CHAPTER IV

WIDTHS OF DEFORMED HARTREE-FOCK STATES

A. INTRODUCTION

In this chapter we discuss the application of the concept of width as a measure for the goodness of Hartree-Fock states of deformed nuclei. Only those nuclei for which the assumption of axial symmetry seems to work well are studied here. This study forms a natural extension of our study of the goodness of HF states of spherical nuclei which was presented in Chapter II. Work on lines similar to ours described in this section has been reported by a few authors\(^1\)\(^-\)\(^4\) earlier. Our purpose here in presenting this study on widths of deformed nuclei is two fold: first, for the sake of completeness of our discussion on widths it is necessary to include in our study a discussion on the widths of deformed nuclei. Secondly, we would like to present here some results of constrained HF and projection calculations for Preedom-Wildenthal and K+12FP interactions and also for a Schematic interaction which have not been reported so far in literature.

In section B of this chapter we discuss the concept of intrinsic wave function and the question what the width
of a deformed intrinsic state tells us. We point out there that the concept of width as a measure for goodness (as discussed in Chapter II) is useful only for those approximate wave functions which have exactly the same symmetries as the real eigenstates of the Hamiltonian. As for the concept of width applied to deformed intrinsic states it has a different meaning and use which we discuss in some detail in the same section. In Section C we describe a schematic interaction for the 0d-1s shell built on considerations of certain symmetries and energy systematics. This interaction is so constructed that it is exactly soluble and it gives exact rotational spectrum. By virtue of these properties this interaction is useful in testing HF approximation together with angular momentum projection. In Section D we describe HF plus angular momentum projection calculations for N=Z even-even nuclei in the 0d-1s shell using the schematic, the Freedom-Wildenthal and the K+12 FP interactions. The results of these calculations are discussed next.

B. DEFORMED HARTREE-FOCK STATES AND WIDTHS

We have already reviewed in Chapter II the Hartree-Fock method for obtaining approximate states. In this
section we consider its application to non closed shell nuclei.

It is well-known that there is a large class of nuclei which exhibit rotational properties at low excitation energies. Many attempts have been made to understand this rotational behaviour of highly deformed nuclei both from the phenomenological and the microscopic view points. The phenomenological model approach consists in using a moment of inertia to describe the states when the spectrum is rotational. On the other hand, the microscopic theories are based on the assumption that one can describe the states of a rotational band starting from a shell model single particle well plus a two-body interaction and then using Hartree-Fock or Hartree-Fock-Bogoliubov theories to obtain the 'best' approximations to the intrinsic wave function and finally employing a projection technique to project out the various good angular momentum states of the rotational band. It is this latter approach using the HF prescription that we have used in this section to study some nuclei in 0d-1s shell under the assumption of axial symmetry.

The concept of an intrinsic wave function has proved to be very useful in the theory of rotational nuclei. The
Idea here is that in a highly deformed nucleus the low lying energy states can be considered compositely by a single deformed wave function called the intrinsic wave function and the states of good angular momentum projected from it describe well the actual eigenstates. Let $\Phi_K$ denote the intrinsic wave function and $\Psi_{MK}^J$ the good angular momentum states projected from it. Then

$$
\Psi_{MK}^J = \frac{2J+1}{c_{JK} \delta \pi^4} \int d\Omega \ R_{MK}^J(\Omega) \ R(\Omega) \ \Phi_K
$$

IVB(1)

Here $R(\Omega)$ is the operator of rotation through Euler angles $\Omega$, $D_{MK}^J(\Omega)$ is a representation of $R(\Omega)$, $J$ is the angular momentum and $M$ its projection in the laboratory frame, $K$ is the projection of $J_Z$ on the $Z$-axis of the intrinsic frame, and $c_{JK}$ is the normalization constant such that

$$
\Phi_K = \sum_J c_{JK} \Psi_{MK}^J
$$

IVB(2)

with $\Phi_K$ and $\Psi_{MK}^J$ normalized. It is assumed that the projected states $\Psi_{MK}^J$ are eigenfunctions of the Hamiltonian $H$: 
The HF procedure has been extensively used to obtain approximate intrinsic wave functions, i.e., one approximates $\Phi_K$ by a Slater determinant $\Phi^\text{HF}_K$ having the lowest energy:

