CHAPTER II

GENERATION MECHANISMS OF IRREGULARITIES

The mechanisms of generation of ionization irregularities observed by the author in the D and E-regions of the ionosphere are the cross-field instability and the neutral turbulence. The development of some of these theories which are applicable to the ionospheric case is discussed here.

2.1 Theories of Cross-field Instability

2.1.1 Linear Theories

The first quantitative theory of the cross-field instability was proposed by Simon (1963) for the case of laboratory plasma. Simon considered a slab of weakly ionised plasma with mutually perpendicular electric and magnetic fields and found that if the electric field was greater than
a critical value, in presence of non-uniformities of plasma density, the study state could become unstable. While discussing the Penning discharge at high pressure Hoh (1963) showed independently that a weakly ionized plasma column, in an axial magnetic field and an inward directed radial electric field, is unstable to macroscopic perturbations. It was shown that the instability is caused by the presence of a third fluid, the neutrals in an ion electron plasma. Hoh came out with a critical magnetic field which agreed well with the experimental results.

Meeda et al. (1963) and Knox (1964) were the first to apply the Simon's theory to the case of ionosphere. Meeda et al. while attempting to explain $E_s - q$ echoes found that a threshold of primary electric field, in W-E direction, was needed. Knox showed that for electrojet region, where crossed electric and magnetic fields are present, the perturbations of electron density can grow when either electron density or electric field gradients is present. The irregularities were found to be moving with wide range of velocities. Further extensions of Simon's theory were done by Morse (1965) who did two dimensional linear analysis for the equatorial ionosphere, and by Tsuda et al. (1966) to explain the temperate latitude sporadic-E.

All the above mentioned work was limited to the 'bounded case' where the electron density and electric field
perturbations are assumed to be present only between two fixed altitudes. In the 'unbounded case' the ambient electron density as well as the perturbations in it vary exponentially with the height for the same scale height. Reid (1968) studied the case of such an unbounded plasma assuming a realistic electron density profile. The salient features of Reid's theory are the following.

Only those wave numbers which were moving towards magnetic west, represented by $k_\perp$ were considered whereas the ones parallel to the magnetic field were neglected. The electric field component parallel to the magnetic field, $E_{\parallel}$ was also neglected. The variation of electron density was assumed to be of an exponential form with a scale height, $H$, such that

$$\frac{1}{H} = -\frac{1}{n_0} \frac{dn_0}{dh}$$

where $n_0$ is electron density and $h$ is the altitude.

The time and space dependence of the electron density perturbation was assumed to be of the following form.

$$n_1(t,x) = n_1(x) \exp \left\{ -i(\omega t - k_z z) \right\}$$

and

$$n_1(x) = n_1 \exp \left\{ -\frac{x \cos \phi}{H} \right\}$$

where $n_1$ is the perturbation in the electron density,
$\omega$ is the frequency of the perturbation,

$k_z$ is the wave number in $z$ direction (magnetic west),

$x$ is the direction pointing towards the vertical, and

$I$ is the magnetic dip angle.

The functional form of the electric field perturbation was same as that of electron density perturbation. The space dependence of the perturbation shows that only field aligned irregularities which form a rippled pattern in east-west direction are considered. The functional form of the perturbation assumed by Simon (1963) and Tsuda et al. (1966) was a sinusoidal of half wave length with amplitude vanishing at $x = 0$ and $x = L$. The spatial averages of electron density and electric field were taken over the distance $L$ in the further analysis. Reid pointed out that these could be true only for the bounded case and were certainly not applicable to the system like ionosphere, and came out with the above mentioned exponential dependence. The analysis of Reid showed that in the presence of a certain critical electric field, the ionospheric $E$-region could be unstable to the growth of electron density perturbations of sizes ranging from a few tens of meters to a few kilometers. The electric fields supporting the large scale irregularities are conducted upto high latitude $F$-region heights along the highly conducting magnetic field lines and there they produce small scale irregularities in presence of density gradients. Cunnold (1969) extended Reid's work by including the effects of finite $k_z$. It was
found that the effect of inclusion of $k_{11}$ is to reduce the growth rate and accelerate the loss rate due to diffusion.

