CHAPTER IV

EXPERIMENTAL RESULTS

IV.1 Shower Data

About 4000 showers have been recorded in the present investigation using SU7 triggers. The rate of chance coincidence for such triggers is estimated to be ~ one per day. Taking into account the detection efficiency of the UG-detector the total rate for the SU7 triggers is $(0.92 \pm 0.02) \text{ m}^{-2} \text{ hr}^{-1}$.

For the present analysis, about 10,000 showers, recorded by triggering the EAS system by EAS pulse only without the requirement of U.G pulse have also been used. These showers, to be called S-trigger showers, were made available by TIFR EAS group and were recorded during the operation of the present experiment.

As mentioned earlier (Chapter II), the selection criterion for the showers required a coincidence between any four of the inner nineteen density detectors. During the selection of the showers for analysis another criterion has been imposed which requires that at least three of the inner four density detectors should have $> 2$ particles each. This helps in removing the showers which are far away from the centre of the EAS array and for which the parameters could not be evaluated very accurately.
In order to determine the size spectra, the recorded showers have been classified in eight different size groups as given in Table IV.1.

In absence of the information on the arrival directions, the showers have been assumed to be vertical. This does not introduce much error because of a very steep angular distribution for the showers. The angular distribution is given by

\[ I(\theta) = I_0 \cos^n \theta \]

where \( n \leq 8 \) for the present level of observation, \( \theta \) is the zenith angle and \( I(\theta) \) and \( I_0 \) are the flux values at an angle \( \theta \) and along vertical respectively.

IV.2 The 100% area for showers:

To obtain the spectrum for the recorded showers it is essential to have an estimate of 100% area for the showers of a given size \( N \) recorded by a given EAS array. The 100% area for showers of given size \( N \) can be defined as an area such that the incident showers having cores within this area are detected with 100% efficiency by the air shower array.

If \( \Delta_i \) represents the particle density in the \( i \)th density detector due to an incident shower, then the probability of occurrence of a four fold coincidence can be written as

\[ \prod_i (1 - \varepsilon^{-\Delta_i S_i}) \]
<table>
<thead>
<tr>
<th>Group No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>$5.0 \times 10^4$</td>
<td>$1.0 \times 10^5$</td>
<td>$2.0 \times 10^5$</td>
<td>$4.0 \times 10^5$</td>
<td>$8.0 \times 10^5$</td>
<td>$1.6 \times 10^6$</td>
<td>$3.2 \times 10^6$</td>
<td>$6.4 \times 10^6$</td>
</tr>
<tr>
<td>Range</td>
<td>$1.0 \times 10^5$</td>
<td>$2.0 \times 10^5$</td>
<td>$4.0 \times 10^5$</td>
<td>$8.0 \times 10^5$</td>
<td>$1.6 \times 10^6$</td>
<td>$3.2 \times 10^6$</td>
<td>$6.4 \times 10^6$</td>
<td>$1.28 \times 10^7$</td>
</tr>
</tbody>
</table>
where $S_i$ is the area of the $i$th density detector. This probability is equivalent to the detection efficiency $\xi(N)$ for the incident shower of size $N$. Thus if we select an area such that for showers of size $N$ incident on this area $\xi(N) \simeq 1$, then this area can be termed as 100% area (to be denoted by $A(N)$). If showers of size $N$ are incident on the array such that the density in any four of the density detectors is $\geq 20$ particles $/m^2$ then for such showers $\xi(N) \simeq 0.99$, a value which is sufficiently close to 1. Thus for computing $A(N)$ the following procedure has been followed:

Using NKG distribution function, given by equation (1.3.4), the value of radius $R_{20}(N)$ for a given size $N$ is determined. $R_{20}(N)$ is the distance from the core, of a shower of size $N$, at which the density is equal to 20 particles $/m^2$. Having evaluated $R_{20}(N)$ an area is mapped out on the EAS array diagram such that each point in this area has at least four detectors within a circle of radius $R_{20}(N)$ m, around the point. This area then represents $A(N)$ and fig. IV.1 illustrates examples of 100% areas for six different sizes. A plot of $A(N)$ as a function of size $N$ is given in Fig.IV.2.