$$\delta \left( \frac{\langle \Phi_K | H | \Phi_K \rangle}{\langle \Phi_K | \Phi_K \rangle} \right) = 0$$  

where $\delta$ denotes first order variations in $\Phi_K$. In general, the intrinsic wave function $\Phi^\text{HF}_K$ obtained by a HF variational calculation, will not be an eigenfunction of the Hamiltonian $H$. This is because, first, the HF determinant being an independent particle wave function is only an approximation to the correlated intrinsic wave function supposing that it exists. Secondly, a deformed intrinsic wave function cannot be used as an approximation to the eigenfunctions of $H$ which have good $J$ symmetry. The third reason is that even if Eq. IVB(3) is true, the eigenvalues $E_\ell$ are in general nondegenerate. Therefore, a linear combination of the $\Psi_{MK}$ will not be an eigenfunction of $H$. 

$$H |\Psi^J_{MK}\rangle = E_J |\Psi^J_{MK}\rangle$$  

IVB(3)
The eigenvalues can be made degenerate by subtracting from the Hamiltonian $H$ a polynomial operator $P(J^2)$ of $J^2$, where the polynomial $P(J(J+1))$ gives the dependence of $E_J$ on $J$. Under the assumption that the states are rotational we have $P(J^2) \sim \alpha J^2$ where $\alpha$ is a parameter. We can write\(^1\)\(^-\)\(^2\)

$$H_\alpha = H - \alpha J^2 \quad \text{IVB}(5)$$

Now by a suitable choice of the parameter $\alpha$ we can make the $J$-states of the band degenerate, i.e.

$$H_\alpha |\Psi_{MK}^J\rangle = E_\alpha |\Psi_{MK}^J\rangle \quad \text{IVB}(6)$$

and

$$H_\alpha |\Phi_\alpha\rangle = E_\alpha |\Phi_\alpha\rangle \quad \text{IVB}(7)$$

Eq.IVB(7) follows from the fact that a linear combination of $\Psi_{MK}^J$ is also an eigenfunction of $H_\alpha$. In Eq.IVB(5) the parameter $\alpha$ is related to the moment of inertia $I$ of the band according to

$$\alpha = \frac{\hbar^2}{2I} \quad \text{IVB}(8)$$
It is not essential that the intrinsic wave function should be an eigenfunction of $H$ in order that its projections be eigenfunctions of $H$. But if $\Phi_\alpha$ is an eigenfunction of $H_\alpha$ then we have that $\Psi_{MK}^J(\alpha)$ are eigenfunctions of $H$. We can see this as follows: we have

$$\Phi_\alpha^K = \sum_J c_{JK}^{\alpha} \Psi_{JK}^{\alpha}$$

$$\Rightarrow \quad H \Phi_\alpha^K = \sum_J c_{JK}^{\alpha} (H_\alpha + \alpha J^2) \Psi_{JK}^{\alpha}$$

$$= \sum_J c_{JK}^{\alpha} [E^{\alpha}_J + \alpha J(J+1)] \Psi_{JK}^{\alpha}$$

Expanding $\Phi_\alpha^K$ on the left side we get

$$\sum_J c_{JK}^{\alpha} H \Psi_{JK}^{\alpha} = \sum_J c_{JK}^{\alpha} [E^{\alpha}_J + \alpha J(J+1)] \Psi_{JK}^{\alpha}$$

from which it follows that

$$H \Psi_{JK}^{\alpha} = [E_{JK}^{\alpha} + \alpha J(J+1)] \Psi_{JK}^{\alpha}$$

The advantage in considering $H_\alpha$ instead of $H$ is that it justifies the use of HF variational procedure to find $\Phi_\alpha$ since in that case Eq. IVB(6) will be satisfied.
To determine the best $\mathcal{F}_\alpha$ and the best value of the parameter $\alpha$, we first define the variance of $\mathcal{F}_\alpha$ with respect to the Hamiltonian $H_\alpha$:

$$
\sigma^2(\mathcal{F}_\alpha) = \langle \mathcal{F}_\alpha | H_\alpha^2 | \mathcal{F}_\alpha \rangle - |\langle \mathcal{F}_\alpha | H_\alpha | \mathcal{F}_\alpha \rangle|^2
$$

