CHAPTER III

AC CHARACTERISTICS

Thus far, we have been concerned exclusively with steady state or DC characteristics of MgO cold cathode diodes. Without a doubt, however, AC measurements on self-sustained, operating MgO diodes furnish much more information about the internal mechanisms in the MgO layer than DC measurements can. In fact, for every point representing a single value of DC voltage and current, a whole series of experiments can be run by superimposing a small perturbing AC signal voltage and observing its effect on the AC component of current produced as a function of frequency. The maximum information can be obtained from this type of experiment by measuring, not only the magnitude ratio of the AC current produced to the applied AC voltage, but also the phase angle difference. In graphical form this could be an Argand diagram of the vector admittance of a diode as a function of frequency, with average DC tube current as a parameter.

In what follows we shall mainly make use of the experimental work done under the direction of Van der Ziel,
of the AC admittance characteristics of operating MgO diodes plotted on the complex admittance plane with applied frequency and average tube current as parameters.\(^{13}\)

Figures 9A and 9B taken from Van der Ziel's report show complex admittance plots of two diodes. Figure 9A is from a diode with good coating characteristics, and 9B is from a diode with poor coating characteristics. The criteria of good and bad coating characteristics are not specified, but it is presumed from our later analysis that bad coating characteristics include inability to sustain a high current and possibly, noisier operation. Both of these undesirable coating characteristics appear to be possessed to a greater degree by the diode used to obtain the admittance characteristics of Figure 9B than the diode used for Figure 9A.

The relative ability to sustain high current (without pop-out), and relative noisiness during operation can apparently be correlated with these complex admittance plots. To do so it is necessary to analyze the admittance plots in terms of a mathematical model. To avoid undue complexity, let us, at the outset, avoid details which are unnecessary for understanding the physical meanings which are attached to this mathematical model.
3.1 Theoretical Interpretation of AC Characteristics

The complex admittance characteristics of Figures 9A and 9B can be represented to good approximation by the sum of two vectors \( Y' \) and \( Y'' \). (Primed quantities will refer to what is experimentally observed, unprimed quantities will refer to our theoretical model.) \( Y' \) and \( Y'' \) have special properties in terms of frequency and average tube current for a given MgO cold cathode diode. We can write

\[
Y' = K' e^{-i\theta'} = K' e^{-i\omega \Delta t'}
\]  
\[
Y'' = G'' + i B''
\]

In these equations \( Y'' \) is a vector which is independent of frequency and is made up of two components, \( G'' \) and \( B'' \). \( G'' \) is a pure conductance which varies parametrically with average tube current and, presumably, from tube to tube. \( B'' \) is a pure capacitive susceptance which, within the limits of experimental error, is independent of average tube current, but varies from tube to tube. \( Y' \) is a vector whose length, \( K' \), is proportional to average tube current and whose angle with the real axis, \( \theta' \), varies with applied...
angular frequency, \( \omega \), in the manner shown by Equation (32). In this equation, \( \Delta t^f \) is approximately a constant for a given average tube current. It has the dimension of time.

Before proceeding further, let us consider how closely Equations (32) and (33) describe the observed complex admittance characteristics of Figures 9A and 9B. In Figure 9A there are approximately 23 plotted points. Our equations yield curves which fit all but three of the plotted points to within the diameter of a point. In Figure 9B the curves from our equations appear to fit all of the 19 plotted points. In order to avoid unnecessary initial complexity, we are confining attention to that portion of the admittance characteristics which contains the plotted points. However, this frequency region, the region of semi-circular loci, is the most interesting region in terms of physical interpretation. Consider the physical meaning of the terms making up the total admittance characteristic. Because these terms are additive admittances, they refer to simultaneous processes going on in adjacent parts of the MgO coating. The information about these processes is carried to the plate by electrons. The problem is to locate
the sources of these electrons and relate them to our theoretical model.