IV.3: The Size Spectra:

For obtaining the size spectrum for $S$-trigger showers a selection of the showers having cores within 100% area is done as follows.
ACCEPTANCE AREAS FOR SHOWERS

**Fig. IV.1**

- **a**: $3 \times 10^5$ particles, Area = 223 m$^2$
- **b**: $2 \times 10^5$ particles, Area = 2000 m$^2$
- **c**: $5 \times 10^6$ particles, Area = 5400 m$^2$
- **d**: $1 \times 10^6$ particles, Area = 11,600 m$^2$
- **e**: $2 \times 10^6$ particles, Area = 34,800 m$^2$
- **f**: $32.3 \times 10^6$ particles, Area = 31,416 m$^2$
FIG. IV. 2

100% AREA (TT²)

SHOWER SIZE
A shower of size $N$ is accepted or rejected according to whether or not a circle of radius $R_{20}(N)$ drawn around the core of the shower contains at least four detectors within it. The method is adopted from one used Clark et al. (1958). Using the area, obtained as described above, and the number of S-trigger showers incident on the area, the differential size spectrum for the S-trigger showers has been obtained.

The same procedure could also be used for obtaining the size spectrum for SU7-trigger showers. However, the number of the showers for each size group, which fall within 100% area, is very small for some of the size groups. A slightly modified method has, therefore, been used for determination of SU7- spectrum. The modification consists in including the showers having cores outside the 100% area, in the analysis and correcting their number for the reduction in the detection efficiency. For this purpose correction factors $C(N)$ are worked out as follows:

For a given size group a comparison is made between the number density (number of showers per unit area) of the S-trigger showers falling within 100% area to the number density of these showers (S-trigger) having cores within a radius $R_a(N)$ of the centre of the array. The ratio of the two densities then yields the correction factor $C(N)$. The SU7-trigger showers having cores within
a circle of radius $R_a(N)$ around the centre of the EAS array have been accepted for the analysis. Different values of $R_a(N)$ have been taken for different size groups. The values are given in Table IV.2 together with the values of $C(N)$. The size group number refers to the number given in Table IV.1.

**Table IV.2**

<table>
<thead>
<tr>
<th>Size Group No</th>
<th>$R_a(N)$</th>
<th>$C(N)$</th>
<th>Size Group No</th>
<th>$R_a(N)$</th>
<th>$C(N)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10 m</td>
<td>2.50</td>
<td>5</td>
<td>100 m</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>20 m</td>
<td>2.38</td>
<td>6</td>
<td>100 m</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>50 m</td>
<td>1.70</td>
<td>7</td>
<td>100 m</td>
<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>80 m</td>
<td>1.38</td>
<td>8</td>
<td>100 m</td>
<td>1.00</td>
</tr>
</tbody>
</table>

The reason for taking different values of the acceptance radius $R_a(N)$ for different size groups is that the detection efficiency decreases more rapidly, with increasing distance of the core from the air shower array centre, for showers of smaller sizes than for showers of larger sizes. This is evident from Fig. IV.3 which shows the distribution of S-trigger showers in distance, $R$, of the shower cores from the array centre, for showers in different size groups. A consideration in selecting
DISTRIBUTION OF S TRIGGER SHOWERS OF VARIOUS SIZES IN CORE DISTANCE R.

FIG.IV.3
R_a (N) has been that the correction factor C(N) should not exceed 2.50 i.e., the overall detection efficiency for showers whose cores land within distance R_a(N) from the centre of the array should not be less than 40%.

Having calculated C(N) for different size groups (using S-trigger showers), the number of SU7-trigger showers, having cores within a radius R_a(N) of the centre of the array, is determined. The number is then corrected using the values of C(N). The effect of the correction factors on the probable errors is taken into consideration. The corrected number, then, corresponds to the number expected if the whole area within a radius R_a(N) were 100% area. Using the corrected number of SU7-trigger showers the size spectrum for these showers is obtained.

