D^2} \left(1 - \frac{(\omega + \omega_x) \beta'}{\omega} \right)$$  \hspace{1cm} (2.78)

so that

$$\chi_i(\omega_e) = -\frac{k^2}{k_D^2}$$  \hspace{1cm} (2.79)

and

$$\chi_e(\omega_e) = \frac{k^2}{k_D^2}$$  \hspace{1cm} (2.79a)

The dielectric function is given by

$$\epsilon = 1 + \frac{k^2}{k_D^2} (2 - \beta' - \frac{\omega_x \beta'}{\omega}) \ ; \ k_D^2/k^2 \gg 1$$  \hspace{1cm} (2.80)

Thus we have

$$\left(\frac{\partial \epsilon}{\partial \omega}\right)_\omega = \omega_e = \left(\frac{k^2}{k_D^2}\right) \frac{(2 - \beta')}{\omega_e}$$  \hspace{1cm} (2.81)

Substituting equations (2.79) to (2.81) into (2.18) and (2.19) we get the values of growth rate and threshold power as

$$\gamma_0^2 = \frac{V_0^2}{v_e^2} \frac{\omega_e}{\omega_0} \frac{\omega_e}{\omega_e} \sin^2 \phi$$  \hspace{1cm} (2.82)

and

$$\frac{V_0^2}{\epsilon^2} = \frac{2}{\sin^2 \phi} \frac{\Gamma_e}{\omega_0} \frac{\Gamma_e}{\omega_e} \frac{k_o^2}{k_D^2}$$  \hspace{1cm} (2.83)

Now as the laser produced plasma is expanding with a blow off velocity $V$, which is comparable to the phase velocity
of the drift wave, the frequency of the drift wave will be
doppler shifted and can be written as

\[ \omega' = \frac{\omega_z \beta'}{2 - \beta'} - k_1 x U(r) = \frac{k_1 y v_d \beta'}{2 - \beta'} - k_1 x U(r) \quad (2.8c) \]

where \( v_d = -\frac{Te}{m_0 e L_0 n} \) is the drift velocity. The
group velocity is given by

\[ v_g = -U(r) \quad (2.85) \]

and \( k_1 x = (\omega_1 - k_1 y v_d) / U(r) \quad (2.85) \)

Then \( k' \) is calculated to be

\[ k' = \frac{\omega_z^2 \sin^2 \theta'/2}{L_0 e^2 k_0 x} \left\{ \frac{1}{\cos \theta} + \frac{2\omega_0^2 L_0}{\omega \epsilon^2 L_U} \right\} \quad (2.87) \]

where

\[ L_U = \frac{1}{U} \frac{dU}{dx} \quad (2.88) \]

Substituting equations (2.38), (2.82), (2.85) and (2.87) in
equation (2.44) we get the amplification factor for the
drift wave as

\[ \alpha = 2\pi \frac{V_0^2}{\nu_e^2} \frac{\omega L_0}{\sin^2 \phi} \sqrt{1 + \frac{2 \cos \theta \omega_0^2 L_0}{\omega \epsilon^2 L_U}} \quad (2.89) \]

For side scattering we have

\[ \alpha_{q,0} = 4\pi \frac{V_0^2}{\nu_e^2} (k_0 L_0) \sin^2 \phi \quad (2.50) \]

and for back scattering we get,

\[ \alpha_{180^0} = 2\pi \frac{V_0^2}{\nu_e^2} (k_0 L_0) \frac{\omega \epsilon^2}{\omega_0^2} \sin^2 \phi \quad (2.51) \]
2.5 Conclusion and discussion.

In this chapter we have calculated the threshold powers, maximum growth rates and amplification factors for scattering of an EM wave by three different electrostatic modes in a magnetized plasma. Our results are summarised in Table 2.1. Recently there has also been other calculations, done independently, in the same general area. Yu et al\(^{(13)}\) have calculated growth rates and threshold powers for scattering of an EM wave by upper hybrid modes, lower hybrid modes and drift modes. Their results for lower hybrid and drift modes agree with those of our's. Lee\(^{(14)}\) considered a two ion species plasma and calculated growth rates and threshold powers for backscattering of an EM wave from a ion ion hybrid wave and found that the threshold was much greater, and the growth rate much smaller, compared to stimulated scattering from upper and lower hybrid waves. Lee\(^{(15)}\) also considered stimulated scattering of EM waves of circular polarization incident on a magnetized plasma from ion waves propagating parallel to the external magnetic field. He\(^{(16)}\) also studied the stimulated scattering of EM ordinary waves from electron plasma waves at the upper hybrid frequency and found that the threshold intensity required for stimulated scattering was higher in a magnetized plasma than in an unmagnetized plasma and that it increased with increasing magnetic field.
In our calculations we have considered scattering from only one ion mode viz. drift modes. It is to be noted that the growth rates for scattering off most low frequency ion modes are comparable to each other. Another interesting feature is the very low threshold field for the hybrid and the Bernstein modes because of their weak damping. These modes however require long parallel wavelengths and might, therefore, be prevented by finite geometry effects.