In terms of the DC model presented in an earlier section, the effect of an AC perturbation voltage on the plate of an operating diode is to modulate the kinetic energy of electrons in the MgO vacuum pores. Of these electrons modulated, some will make inelastic collisions with the MgO and thus modulate the rate of hole production. Other electrons will either strike the MgO elastically or else arrive at the plate without striking the MgO. All of these electrons, the inelastic, elastic, and direct electrons will suffer negligible time delay and arrive at the plate essentially in phase with the AC modulation voltage. (The time involved in secondary multiplication is of the order of $10^{-11}$ sec.) These electrons will make up the $G''$ component. Electrons which make up the $K' e^{-i\omega \Delta t}$ component are electrons which result from modulation of the avalanching interface by holes which have migrated from inelastic collision sites at the surface of the MgO. The $iB''$ electrons are probably produced by capacitive coupling between the plate and the interfacial layer through inactive areas of the MgO coating. (Inactive areas are areas not struck by
electrons from the MgO pores.) If this interpretation of $iB''$ is correct, then the tube with poor coating characteristics (i.e., more inactive area) should show a larger $iB''$. It is indeed observed that Figure 9B has a larger $iB''$ than Figure 9A. Also the magnitude of $iB''$ is unaffected by changes in tube current over the range of currents shown. This would be expected if all of the avalanching areas were active over this range of currents. However, the exact interpretation of $iB''$ is difficult. It is not apparent that any simple equivalent circuit could make the magnitude of $iB''$ independent of frequency and still retain its pure capacitive susceptance characteristic. However, the barrier layer is probably not susceptible to representation by a simple equivalent circuit.

Let us now consider the admittance characteristic of the process postulated as the mechanism responsible for self-sustained emission. To recapitulate, primary electrons from the MgO vacuum pores cause secondaries to be emitted with $\delta > 1$. The residual holes drift through the coating under the influence of the applied external field, where they accumulate against an interfacial barrier. Additional primaries enter from the bottom of the vacuum
pores because of the increased field strength from hole accumulation at the interfacial barrier. The only important time lag in this process is the average time, $\Delta t$, for a hole to drift across the coating to the interfacial barrier. It is to be expected that this time lag will be dependent on the average voltage across the MgO layer and also on the average hole mobility. The average hole mobility will be dependent on temperature through the mechanism of scattering involved. Now let us suppose that the DC voltage across the coating is perturbed by superimposing a small AC voltage on it. Furthermore, let us compare the resulting secondary current in phase and magnitude with the applied AC perturbation voltage. We have

$$V = V_0 e^{i\omega t}$$  \hspace{1cm} (34)

applied at the coating and the resulting AC current lags by a time interval $\Delta t$, that is,

$$I = I_0 e^{i\omega (t - \Delta t)}$$  \hspace{1cm} (35)

From these equations therefore,

$$Y = \frac{I}{V} = \frac{I_0}{V_0} e^{-i\omega \Delta t} = \frac{I_0}{V_0} e^{-\frac{2\pi i \Delta t}{\tau}}$$  \hspace{1cm} (36)
This is the admittance characteristic to be expected from our model. This derivation has been simplified by neglecting individual fluctuations in hole transit time, and using an average time $\bar{\Delta t}$. In terms of the admittance characteristic of our model,

$$\theta = \omega \bar{\Delta t} = 2\pi \left( \frac{\bar{\Delta t}}{\tau} \right) , \quad (37)$$

where $\theta$ is the angle made by the admittance vector $Y$ with the real axis, $\omega$ is the angular frequency of the applied voltage, and $\tau$ is the period of the applied voltage.

Appendix B demonstrates that Equation (36) explains the fact that MgO cold cathode diodes are, in some respects, electrically equivalent to a resistance and inductance in series.

To determine the effect of individual fluctuations in transit time, write each individual transit time, $\Delta t_k$, in the form

$$\Delta t_k = \bar{\Delta t} \pm \epsilon_k \quad (38)$$

Therefore, for each site characterized by a $\Delta t_k$ we can write
This is the admittance characteristic of each site characterized by a $\Delta t_k$. To get the macroscopic admittance characteristic we must form the sum of the $Y_k$, because the processes are occurring in parallel.

$$Y = \sum_{k} Y_k = \frac{I_0}{nV_o} \sum_{k} e^{i\omega(-\Delta t + \epsilon_k)}$$

$$Y = \frac{I_0}{V_o} e^{-\frac{2\pi i \Delta t}{\tau}} \frac{1}{n} \sum_{k} e^{\frac{+2\pi i \epsilon_k}{\tau}}$$

From this expression we see that as long as the individual fluctuations $\epsilon_k$ are small compared with the period $\tau$ of the applied voltage, the macroscopic admittance will have the simple form of Equation (36), i.e.,

$$Y = \frac{I_0}{V_o} e^{-\frac{-2\pi i \Delta t}{\tau}}$$
This is the admittance characteristic which appears to hold for all but the highest frequencies. However, if the $\epsilon_k$ are not negligible compared to $\tau$, and if the $\Delta t_k$ are symmetrically distributed about $\Delta t$, $\Delta t$ will be chosen in such a way that there are an equal number of $+\epsilon_k$ and $-\epsilon_k$ terms.