Fig.IV.4 shows the differential size - spectra for S-trigger and SU7-trigger showers. Representing the S-trigger spectrum as

\[ F(N) \, dN = K \left( \frac{N}{10^5} \right)^{-\gamma} \, dN \, \text{m}^{-2} \text{Sec}^{-1} \quad \ldots \quad (4.3.1) \]

and SU7-trigger spectrum as

\[ d(N) \, dN = K' \left( \frac{N}{10^5} \right)^{-\gamma'} \, dN \, \text{m}^{-2} \text{Sec}^{-1} \quad \ldots \quad (4.3.2) \]

Where \( K, \gamma \) and \( K', \gamma' \) are constants, we can write

\[ K = (1.01 \pm 0.10) \times 10^{-10} \quad ; \quad \gamma = 2.78 \pm 0.04 \quad \ldots \quad (4.3.3) \]

\[ K' = (1.15 \pm 0.28) \times 10^{-13} \quad ; \quad \gamma' = 2.30 \pm 0.09 \quad \ldots \quad (4.3.4) \]
**DIFFERENTIAL SPECTRA FOR THE SHOWERS**

**FIG. IV.4**

**S-TRIGGER SPECTRUM**

\[ K = (1.01 \pm 0.11) \times 10^{10} \]

\[ \gamma = (2.78 \pm 0.04) \]

**SU7-TRIGGER SPECTRUM**

\[ K = (1.15 \pm 0.29) \times 10^{13} \]

\[ \gamma = (2.30 \pm 0.09) \]
While fitting eqn. (4.3.2) to the SU7-trigger spectrum the point corresponding to the first size group has been ignored because a change in slope around this point is indicated.

The difference \( \Delta \) in the Power indices \( \gamma \) and \( \gamma' \) can be accounted for by the increase in the probability of association, of a shower with a muon at underground level, with increasing shower size resulting in a SU7-trigger spectrum which is flatter than S-trigger spectrum. A size dependence of muon number \( \eta_\mu \) of approximately following type is indicated

\[
\eta_\mu \propto N^{0.48 \pm 0.02}
\]

A more exact relation between \( \eta_\mu \) and \( N \) can, however, be obtained from the spectra derived above using the procedure outlined in the following section.

**IV.4 Number of muons of energy \( \geq 150 \text{ GeV} \) in showers of different sizes:**

Let \( S(N) \, dN \) denote the total rate of showers of sizes between \( N \) & \( N + dN \) recorded by the EAS array, and let \( S U(N) \, dN \) be the rate of the showers in the same size range associated with a muon detected at a depth \( d \) underground. Then the probability,

\[
p_\mu \left( \geq E_\mu, N \right) = \frac{S U(N) \, dN}{S(N) \, dN} \quad \ldots \quad (4.4.1)
\]
that a shower of size $N$ is recorded in association with a muon detected at the depth $d$, is related to

$$\eta_{\mu}(\geq E_{\mu}, N),$$

the average total number of muons, of energy \( \geq E_{\mu} \), present in shower of size $N$. Here $E_{\mu}$ is the minimum energy which a muon must possess to traverse a depth $d$.

In the present experiment the depth $d = 194$ m corresponds to 590 m.w.e. of standard rock. Using the range-energy relation for muons given by Menon and Ramanamurthy (1967) the value of $E_{\mu}$ for present case is found to be 150 Gev.

In order to obtain the relation between $\eta_{\mu}(\geq E_{\mu}, N)$ and $P_{\mu}(\geq E_{\mu}, N)$ let us consider the geometry of the experimental set up (Fig.IV.5). $S'$ and $S$ are two parallel planes at surface and underground level respectively, separated by a vertical distance $d$. $O'$ is the array centre at surface and $O$ its vertical projection on $S$. Let $F(N)dN$ represent the vertical flux of showers having size between $N$ & $N + dN$. Let us consider showers in this size group incident on $S'$ at zenith angle $\Theta$ such that their axes pass through an elemental area $dA'$ around $C'$. The axes of these showers will fall on an elemental area $dA$. 