IVB(9)

Then in the ideal situation where $E_J = E_0 + \alpha J(J+1)$ and the good-$J$ states projected from $\mathcal{F}_\alpha$ are the corresponding eigenstates we would have $\sigma^2(\mathcal{F}_\alpha) = 0$. In practice, however, the best one can do is to minimize $\sigma^2(\mathcal{F}_\alpha)$ so as to get the best $\alpha$ and $\mathcal{F}_\alpha$. The procedure then is as follows: We choose a certain value for $\alpha$, then carry out HF variational calculation to obtain the intrinsic wave function $\mathcal{F}_\alpha$ and then we evaluate its width (Eq. IVB(9)). We repeat this procedure varying the parameter $\alpha$ until the minimum value of $\sigma^2(\mathcal{F}_\alpha)$ is found. Finally we project out good-$J$ states from this minimum width intrinsic wave function $\mathcal{F}_\alpha$. The above method has the further advantage that it gives bounds for errors of the eigenvalues and thus indicates whether the assumption of axial symmetry is good or not (see Chapter II, Section A, for the discussion on bounds).
The concept of width of an intrinsic state introduced in Eq.IVB(9) should be clearly distinguished from the width of an approximate state of the Hamiltonian \( H \) introduced in Chapter II. This latter width is a measure of goodness of an approximation to an eigenstate of \( H \) and this approximate state has all the exact symmetries of the actual eigenstate. Only, it is an approximation. This was the situation with the spherical Hartree-Fock states whose widths we evaluated and discussed in Chapter II and further minimized in Chapter III. In this chapter we are dealing with intrinsic HF states which are deformed and as such these can never be used as approximations to the actual eigenstates of the Hamiltonian which have good \( J \).

The width defined in Eq.IVB(9) gives the error we have made in approximating the deformed intrinsic wave function \( \Phi \) by the determinant \( \Phi_\alpha \) and to find the best \( \alpha \) and \( \Phi_\alpha \), we have to minimize this error. However, we can apply the concept of width as defined in Chapter II to the states of good-\( J \) projected from the intrinsic wave function i.e. we may define

\[
\sigma^2(\Psi_{MK}^J) = \langle \Psi_{MK}^J | H^2 | \Psi_{MK}^J \rangle - \langle \Psi_{MK}^J | H | \Psi_{MK}^J \rangle^2
\]

IVB(10)
It is possible to relate these variances for projected $J$ states to the variance of the intrinsic wave function $\Phi_K$ defined with respect to $H$:

$$\sigma^2(\Phi_K) = \langle \Phi_K | H^2 | \Phi_K \rangle - \langle \Phi_K | H | \Phi_K \rangle^2 \quad \text{IVB(11)}$$

With the variance defined as in Eq. IVB(10) for a projected $J$-state it is then straightforward to consider its application to the deformed nuclei in exactly the same way as was done for spherical nuclei in Chapters II and III. However, these are aspects which we have not considered here.

C. A SCHEMATIC INTERACTION FOR THE Od-1s SHELL

We described in the previous section a method for obtaining good approximations to the intrinsic wave function for highly deformed nuclei. We describe in this section a schematic Hamiltonian in the Od-1s shell which when used in conjunction with the above method yields very good intrinsic states. This interaction has been so constructed that it is exactly soluble and gives pure rotational spectrum for nuclei in Od-1s shell. Hence we can use this interaction to illustrate the method we have discussed in the previous section.
The interaction part of the schematic Hamiltonian is chosen to be of the following form:

\[ V_2 = \sum_{ij} V_{ij} = A \binom{n}{2} + B G(SU(6)) + C G(SU(3)) \]

\[ + DL^2 + E S^2 + F_1 J^2 + F_2 T^2 \]