It therefore appears unjustified to ignore the evolution of magnetized modes in laser plasma situations - especially while considering the non-linear saturation levels of the scattered electromagnetic waves. In conclusion we suggest that the inclusion of spontaneously generated magnetic field is essential for a realistic estimate of the stimulated scattering of laser beams in pellet fusion systems.
### TABLE 2.1 - Growth rates, threshold powers, amplification factors for scattering of an EM pump wave off different ES modes.

<table>
<thead>
<tr>
<th>Electrostatic Mode</th>
<th>Threshold Power ( V_0 \gamma / c^2 )</th>
<th>Maximum Growth Rate ( \gamma_0 )</th>
<th>Spatial Amplification factor ( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drift Wave</td>
<td>( \frac{2}{\sin^2 \phi} \frac{\Gamma \gamma}{w_0 \omega_0 \omega_e \kappa^2} )</td>
<td>( \frac{V_0^2 \omega_e \omega_2 \sin^2 \phi}{\psi^2 \omega_0 \sigma_e} )</td>
<td>( \frac{4\pi V_0^2 (\Omega_0 n \eta)}{V_e^2} \frac{\omega_{pe}^2}{\sin^2 \phi} )</td>
</tr>
<tr>
<td>Electron Bernstein Modes, ( n = 1 )</td>
<td>( \frac{1}{\psi^2} \frac{\Gamma \gamma}{w_0 \omega_0 \omega_e} )</td>
<td>( \frac{V_0^2 \omega_2 \omega_0 \Omega_e}{c^2 \psi^2 \omega_e} )</td>
<td>( \frac{2\pi V_0^2 (\Omega_0 n \eta)}{V_e^2} \frac{\omega_{pe}^2}{\sin^2 \phi} )</td>
</tr>
<tr>
<td>(i) ( \omega_{pe}^2 \ll \Omega_e^2 )</td>
<td>( \frac{2}{\psi^2} \frac{\Gamma \gamma}{w_0 \omega_0 \omega_e} )</td>
<td>( \frac{2V_0^2 \omega_0 \Omega_e}{c^2 \psi^2} )</td>
<td>( \frac{16\pi (\Omega_0 n \eta)}{3} \frac{\omega_{pe}^2 - \Omega_e^2}{\omega_e \omega_0 / \omega_{pe}} )</td>
</tr>
<tr>
<td>(ii) ( \omega_{pe}^2 \gg \Omega_e^2 )</td>
<td>( \frac{2}{\psi^2} \frac{\Gamma \gamma}{w_0 \omega_0 \omega_e} )</td>
<td>( \frac{2V_0^2 \omega_0 \Omega_e}{c^2 \psi^2} )</td>
<td>( \frac{8\pi (\Omega_0 n \eta)}{q} )</td>
</tr>
</tbody>
</table>

(\( \psi \) is the wave number)
| Lower Hybrid Waves | | | | |
|---------------------|-----------------|-----------------|-----------------|
| $\frac{1}{\Psi^2} \frac{\Gamma \Pi_c}{\omega \omega_p e} \frac{A^{3/2}}{\psi^2}$ | $\frac{V_0}{\omega_0 \omega_p e} \frac{\psi^2}{\psi^{3/2}}$ | $\frac{4\pi V_0^2 \Omega_e^2}{c^2 A} (\Omega_c) \frac{\omega_0^2}{\omega_p e^2} \frac{\psi^2}{\psi^{3/2}} \frac{\sin^2 \phi}{k_z^2}$ | $\frac{4\pi V_0^2 \Omega_c}{c^2} \frac{\omega_0^2}{\omega_p e^2} \frac{\psi^2}{\psi^{3/2}} \frac{\sin^2 \phi}{k_z^2}$ |
| $\frac{\Omega_e^2 \gg \omega_p^2}{k_z^2 >> \frac{m}{M}}$ | $\frac{1}{\Psi^2} \frac{\Gamma \Pi_c}{\omega \omega_p e} \frac{k_z}{k_z^2} \frac{\psi^2}{\psi^{3/2}}$ | $\frac{4\pi V_0^2 \Omega_c}{c^2 \omega_0^2} \frac{\omega_p e^2}{\omega_p e^2} \frac{\psi^2}{\psi^{3/2}} \frac{\sin^2 \phi}{k_z^2}$ | $\frac{4\pi V_0^2 \Omega_c}{c^2 \omega_0^2} \frac{\omega_p e^2}{\omega_p e^2} \frac{\psi^2}{\psi^{3/2}} \frac{\sin^2 \phi}{k_z^2}$ |
| Remarks | $L_n = \frac{1}{\omega_p e} \frac{d}{dx} \frac{\psi^2}{\psi^{3/2}}$ | $L_U = \frac{1}{U} \frac{dU}{dx}$ | $U$ is the blow off velocity. |
| | $P = (\omega_p e - 3\Omega_e^2) \frac{V_0^2}{\omega_p^2 \omega_p e^2} \frac{\sin^2 \phi}{V_e^2}$ | $Q = \frac{1}{k_z^2 \omega_p e} \frac{\omega_p e^2}{\omega_p e^2} \frac{\psi^2}{\psi^{3/2}} \frac{\sin^2 \phi}{k_z^2}$ | $A = \left(1 + \frac{\omega_p e^2}{\Omega_e^2} \right) \frac{k_z^2}{k_z^2 + m} \frac{m}{M}$ |
References


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