around C in underground plane S. The total number of
such showers is then given by

\[ F(N) \frac{dA \cdot \cos \Theta \cdot dA'}{L^2} \cos^2 \Theta \cos^n \Theta \quad \ldots \quad (4.4.2) \]

where \( L \) is the distance CC' and \( \cos^n \Theta \) term comes
because of zenith angle dependence of shower flux. Now
if \( r \) is the perpendicular distance of the detector at
D from the shower axes and \( \Delta \mu (N) \) the density of
muons at distance \( r \) from the shower axes at underground
level then the probability that such showers will be
recorded in association with a muon at underground
detector D is

\[ \left( 1 - e^{-s \cos \Theta} \cdot \Delta \mu (r) \right) \quad \ldots \quad (4.4.3) \]

where \( s \) is the area of the detector D. This probability
expression is based on the assumption that the number of
muons in the showers is subject to Poisson fluctuations.
Combining (4.4.2) and (4.4.3) we get

\[ SU(N)dN = \int \int F(N)dN \frac{dA \cdot dA'}{2} \cos^{n+2} \Theta (1 - e^{-s \Delta \mu (r) \cos \Theta}) \]

\[ \ldots \quad (4.4.4) \]

Here the integration is carried over the array
area \( A' \) and over an infinite plane passing through under-
ground detector D. For the total rate of the Air showers
of sizes between \( N \) & \( N + dN \) we can write:
GEOMETRY OF EXPERIMENTAL ARRANGEMENT

FIG. IV·5
\[ S(N) dN = \int_A \int_{\Omega} F(N) dN \cdot \cos^2 \theta \cdot dA \cdot \cos \theta \cdot d \Omega \quad (4.4.5) \]

where \( A' \) is same as above and \( \Omega \) is the solid angle of the array. We can then write
\[
P_\mu (\geq E_\mu, N) = \frac{\int \int dA' \cdot dA \cdot \cos^2 \theta \left( 1 - e^{-s \Delta \mu \cos \theta} \right)}{\int \int dA' \cdot \cos^2 \theta \cdot d \Omega} \quad (4.4.6)\]

This gives us a relation between \( P_\mu (\geq E_\mu, N) \) and \( \eta_\mu (\geq E_\mu, N) \). However for this we must assume an explicit form of lateral distribution of the muons of energy \( \geq E_\mu \) in the EAS. A likely form for lateral distribution of high energy muons in EAS is the exponential distribution given by
\[
\Delta \mu (\gamma) = \frac{\eta_\mu (\geq E_\mu, N)}{2 \pi \gamma_o^2} \exp \left( -\frac{\gamma}{\gamma_o} \right) \quad (4.4.7)\]

this distribution is based on the transverse momentum distribution, for pions produced in Nucleon - nucleon interactions, as given by Cocconi et al. (1961) viz.
\[
f(p_\perp) \propto \frac{p_\perp}{p_o} \exp \left( -\frac{p_\perp}{p_o} \right) \quad (4.4.8)\]
where \( \langle p_\perp \rangle = 2 p_o \). Expression (4.4.7) is derived from (4.4.8) for the case of monoenergetic muons produced at a given level in the atmosphere. However this can also be used for the case of high energy muons as their production
levels are confined to a narrow region of atmosphere and also because they have a steep energy spectrum.

Substitution of equation (4.4.7) in right hand side of eqn. (4.4.6) yields a relation, which in conjunction with equation (4.4.1) can be used to obtain values of \( \eta_{\mu}(>E_{\mu},N) \) for a given value of \( E_{\mu} \). Using S-trigger and SU7 - trigger spectra derived above/equation (4.3.3; 4.3.4)/which correspond to \( S(N) \) and \( SU(N) \) respectively, the values of \( P_{\mu}(>E_{\mu},N) \) for different values of \( N \) have been obtained using eqn. (4.4.1). Also integrals on R.H.S. of Eqn. (4.4.6) have been solved numerically using \( \eta_{\mu} \) and \( r_0 \) as free parameters. Before carrying out the integration \( dA', dA, \Theta, r & l \) were expressed in terms of \( R, \phi, R', \phi' \), and \( R_0 \) using the geometry of the set up (Fig. IV.5). The area \( A' \) is the 100\% area for the size group. However, the dependence of \( P_{\mu}(>E_{\mu},N) \) on \( A' \) is negligible. The area \( A \) was taken to be the area of a circle of radius 500 \( r_0 \) around the underground detector D. The integration was carried out using Gaussian Quadrature formulae for \( R \) and \( R' \) and the trapezoidal rule for angles \( \phi \) and \( \phi' \).