where \( A, B, C, D, E, F_1, F_2 \) are coefficients to be determined and \( n \) is the number operator. The Casimir operators \( G(SU(6)) \) and \( G(SU(3)) \) have the well-known forms:

\[ G(SU(6)) \equiv \mathcal{M} = \sum_{ij} P_{ij} \], the space-exchange Majorana operator

\[ G(SU(3)) = \frac{3}{4} \mathcal{L}^2 + \frac{1}{4} \mathcal{Q} \cdot \mathcal{Q} \]

where \( \mathcal{L} \) is the orbital angular momentum operator and \( \mathcal{Q} \) is the quadrupole operator. It is clear that \( V_2 \) preserves space symmetry, \( SU(3) \) symmetry, and \( L, S, J, T \) symmetries.

Now the standard way of labelling the representations of \( U(6) \) and \( SU(3) \) is as follows:
Representation of $U(6)$: $[f] = [f_1 \ldots f_6]$

Representation of $SU(3)$: $(\lambda \mu^i)$

The expectation values of the Casimir operators in these representations are given by

$$\langle G(SU(6)) \rangle [f] = \frac{1}{2} \sum_{i=1}^{6} f_i (f_i - 1)$$  \hspace{1cm} \text{IVC(4)}$$

$$\langle G(SU(3)) \rangle [\lambda \mu] = (\lambda + \mu) (\lambda + \mu + 3) - \lambda \mu$$  \hspace{1cm} \text{IVC(5)}$$

The coefficient $A$ in Eq.IVC(1) represents the average nucleon nucleon interaction. Coming to the one-body part of the Hamiltonian this has the following form:

$$V_{1}\text{(one-body)} = 10C+D \ 1(1+1) + \frac{3}{4} E+F_1 j(j+1)+F_2 s(s+1)$$  \hspace{1cm} \text{IVC(6)}$$

The parameters $C$, $D$, $E$, $F_1$ and $F_2$ are the same as those in Eq.IVC(1).

The parameters $A$, $B\ldots F_2$ in Eq.IVC(1) are chosen so that:

i) the binding energies of the nuclei come out reasonably well.
ii) the centroid spacings of SU(6) representations correspond to those given by a 'realistic' two-body interaction.

iii) the centroid spacings of SU(3) representations contained in the lowest SU(6) representation is roughly what one expects from the assignments of SU(3) representations to levels in $^{20}$Ne.

iv) the coefficients of $L^2$, $S^2$, $J^2$ and $T^2$ are chosen in such a way that the

(a) states of lower $J$ lie lower in energy
(b) for ($4n+2$) nuclei the $T< states in the maximum space symmetry representation are lower in energy by about 2 MeV than $T>$ states.

We give below several useful numbers for this interaction:

(i) $E(2^+)-E(0^+)=1.5$ MeV for all even even N=Z nuclei

(ii) $E_c([f]_{\text{max-1}})-E_c([f]_{\text{max}}) = +7.2$ MeV

Here $[f]_{\text{max}}$ is the leading SU(6) representation and $[f]_{\text{max-1}}$ the next lower symmetry.

(iii) $E_c(\lambda\mu = (42)) - E_c(\lambda\mu = (80)) = 8.4$ MeV
From the above mentioned considerations it turns out that the coefficients \( A, B, \ldots, F_2 \) in Eq. IVC(1) have the following values:

\[
\begin{align*}
A &= -2.00 \text{ for } ^{20}\text{Ne} \\
B &= -1.8 \\
C &= -0.2 \\
D &= -0.35 \\
E &= 1.08 \\
F_1 &= 0.6 \\
F_2 &= 2.67
\end{align*}
\]

Since \( A \) varies from nucleus to nucleus the first term in Eq. IVC(1) contains many-body effects. Substituting the values of the coefficients from Eq. IVC(8) we get

\[
V_2 = \left( \frac{n}{2} \right) -1.8 G(SU(6)) -0.2 G(SU(3)) -0.35 L^2 +1.08 S^2 \\
+ 0.6 J^2 + 2.67 T^2
\]

and the one-body energies (Eq. IVC(6)) are:
Note the queer sequence of d-levels. For a given nucleus it is easy to calculate the energies of its states from a knowledge of the representations \([\mathbf{f}]\) and \((\lambda^\mu)\) and the values of the quantum numbers \(L, S, J\) and \(T\). The wave functions for the states follow from the representations used. Thus this interaction is exactly soluble and yields \(J(J+1)\) spectrum for the nuclei.