Therefore, we have

$$1 \sum_{k}^{n} \frac{\epsilon_k}{\tau} = \frac{1}{n} \left[ \sum_{k}^{n/2} \frac{2\pi i \epsilon_k}{\tau} - \sum_{k}^{n/2} \frac{-2\pi i \epsilon_k}{\tau} \right]$$

$$= \frac{2}{n} \sum_{k}^{n/2} \cosh \frac{2\pi i \epsilon_k}{\tau} \quad \text{(45)}$$

But by means of the identity,

$$\cosh (i \omega \epsilon_k) = \cos (\omega \epsilon_k), \quad \text{(46)}$$

we can write the macroscopic admittance in the form,

$$Y = \frac{I_0}{V_0} e^{-i \omega \Delta t} \left( \frac{2}{n} \sum_{k}^{n/2} \cos (\omega \epsilon_k) \right) \quad \text{(47)}$$

To see the effect of the distribution in the individual transit times on the macroscopic admittance characteristic, write Equation (47) as
\[
Y = \frac{I_o}{V_o} (\cos \omega \Delta t - i \sin \omega \Delta t) \sum_{k}^{n/2} \cos (\omega \epsilon_k) 
\]

(48)

Therefore, the summation in \( \epsilon_k \) will have no effect on the angle that \( Y \) makes with the real axis, since the ratio of the real and imaginary parts of \( Y \) is independent of the value of the summation. However, the value of the summation will affect the absolute value of \( Y \). We have

\[
|Y| = \frac{I_o}{V_o} \sqrt{\cos^2 \omega \Delta t + \sin^2 \omega \Delta t} \left| \sum_{k}^{n/2} \cos (\omega \epsilon_k) \right| 
\]

(49)

\[
|Y| = \frac{I_o}{V_o} \left| \sum_{k}^{n/2} \cos (\omega \epsilon_k) \right| 
\]

(50)

However, there is no a priori reason to expect that the individual \( \Delta t_k \) are symmetrically distributed about \( \Delta t \). The way to test this point is to plot experimental values of \( \theta' \) against \( f \). Therefore, from Equation (44), \( \Delta t' \) can be determined from the slope of this graph. We will discuss this more fully later.

At low frequencies \( \omega \epsilon_k \approx 0 \) and the summation will have a relative weighting value of unity. As the frequency becomes higher some of the \( \omega \epsilon_k \) terms become appreciably \( > 0 \). Therefore, the relative weighting effect of the summation...
at higher frequencies is to decrease the absolute value of Y. This is the macroscopic influence. Microscopically, the effect of the fluctuations \( \epsilon_k \) would be to increase the observed noise level through its effect on the observed current from Equation (40). This increased noise level would manifest itself, if strong enough to be observed above the noise due to other processes, as a "hump" in the total noise characteristic. There should be a correlation between the frequency at which Y has its minimum and the frequency at which the "hump" in the noise characteristic has its maximum. Although it appears that there might be such a correlation, we are handicapped here because we do not know if the noise characteristics versus frequency are taken on the same tube as used for admittance versus frequency characteristics. It was, therefore, necessary to obtain these characteristics on the same tube. A detailed description of the experimental procedure adopted for admittance measurement is presented on pages (53-54).

The noise voltage was measured with a Quan Tech Wave Analyzer (Model 303) between 100 c/s and 100 kc/s. Figures 10a and 10b illustrate the behavior of admittance as well as noise characteristics on the same tube.
Comparing Figures 10a and 10b, we observe a very good correlation between the frequency at which the noise characteristics peak and the frequency at which Y has minimum values.

In Figures 9A and 9B showing complex admittance characteristics, a noticeable shortening of the Y vector is observed in the high frequency region which contains plotted points. This is to be expected from Equation (50). However, the point to be made here is that above the frequency at which the shortening of the Y vector occurs, we should not expect to see a cyclic effect from the summation in Equation (50). In this frequency region and above, the ε_k are of the same order of magnitude as the period τ, so that complete randomness begins to prevail. The phase of the secondary current is no longer related to the applied AC perturbation voltage through the time lag involved in hole migration. That is, at the higher frequencies it is to be expected that displacement current effects, rather than conduction current effects, would dominate the observed admittance characteristic. We have not explored this region in detail because it is of less interest from the
standpoint of the processes of self-sustained emission. Also, there are no plotted points in the region where displacement current effects are important, although an extension of the admittance curves is shown.