Before deriving the values of \( \eta_{\mu}(>150,N) \) by comparison of experimental values of \( P_{\mu}(>150,N) \) with the values of \( P_{\mu} \) calculated using (4.4.6) it is necessary to specify the values of \( r_\alpha \) appropriate to
\[ E_{\mu} = 150 \text{ Gev.} \] This value cannot be derived from the present experimental data. We have, therefore, made an estimate of the value of \( Y_0 (\geq E_{\mu}) \) as follows:

We note that for a given energy \( E_{\mu} \), \( Y_0 (\geq E_{\mu}) \) is related to the mean spread, \( \langle r (\geq E_{\mu}) \rangle \), of the muons by the relation

\[ \langle r (\geq E_{\mu}) \rangle = 2 \ Y_0 (\geq E_{\mu}) \quad \ldots (4.4.9) \]

Fig. IV.6 gives the values of \( \langle r (\geq E_{\mu}) \rangle \) for four different values of \( E_{\mu} \). The points at 50 Gev and 100 Gev are calculated from the lateral distributions obtained by Earnshaw et al. (1968). It is to be noted that \( \langle Y (\geq E_{\mu}) \rangle \) decreases with increasing energy and a smooth curve drawn through the points indicates a value of \( \langle Y (\geq 150) \rangle \simeq 24 \text{ m} \) giving an \( r_0 (\geq 150) \) value of 12 m.

Fig. IV.7 gives the variation of \( P_{\mu} \) with \( n_{\mu} \) for six different values of \( r_0 \). It is seen that the \( P_{\mu} - n_{\mu} \) relation does not change significantly, for values of \( r_0 \) from 8 m to 14 m and for \( 10 \leq n_{\mu} \leq 200 \). The experimental values of \( P_{\mu} (\geq 150, N) \) lie between \( 0.9 \times 10^{-3} \) and \( 1.6 \times 10^{-2} \) and for this range of \( P_{\mu} \) values, the \( P_{\mu} - n_{\mu} \) relationship does not change significantly for \( 8 \text{ m} \leq r_0 \leq 14 \text{ m} \).
FIG. 7: VARIATION OF ASSOCIATION PROBABILITY WITH NUMBER OF MUONS. DNA SWARMS. 582. FIG. 6: MEAN LATERAL DISTANCE OF MUONS OF ENERGY E_\gamma. IN EAS. DNA SWARMS. 582.
Variation of number of muons of energy > 150 GeV with shower size.

\[ n_\mu(N) = (27 \pm 7) \left( \frac{N}{10^5} \right)^{0.47 \pm 0.05} \]

\( 10^5 \leq N \leq 5 \times 10^6 \)

Energy spectrum of muons in EAS of size \( 10^5 \) particles.

\[ N = 1.30 \pm 0.16 \]

\( \beta \geq 100 \)
Using the curves in Fig. IV.7 and the experimental values of \( P_\mu ( \geq 150, N) \), the values of \( n_\mu ( \geq 150, N) \) have been calculated for \( r_0 = 12 \) m. Fig. IV.8. shows the variation of the number of muons \( n_\mu ( \geq 150, n) \) with the shower size \( N \). The relation between \( n_\mu ( \geq 150, N) \) and \( N \) can be expressed by a power law of the type

\[
n_\mu ( \geq 150, N) = (27 \pm 7) \left( \frac{N}{10^5} \right)^{0.47 \pm 0.05} \quad (4.4.11)
\]

for \( 10^5 \leq N \leq 5 \times 10^6 \).