D. NUMERICAL RESULTS AND DISCUSSION

In this section we describe some of our HF and width calculations and results for \(N=Z\) even even nuclei in the \(0d-1s\) major shell using the method described in Section B.

We show in Table IV-1 the results of deformed HF calculations in \(0d-1s\) shell for the nuclei \(^{20}\text{Ne}, ^{28}\text{Si}\) and \(^{36}\text{Ar}\). The assumption of axial symmetry has been made. These calculations have been done using the Schematic \(^5\) interaction described in the previous section. In Table IV-1 the second column shows \(\chi\) the moment of inertia.

\[
\begin{align*}
\mathcal{E}(0d_{5/2}) &= 3.96 \text{ MeV} \\
\mathcal{E}(1s_{1/2}) &= 1.26 \text{ MeV} \\
\mathcal{E}(0d_{3/2}) &= 0.96 \text{ MeV} \\
\end{align*}
\]
parameter (Eq.IV B(8)), the third column gives the energy of the intrinsic state \( \Phi_\alpha \) and the fourth column the expectation value of the quadrupole operator in the state \( \Phi_\alpha \).

The \( \sigma^2 \) in the fifth column is the variance of the HF state with the Hamiltonian \( H_\alpha \) (Eq.IV B(9)). In the last column is shown the HF gap which is defined as the energy difference between the highest occupied and the lowest unoccupied levels in the HF single-particle spectrum.

We see that the schematic interaction gives large values for widths for the nuclei listed when an unmodified Hamiltonian is used (\( \alpha = 0 \)). The HF gaps are twice as large as what one usually finds (\( \sim 8 \text{ MeV} \)) with realistic interactions in the \( 0d-1s \) shell (see Tables IV-2 and IV-3). We find that the widths and HF gaps of \(^{20}\text{Ne} \) and \(^{36}\text{Ar} \) are the same which indicates that there is a particle-hole symmetry for widths and HF gaps with the schematic interaction. Coming to the discussion on results with the modified Hamiltonian \( H_\alpha \), we observe that the widths of HF states of all the nuclei shown are zero. Thus we have here intrinsic states which represent bands of collapsed J-states and hence these intrinsic states as well as their projected states are eigenstates of the Hamiltonian \( H \). Further we note that the moment of inertia parameter \( \alpha \) is the same for all the three nuclei. This value of \( \alpha = 0.25 \) is considerably
larger than the values of $\alpha$ one usually finds for $^{20}\text{Ne}$ and $^{28}\text{Si}$. We show in Table IV-4 the spectrum of the ground state band of J-states. The second column in Table IV-4 shows the energies of the J-states projected from the $\alpha = 0$ HF solution and the third column shows the same for $\alpha = 0.25$ HF solution. The last column gives the exact spectrum (see the previous section). We see that the spectrum projected from $\alpha = 0$ HF intrinsic state is already close to being a $J(J+1)$ spectrum. With the modified Hamiltonian $H_\alpha$ ($\alpha = 0.25$) we get a pure $J(J+1)$ spectrum which is identical with the exact spectrum.

Let us now consider the results of our calculations done using the Preedom-Wildenthal\textsuperscript{6} interaction. This two-body interaction is a modification of the well-known Kuo\textsuperscript{8} interaction in the Od-1s shell and gives good fit to the experimental data for nuclei in the mass region $A=18-22$. The complete Hamiltonian consists of 63 two-body matrix elements plus the 3 single particle energies viz. $(0d_{5/2}) = 0.0, (1s_{1/2}) = 0.87 \text{ MeV}$ and $(0d_{3/2}) = 5.08 \text{ MeV}$. We show in Table IV-2 results of HF calculations for $^{20}\text{Ne}$, $^{28}\text{Si}$ and $^{36}\text{Ar}$. Considering first the results of unconstrained calculations ($\alpha = 0$) we see that the HF solutions of $^{20}\text{Ne}$, $^{28}\text{Si}$ are quite stable as indicated by their large HF gaps. The HF gap in $^{36}\text{Ar}$ is small. The width in all the
cases is fairly large. We next investigate the contribution to the widths coming from the non-degeneracy of the J-levels in the spectrum.