The observed experimental relationship shown in Figure 11 between the angle \( \theta^1 \) which the \( Y^1 \) vector makes with the real axis and the applied frequency is interesting. Equation (36) whose derivation is based on the assumption that the \( \Delta t_k \) are symmetrically distributed about \( \Delta \bar{t} \), predicts a direct proportionality between \( \theta^1 \) and angular frequency \( \omega \), that is,

\[
\theta = \omega \Delta \bar{t} \tag{51}
\]

This derivation also assumes that the interfacial barrier responds to the AC perturbation voltage without adding a significant amount of time lag or phase shift to the primary current produced.

Figure 11 shows that there is a fairly linear relation observed for the 10 ma curve up to about 20 kc. At higher frequencies, the curve bends in the direction of shorter \( \Delta \bar{t}^1 \). At lower average tube currents \( \Delta \bar{t}^1 \) is more than the \( \Delta t^1 \) for
the 10 ma curve. This, of course, is to be expected since lower average tube current implies lower voltage across the MgO layer and hence a longer hole transit time. Another observation is that the curves depart from linearity sooner for lower average tube currents.

We believe there is a fairly simple explanation for the observed departures from linearity. The explanation involves a frequency selective weighting of $\Delta t^i$. At any point on the $\theta$ versus $f$ curve the slope determines the average $\Delta t^i$ at that particular frequency. This average $\Delta t^i$ is the average of all the $\Delta t_k$ over the surface of the cathode. Even if we assume that all of the holes drift with the same mean velocity, the $\Delta t_k$ will vary because of different distances from hole creation sites to the interfacial barrier.

An applied AC perturbation voltage will almost instantaneously create an equivalent sinusoidal perturbation in the rate of hole generation over the whole cathode surface. This perturbation in hole generation rate could be represented by a sine wave whose area is proportional to the perturbation in the number of holes about the mean value. Divide the sine wave into a large number of rectangles of
infinitesimal width $dt$. Consider the behavior of the holes comprising one small rectangle. These holes will arrive at the barrier layer after individual transit times $\Delta t_k$. If the time spread in the $\Delta t_k$ is small compared to a fraction of a cycle of the AC perturbation then we can to good approximation ascribe the effect of the $\Delta t_k$ to an average $\bar{\Delta t}$. This is what we observe at low frequencies. It is apparent that $\bar{\Delta t}$ will be a poorer approximation for the $\Delta t_k$ at a given frequency if the voltage across the MgO layer is less (i.e., at lower average DC tube currents).

At higher frequencies we will begin to notice a frequency selective effect on the observed $\bar{\Delta t}$. This occurs when the longest $\Delta t_k$ are equal to or longer than a half cycle of the AC perturbation. When that occurs, holes of the longest $\Delta t_k$ will arrive at the barrier too late to contribute to the barrier modulation in the correct half cycle. Instead they will act in opposition to the majority of holes modulating the barrier during the next half cycle. One effect of frequency selection on holes of longest $\Delta t_k$ will be to shorten the length of the $Y'$ vector. We have already considered this effect under the less general assumption of a symmetrical distribution of the $\Delta t_k$ around $\bar{\Delta t}$. The second effect,
however, does not occur for a symmetrical distribution of the $\Delta t_k$ around $\Delta t$. That is, at progressively higher frequencies more and more of the longest $\Delta t_k$ do not contribute to determining the average $\Delta t$. This means that the average $\Delta t$ as observed from the slope of Figure 11 progressively decreases in the direction of higher frequencies.

Over the frequency range of good semi-circular admittance loci and down to about 1000 cps, this behavior appears to hold. Below about 1000 cps, however, we see a relatively drastic change in behavior. That is, the slope is approximately constant from $\sim 1000$ cps to 50 cps (the lower measurement limit). However, the slope is much greater, implying a longer time constant, below about 50 cps. At present, it appears that this change in behavior may be either a temperature effect in the barrier from low frequency modulation of barrier current, or else an effect due to diffusion of holes along the barrier layer.