The experimental point corresponding to the first size group (Table IV.I) is much lower than expected on the basis of equation (4.4.11) and may indicate an increase in the power index for \( N \leq 10^5 \) particles; in this case the error factor is also much larger.

### IV.5 Energy Spectrum of Muons in EAS

Sivaprasad (1970) has obtained the following relations for the variation of number of muons, of energy \( E_\mu \geq 220 \) GeV and \( E_\mu \geq 640 \) GeV, with the shower size

\[
n_\mu ( \geq 220, N) = (16 \pm 3) \left( \frac{N}{10^5} \right)^{0.41 \pm 0.09} \quad (4.5.1)
\]

\[
n_\mu ( \geq 640, N) = (4.1 \pm 1.2) \left( \frac{N}{10^5} \right)^{0.41 \pm 0.15} \quad (4.5.2)
\]

The power indices for \( n_\mu - N \) relation in this case are in agreement with the power index obtained in present experiment within experimental errors and hence these values
of $n_{\mu}$ can be used with the value of $n_{\mu}(\geq 150, n)$, obtained in present experiment, to obtain the energy spectrum of the muons of energy $\geq 150$ Gev.

Fig. IV.9 gives the energy spectrum which can be represented by a power law of the type

$$n_{\mu}(\geq E_{\mu}) = A \left(\frac{E_{\mu}}{150}\right)^{-\beta} \ldots \tag{4.5.3}$$

where $\beta = 1.30 \pm 0.16$

$$A = (27 \pm 7) \quad \& \quad N = 10^5 \text{ particles}$$

Combining equation (4.4.11) and equation (4.5.3) we can write

$$n_{\mu}(\geq E_{\mu}, N) = (27 \pm 7) \left(\frac{N}{10^5}\right)^{0.47 \pm 0.05} \left(\frac{E_{\mu}}{150}\right)^{-1.30 \pm 0.16}$$

for $E_{\mu} \geq 150$ Gev and $10^5 \leq N \leq 5 \times 10^6$

IV.6 Primary Cosmic Ray Spectrum

The differential spectrum for S-trigger showers obtained as described in section IV.3 and expressed by equation (4.3.1) can be used for obtaining the energy spectrum of the primary cosmic rays. For this purpose, however, it is necessary to have a knowledge of the relationship between the energy, $E_p$, of the primary cosmic ray particle and the size $N$, of the EAS generated by the primary particle, at the level of observation.
Lal (1967), using the curves given by Bradt et al. (1965), has obtained a relationship between $E_p$ and $N$ at 920 gm/cm$^2$ (the observation level of the present experiment). The relationship, shown in Fig. IV.10, can be represented by a power law of the type

$$N \propto E_p^\beta$$

where $\beta \approx 1.2$.

Using this relationship and equation (4.3.1), the integral energy spectrum for primary cosmic rays has been obtained and is shown in Fig. IV.11. The spectrum can be represented by

$$F(>E_p) = K_0 \left( \frac{E_p}{10^{15}} \right)^\gamma \text{Cm}^{-2} \text{Sec}^{-1} \text{Sr}^{-1} \quad (4.6.1)$$

where $K_0 = (3.74 \pm 0.19)10^{-10}$ \quad (4.6.2)

and $\gamma = (2.16 \pm 0.05)$ \quad (4.6.3)

The energy $E_p$ is expressed in eV and

$$5 \times 10^{14} \text{ eV} \leq E_p \leq 2 \times 10^{16} \text{ eV}.$$
INTEGRAL ENERGY SPECTRUM OF PRIMARY COSMIC RAYS

![Graph of integral energy spectrum of primary cosmic rays](image)

**Figure III.11**

PRIMARY ENERGY (GeV)

INTEGRAL FLUX F(E) (cm⁻² sec sr⁻¹)

**Figure IV.10**

PRIMARY ENERGY (GeV)

SHOWER SIZE AT 920 gms/cm²

![Graph of primary energy vs. shower size](image)