Table IV-2 also shows the results of constrained HF calculations ($\alpha \neq 0$). We find that the widths are considerably reduced when $\alpha$ is varied. But the minimum widths are not close to zero which may be taken as an indication that the assumption of $J(J+1)$ sequence for the HF projected spectrum is not good and/or that the Slater determinantal HF description of the intrinsic states is inadequate.

In the case of all the nuclei listed (Table IV-2) the HF gaps for the two HF solutions i.e. for $\alpha = 0$ and $\alpha \neq 0$ turn out to be nearly equal. We next show in Tables IV-5 and IV-6 the J-projected spectra of $^{20}\text{Ne}$, $^{28}\text{Si}$ and $^{36}\text{Ar}$ using the HF solutions shown in Table IV-2. The absolute energies of the $0^+$ states are shown in brackets in Tables IV-5 and IV-6. The energies of the excited states of the ground state band are shown relative to the $0^+$ state which is taken as zero. The projected ground state of $^{20}\text{Ne}$ from the HF solution for $\alpha = 0.177$ shows a definite improvement over the one projected from $\alpha = 0$ solution. For comparison the experimental numbers are shown in the last column of Table IV-5. There is a similar marked improvement in the case of $^{28}\text{Si}$ and $^{36}\text{Ar}$ when the
modified Hamiltonian is used (Table IV-6).

The results of calculations done using the $K+12FP'$ interaction are presented in Tables IV-3, IV-7 and IV-8. The $K+12FP$ Hamiltonian is a modification of the Kuo effective Hamiltonian for the $0d-1s$ shell and has been adjusted to fit the ground state binding energies and level excitations in the nuclei $A=17-22$. The adjusted parameters in this Hamiltonian are the three single particle energies plus nine two-body matrix elements involving only $0d_{5/2}$ and $1s_{1/2}$ orbits. All other two-body matrix elements are held fixed at the Kuo$^8$ values. The $K+12FP$ results for $^{20}\text{Ne}$ are similar to the results obtained with Freedom-Wildenthal (PW) interaction. The moment of inertia is the same as in the case of PW interaction but the minimum width is larger than the PW width. If we look at the projected spectrum of $^{20}\text{Ne}$ (Table IV-7) the $\alpha = 0.177$ HF projected spectrum is definitely better than the ordinary ($\alpha = 0$) HF projected spectrum except for the $4^+$ state. In the case of $^{28}\text{Si}$ the lowest HF solution is oblate and has a large width. When $\alpha$ is varied the width reduces substantially but the absolute magnitude of the minimum width still remains large. The projected spectrum of $^{28}\text{Si}$ (Table IV-7) obtained from $\alpha = 0.16$ HF intrinsic state shows a slight improvement
over the $\alpha = 0$ spectrum in that with $\alpha = 0.16$, the energy of the $0^+$ state is lowered a little. The large intrinsic width with $\alpha = 0.16$ and only slight improvement in the $0^+$ energy with $\alpha = 0.16$ essentially imply that the single determinant assumption does not work well for $^{28}$Si with the K+12FP Hamiltonian. In the case of $^{36}$Ar also (Tables IV-3 and IV-8) we find that $\alpha = 0$ HF projected ground state is better than the projected $J = 0$ state obtained from the constrained HF solution ($\alpha = 0.23$).
Some properties of deformed HF states of even-even $N=Z$ $0d-1s$ shell nuclei with the Hamiltonian $H = H - \alpha J^2$.