3.2 Low Frequency AC Admittance Characteristics

Admittance measurements were extended to the low frequency region. Van der Ziel's data\textsuperscript{(13)} on admittance characteristics are limited to one kc and above. During the present investigations, measurements were made at
frequencies down to 50 cps using a very narrow band (~ 4 cps) detector with tunable crystal filter (General Radio Type, 736-A) in conjunction with an impedance-admittance bridge (General Radio Type, 1603-A). The schematic of the experimental setup for such measurements is shown in Figure 12. The difficulty in measuring at frequencies lower than 50 cps is that it becomes increasingly difficult to determine the bridge balance condition due to the 1/f noise characteristic of an MgO diode.

The coupling capacitor C, between the MgO tube and the ungrounded end of the bridge had a large enough value of capacitance so that its impedance down to low frequencies (~ 50 cps) was negligible compared to the tube impedance. This capacitor was necessary to isolate the bridge from the DC voltage across the tube. Since an MgO tube should be operated with a high anode impedance to ensure stable operation, a series resistance R was required between the tube and the DC power supply. The admittance seen by the bridge is, therefore, the MgO diode in parallel with a series circuit consisting of R and the output impedance of the DC power supply. The impedance of the power supply is negligible compared with the high resistance (~ 50 K) of R.
Therefore, correction of the observed values of admittance to the true MgO tube values requires a subtraction of 
$G = 1/50,000 = 20 \mu \Omega$ on the pure $G$-axis. This correction was made before the values were plotted on the admittance chart. Typical admittance plots, when the anode current was stabilized at 2.5 ma, 1.5 ma, and 1.0 ma, respectively, are shown in Figure 13. We see from this figure that the conductance component $G$ and the length of the $Y'$ vector increase with average DC anode current.

According to our model, the hole transit time $\Delta t$ in an MgO diode is connected to the experimentally observed $Y'$ vector by the relation,

$$Y' = ke^{-i\omega\Delta t}$$

(52)

That is, in terms of the admittance characteristic of our model, the angle $\theta'$ made with the real axis by the observed admittance vector $Y'$ is related to the hole transit time $\Delta t$ by

$$\theta' = \omega\Delta t = 2\pi f (\Delta t),$$

(53)

where $\omega$ is the angular frequency of the applied voltage. We can define frequency response as the reciprocal of $\Delta t$. In terms of our definition, it is observed from Figure 13 that
frequency response improves at higher average tube currents. This means for a given frequency, $\theta$ decreases at higher tube currents. These results are compatible with the observations of Van der Ziel.\(^{(13)}\)

3.3 AC Admittance Characteristics of Heated MgO Diodes

We have observed that heating the cathode affects the admittance characteristics as shown in Figures 14a and 14b. These figures show the admittance behavior at 0.9 ma, and 1.46 ma, when the cathode is heated to different extents (heater voltages 0.5 volt and 1 volt). Under these conditions, the admittance characteristics show an increase with heating in the conductance component $G$ as well as the length of the $Y_r$ vector. The reason for such behavior may be that the MgO conductivity increases with temperature when allowance has been made for the effect of heating on the barrier by taking heated and unheated admittance characteristics at the same current. Firth, Mayer, and Johnson\(^{(8)}\) have reported an appreciable increase in MgO conductivity between room temperature and 400°C. However, the admittance changes we have observed with heating are contrary to what Van der Ziel has observed. Van der Ziel's\(^{(13)}\)
measurements show a decrease in the conductance component and length of the Y₁ vector.

In order to determine the effect which heating alone has on the admittance characteristics, it is necessary to take admittance measurements at different temperatures, but at the same average anode current. That is, the anode voltage must be less at higher temperatures. The behavior of such curves is shown in Figure 15. It is observed that both the conductance component and the length of the Y₁ vector are increased with increasing temperature.

Measurements were also made of admittance characteristics of diodes at currents less than the 1 mA reported by Van der Ziel. In general, the admittance characteristics at low current levels are less stable than at current levels in the milliampere range. However, it appears safe to say that at low currents the length K₁ of the Y₁ vector decreases relative to the length of the G₁'' vector. At high currents K₁ is equal to or longer than G₁''. At low currents K₁ may be only a fraction as long as G₁''. (It must be realized that G₁'' also decreases at lower currents, but not as fast as K₁'.)

This behavior of K₁ and G₁'' is readily understandable in terms of our model. The lengths of K₁ and G'' are
ratios of AC current to AC voltage for two different processes. At higher tube currents, the DC voltage across the MgO layer must be higher to insure an adequate hole generation rate. This means that at higher tube currents the length $K^3$, which is dependent on the secondary emission ratio of the MgO surface, must increase more rapidly than $G^1$ which is dependent only on the vacuum pore electron current.