APPENDIX-II

Considering a biased Monte Carlo scheme where exactly one particle comes out of each collision with the corresponding modified weight and the scoring is defined by

\[ s_0 = 1 \quad \text{...(A-44)} \]

\[ p(\mathcal{E}, s) = \mathcal{E}(s) \quad \text{...(A-45)} \]

the equation for \( M_2 \) becomes

\[
M_2(x, u) = \frac{\left[ 1 - \int dx' C(x, x' ; u) \right]^2}{\left[ 1 - \int dx' C(x, x' ; u) \right]} \mathcal{C}(u) + \\
+ \int \int dx'd u' \frac{K^2(x, u, x', u')}{K(x, u, x', u')} M_2(x', u') \quad \text{...(A-46)}
\]

Where

\[ K(x, u, x', u') = C(x, x'; u) \mathcal{B}(u, u'; x') \quad \text{...(A-47)} \]

and

\[ \tilde{K}(x, u, x', u') = \tilde{C}(x, x'; u) \tilde{B}(u, u'; x') \quad \text{...(A-48)} \]

Now if

\[ \frac{\tilde{K}(x, u, x', u')}{K(x, u, x', u')} = \frac{M_1(x, u)}{M_1(x', u')} \quad \text{...(A-49)} \]

and

\[ \frac{1 - \int dx' C(x, x'; u)}{1 - \int dx' C(x, x'; u)} = M_1(x, u) \quad \text{...(A-50)} \]
then

\[
\frac{\epsilon_2(x,u)}{\epsilon_1(x,u)} = \left[1 - \int dx' \left( x, x', u \right) G(u) + \int dx' d u' \mathcal{K}(x, u, x', u') \right] \epsilon_2(x', u') \epsilon_1(x, u')
\]

which satisfies the same equation as \( \epsilon_1(x, u) \)

Therefore,

\[
\frac{\epsilon_2(x, u)}{\epsilon_1(x, u)} = \epsilon_1(x, u)
\]

or,

\[
\epsilon_2(x, u) = \epsilon_1^2(x, u)
\]

Hence,

\[
\text{Var} \left[ \epsilon_1(x, u) \right] = 0 \quad \text{...(A-52)}
\]

Eq.(A-49) can be written as

\[
\int dx' d u' \mathcal{K}(x, u, x', u') = \frac{1}{\epsilon_1(x, u)} \int dx' d u' \mathcal{K}(x, u, x', u') \epsilon_1(x', u')
\]

\[
= \frac{1}{\epsilon_1(x, u)} \left\{ \epsilon_1(x, u) - \left[1 - \int dx' G(x, x'; u) \right] \theta(u) \right\}
\]

\[
= 1 - \frac{\left[1 - \int dx' G(x, x'; u) \right] \theta(u)}{\epsilon_1(x, u)} \quad \text{...(A-53)}
\]

Now, since there is no absorption in the altered game

\[
\int dx' d u' \mathcal{K}(x, u, x', u') + \left[1 - \int_0^b \mathcal{C}(x, x'; u) dx' \right] = 1 \quad \text{...(A-54)}
\]
There

\[ a = x \text{ and } b = x_0 \text{ for } u > 0 \]
\[ a = 0 \text{ and } b = x \text{ for } u < 0 \]

Using Eqs. (A-53) and (A-54)

\[ 1 - \left[ 1 - \int_{a}^{b} \phi(x, x'; u) \right] M_1(x, u) \Omega(u) = \int_{a}^{b} \tilde{\phi}(x, x'; u) \right. \]
\[ \left. \int_{a}^{b} \tilde{\phi}(x, x'; u) \right) \]

Therefore,

\[ 1 - \int_{a}^{b} \tilde{\phi}(x, x'; u) dx = 0 \text{ for } u < 0 \]

...(A-56)

which implies that there should be no escape of particles except through the boundary of interest.

Playing an altered game as per Eq. (A-49) leads to a zero variance for the quantity \( M_1(x, u) \). Zero variance for any source distribution can be obtained with the help of proper biased source

\[ \tilde{S}(x, u) = S(x, u) \frac{M_1(x, u)}{\int_{a}^{b} dx u S(x, u) M_1(x, u)} \]

...(A-57)

where \( S(x, u) \) is the true source distribution.