Interaction: Schematic

S.P. Energies (MeV): $(0d_{5/2})=3.96$, $(1s_{1/2})=1.26$, $(0d_{3/2})=0.96$.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$\alpha$</th>
<th>Energy (MeV)</th>
<th>$\langle Q^{(2)} \rangle$</th>
<th>$\sigma^2$</th>
<th>HF gap (MeV)</th>
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<tbody>
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<td>12.56</td>
<td>16.97</td>
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<td></td>
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<tr>
<td></td>
<td>0.0</td>
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<td>18.47</td>
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<tr>
<td>$^{28}\text{Si}$</td>
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<td>+24.00</td>
<td>0.0</td>
<td>18.41</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>-229.19</td>
<td>-15.72</td>
<td>12.59</td>
<td>16.97</td>
</tr>
<tr>
<td>$^{36}\text{Ar}$</td>
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<td>-228.96</td>
<td>-16.00</td>
<td>0.0</td>
<td>17.34</td>
</tr>
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### Table IV-2

Some properties of deformed HF states of even-even N=Z 0d-1s shell nuclei with Hamiltonian $H_H = H - \alpha J^2$.

**Interaction:** Freedom-Wildenthal.

S.P. Energies (MeV): $(0d_{5/2})=0.0$, $(1s_{1/2})=0.87$, $(0d_{3/2})=5.08$.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$\xi$</th>
<th>Energy (MeV)</th>
<th>$\langle Q_{20} \rangle$</th>
<th>$\sigma^2$</th>
<th>HF gap (MeV)</th>
</tr>
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</tbody>
</table>
TABLE IV-3

Some properties of deformed HF states of even-even N=Z 0d-1s shell nuclei with the Hamiltonian $H_{\alpha} = H - \alpha J^2$.

Interaction: K+12 FP

S.P. Energies (MeV): $(0d_5/2)=0.0$, $(1s_{1/2})=0.90$, $(0d_{3/2})=5.75$.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$\alpha$</th>
<th>Energy (MeV)</th>
<th>$\langle Q20 \rangle$</th>
<th>$\sigma^2$</th>
<th>HF gap (MeV)</th>
</tr>
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<td>$^{20}\text{Ne}$</td>
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<td>+15.56</td>
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<td>-23.00</td>
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<td>-22.88</td>
<td>13.56</td>
<td>9.62</td>
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<td>15.80</td>
<td>7.52</td>
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<td></td>
<td>0.23</td>
<td>-177.57</td>
<td>-15.58</td>
<td>5.39</td>
<td>8.77</td>
</tr>
</tbody>
</table>
TABLE IV-4

HF projected spectra of $^{20}$Ne with the Hamiltonian $H=H_0-\alpha J^2$

Interaction: Schematic.

S.P. Energies (MeV): $(0d_{5/2})=3.96$, $(1s_{1/2})=1.26$, $(0d_{3/2})=0.96$

\(\kappa=0\) (Prolate solution) (All energies are in MeV)

<table>
<thead>
<tr>
<th>(J^\pi)</th>
<th>(\alpha=0) (E_J) (HF)</th>
<th>(\alpha=0.25) (E_J) (HF)</th>
<th>(E_J) (Exact)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0(^+)</td>
<td>-39.97</td>
<td>-40.40</td>
<td>-40.40</td>
</tr>
<tr>
<td>2(^+)</td>
<td>-38.56</td>
<td>-38.90</td>
<td>-38.90</td>
</tr>
<tr>
<td>4(^+)</td>
<td>-35.21</td>
<td>-35.40</td>
<td>-35.40</td>
</tr>
<tr>
<td>6(^+)</td>
<td>-29.82</td>
<td>-29.90</td>
<td>-29.90</td>
</tr>
<tr>
<td>8(^+)</td>
<td>-22.15</td>
<td>-22.28</td>
<td>-22.40</td>
</tr>
</tbody>
</table>
**TABLE IV-5**

HF projected spectra of $^{20}\text{Ne}$ with the Hamiltonian $H_{\omega} = H - \alpha J^2$.