The theories of cross-field instability got a momentum with the work of Rogister and D'Angelo (1970). It was pointed out by them that the neglect of ion inertia, by previous workers, was not justified as it leads to dissipative effects. They developed the theory for 95 km and 110 km regions, separately because the collision frequencies and other scaling factors are quite different in both these altitude regions. Although they did not write the Vlasov equation and considered only the fluid picture for both electrons and ions, the results of Farley-Buneman instability were recovered. The effects of ionization and recombination were neglected because the time constants associated with these processes were of the order of minutes whereas the period of the waves considered was of the order of milliseconds. The ionosphere was assumed to be stratified both in east-west as well as north-south direction. It was further assumed that the presence of waves did not modify the magnetic field itself i.e. the perturbation were electrostatic in nature. $k_{11}$ was assumed to be much smaller than $k_{1}$. All the perturbed variables were expanded as a power series in terms of a small parameter $\varepsilon$, which was the smallest parameter of the entire system. The density perturbation $\delta n$ was given by

$$\delta n = \delta n^{(1)} + \varepsilon \delta n^{(2)} + \varepsilon^2 \delta n^{(3)} + \cdots$$
The time dependence of these perturbations is given by

$$\tilde{C} \eta (t, \chi) = \tilde{C} \eta (\chi) \exp \left(-i \omega t\right)$$

where $\omega$ is the frequency of the perturbation and is in turn represented by

$$\omega = (\omega^{(1)} + \epsilon (\omega^{(2)}) + \epsilon^2 (\omega^{(3)}) + \ldots$$

The space dependence of the perturbation was described by W.K.B. approximation as follows:

$$\tilde{C} \eta (\chi) = \tilde{C} \eta (\epsilon \chi) \exp \left\{ \frac{i \phi (\epsilon \chi)}{\epsilon} \right\}$$

where $\tilde{C} \eta$ represents the average value.

Using these generalized dependences Register & D'Angelo (1970) derived the expressions for the growth rate for lower as well as the upper region. The expression for the growth rate in the upper region was similar to that of lower region except that it had an additional term which represented the effect of shear of electron flow. The growth rate for the lower region is given by

$$\Gamma^{(1)} = \frac{K \cdot V_{eo}}{1 - \frac{\nu_e \nu_i}{\omega_e \eta_i}} \frac{\nu_e}{\eta_i} \frac{k_y}{K_z^2} \frac{1}{\eta_0} \frac{\partial n_0}{\partial \epsilon \chi^2} - \frac{\nu_e}{\eta_c} \Omega \left( \frac{\omega^{(1)} - K_z^2 \zeta_z^2}{\Omega \left( 1 - \frac{\nu_e \nu_i}{\omega_e \eta_i} \right)} \right)$$

The first term on the R.H.S. is the manifestation of the cross-field instability and has to be positive for the growth of perturbation. This term can be positive when either, both $K \cdot V_{eo}$ and $\frac{\partial n_0}{\partial \epsilon \chi^2}$ are positive or, are negative. The first condition is satisfied during daytime when electron
flow \((\vec{K}.\vec{V}_{eo})\) is towards west and the gradients are positive (electron density increasing with altitude) whereas the second condition is satisfied during night time. Rogister and D'Angelo (1970) clearly demonstrated that the cross-field instability would occur for the situation prevailing in equatorial electrojet and further attributed the cause of the type II backscatter echoes to the irregularities generated through this mechanism.

The second term is the manifestation of Farley-Buneman instability. As \(\Omega_e\) was assumed to be negative, a positive contribution to the growth rate from this term is possible when

\[
\omega^2 \geq k_z v_e^2 \text{ or } K.V_{eo} \geq (1 - \frac{\nu_e}{\Omega_e})k_z
\]

This indicates the necessity of a threshold of electron drift velocity for excitation of two stream instability. This instability is generally believed to be responsible for the generation of irregularities that cause type I radar echoes in the equatorial electrojet (Farley 1963). The role of linear cross-field instability in generation of irregularities in the equatorial electrojet has also been discussed by Whitehead (1971).

We note here that none of the above mentioned theories could explain the generation of 3-meter irregularities associated with type II echoes observed when electron drift velocity, \(V_d\) is smaller than the ion-acoustic velocity \(C_s\).
The two stream instability occurs when $V_d > C_s$ and, therefore, cannot account for the type II echoes. On the other hand, cross-field instability favours the generation of large scalesizes and, therefore, cannot excite 3-meter irregularities directly (Farley and Balsley, 1973). The smallest scale-size that can be generated by cross-field by cross-field instability is given by Sudan et al. (1973) as $\lambda_c \geq \frac{300}{V_d}$.

Thus the radar signals at 3-meter cannot be explained when two-stream threshold is not reached. To resolve this difficulty Sudan et al. (1973) have proposed a two step mechanism for the generation of small scale irregularities, which we describe below.