S.P. Energies (MeV): (0d$_{5/2}$)$^0=0.0$, (1s$_{1/2}$)$^0=0.87$, (0d$_{3/2}$)$^0=5.08$

---

<table>
<thead>
<tr>
<th>$K=0$ (Prolate solution) (All energies are in MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J^\pi$</td>
</tr>
<tr>
<td>$E_J$(HF)</td>
</tr>
<tr>
<td>$0^+$</td>
</tr>
<tr>
<td>$2^+$</td>
</tr>
<tr>
<td>$4^+$</td>
</tr>
<tr>
<td>$6^+$</td>
</tr>
<tr>
<td>$8^+$</td>
</tr>
</tbody>
</table>

\textsuperscript{a}Ref. 9.
TABLE IV-6

HF Projected spectra with the Hamiltonian $H_\alpha = H - \alpha J^2$
Interaction: Preedom-Wildenthal
S.P. Energies (MeV): $(0d_{5/2}) = 0.0$, $(1s_{1/2}) = 0.87$, $(0d_{3/2}) = 5.08$

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$^{28}\text{Si}$</th>
<th>$^{36}\text{Ar}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>K=0 (Oblate solution)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^\pi J$</td>
<td>$E_j$(HF)</td>
<td>$E_j$(HF)</td>
</tr>
<tr>
<td>$0^+$</td>
<td>0.00(-86.37)</td>
<td>0.00(-86.62)</td>
</tr>
<tr>
<td>$2^+$</td>
<td>1.60</td>
<td>1.58</td>
</tr>
<tr>
<td>$4^+$</td>
<td>4.72</td>
<td>4.72</td>
</tr>
<tr>
<td>$6^+$</td>
<td>9.46</td>
<td>9.50</td>
</tr>
<tr>
<td>$8^+$</td>
<td>15.40</td>
<td>15.63</td>
</tr>
</tbody>
</table>

a) Ref.10
b) Ref.11
**TABLE IV-7**

HF projected spectra with the Hamiltonian $H' = H - \alpha J^2$

Interaction: $K+12FP$

S.P. Energies (MeV): $(0d_{5/2})=0.0$, $(1s_{1/2})=0.90$, $(0d_{3/2})=5.75$

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$^{20}\text{Ne}$</th>
<th>$^{28}\text{Si}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K=0$(Prolate solution)</td>
<td>$K=0$(Oblate solution)</td>
</tr>
<tr>
<td>$J^\pi$</td>
<td>$\alpha=0$</td>
<td>$\alpha=0.177$</td>
</tr>
<tr>
<td></td>
<td>$E_J$(HF)</td>
<td>$E_J$(HF)</td>
</tr>
<tr>
<td>0$^+$</td>
<td>0.00(-22.66)</td>
<td>0.00(-22.81)</td>
</tr>
<tr>
<td>2$^+$</td>
<td>1.27</td>
<td>1.37</td>
</tr>
<tr>
<td>4$^+$</td>
<td>4.00</td>
<td>3.82</td>
</tr>
<tr>
<td>6$^+$</td>
<td>8.04</td>
<td>8.16</td>
</tr>
<tr>
<td>8$^+$</td>
<td>11.57</td>
<td>11.82</td>
</tr>
</tbody>
</table>

$^a$Ref. 9

$^b$Ref. 10
HF projected spectra with the Hamiltonian $\hat{H} = \hat{H} - \alpha J^2$

Interaction: K+12 FP

S.P. Energies (MeV) $(0d_{5/2})=0.0$, $(1s_{1/2})=0.90$, $(0d_{3/2})=5.75$

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$^{36}$Ar</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K=0$ (Oblate solution)</td>
<td></td>
</tr>
<tr>
<td>$\alpha$ = 0</td>
<td>$E_J^{(HF)}$</td>
</tr>
<tr>
<td>$0^+$</td>
<td>0.00(-180.50)</td>
</tr>
<tr>
<td>$2^+$</td>
<td>1.50</td>
</tr>
<tr>
<td>$4^+$</td>
<td>4.45</td>
</tr>
<tr>
<td>$6^+$</td>
<td>7.96</td>
</tr>
<tr>
<td>$8^+$</td>
<td>8.80</td>
</tr>
</tbody>
</table>

a. Ref. 11
REFERENCES FOR CHAPTER IV

5. J.C. Parikh, private communication.