2.1.2 Non-linear Theories

The theory of Sudan et al. (1973) is a multilinear theory in which small scalesizes are generated on the top of large scalesizes due to second stage two-stream and cross-field instabilities. In the first step, long wavelengths are generated through the linear cross-field instability mechanism. It is shown that these irregularities do not steepen appreciably because even though they are non-dispersive waves, the phase velocity of these waves does not significantly depend on local parameters such as density or temperature. Since the waves do not steepen in the direction of propagation they attain large amplitudes. The existence of such large amplitude long wavelengths was experimentally
observed by Prakash et al. (1973). The large amplitudes attained by these waves are of the order of a few per cent and, therefore, the associated gradients and the vertical drifts because of local wave electric fields could be much larger than the original vertical gradients and the westward drifts. These gradients and drifts are responsible for small scale perturbations propagating vertically upwards. The quantitative estimates for the generation of small scalesizes show that the minimum scalesize that can be generated by this mechanism is given by \( \lambda_z \) where,

\[
\lambda_z \sim 8 A^{-1} v_d^{-3/4} \text{ meters}
\]

where \( A \) is the percentage amplitude of the large scalesize irregularity. For a drift velocity of 100 meters/sec and a percentage amplitude of 4\% (\( A = 0.04 \)) one finds the minimum \( \lambda_z \) to be about 6 to 7 meters.

Several theories have been advanced in the past for the non-linear evolution of the cross-field instability. For the saturation of the instability, these theories consider effects like quasilinear flattening of the density gradient and mode coupling to linearly stable higher wave numbers. Thus in a non-linear theory presented by Rogister (1972) for one dimensional cross-field instability, the energy is transferred by mode coupling from large to small wavelengths, where it is absorbed by linear diffusion damping. This theory predicts a suprathermal fluctuation level for a range of
linearly stable wavelengths and further the amplitude of the density fluctuations is found to be $\sim 20\%$ of the background density for the modes with maximum linear growth rate.

For the case of laboratory plasmas Kim and Simon (1969) have developed a quasi-linear theory in which the instability is quenched by quasi-linear reduction of externally applied electric field. For the case of reflex arc plasmas Hooper (1970) has shown that a quasi-linear flattening of the density gradient would shut off the cross-field instability.

Sato (1971, 74, 76) has shown that for electrojet situation, quasi-linear shielding of both electric field as well as the density gradient would contribute to the saturation of the instability in a weakly unstable situation. Sato (1971) has also shown that for a strongly unstable system mode-coupling effects become important and the final situation is quite turbulent in which smaller scalesizes are generated by large scalesizes in a kind of cascading down the energy from small $k$'s to large $k$'s. In the latter case it was found that turbulent spectrum approaches $k^{-3}$. In this work Sato (1971) has shown that a critical electric field $E_{c1}$ exists in the system such that if the applied external electric field $E_x < E_{c1}$, then quasi-linear effects saturate the instability whereas if $E_x > E_{c1}$ the turbulent situation arises.

Sato and his coworkers have also carried out numerical investigations for the non-linear cross-field instability.
Sato and Tsuda (1967), Tsuda and Sato (1968) and Sato et al. (1972) performed numerical analysis of the instability in one dimensional model and in general found that when $E_x < E_{c_1}$ then the evolved waveform has a sawtooth shape and for $E_x < E_{c_1}$ the system becomes strongly turbulent. Later on Sato and Tsuda (1968) and Tsuda et al. (1969) extended the analysis to two dimensional model. These computations showed that the important quenching mechanism was cancellation of the external electric field for smaller $E_x$. The mode coupling effects leading to this turbulent situation were found to be important again for the high values of $E_x$ such that $E_x > E_{c_1}$. Ogawa (1972) has further shown through his numerical computations for two-dimensional case, that the density flattening is also a competing saturation mechanism for the nonlinear evolution of cross-field instability.

Recently, McDonald et al. (1974, 75) have performed a two-dimensional numerical solution of the basic fluid equations believed to describe the equatorial electrojet situation. In their computer simulation experiment, it is found that long wavelength horizontally propagating gradient drift instabilities excite short wavelength vertically propagating instabilities and the final state is highly turbulent two dimensional state which supports the two step theory of Sudan et al. (1973). It is further found that the density fluctuations' power spectra for short wavelength irregularities go as $k^{-3.5}$. 
In an attempt to explain some of the rocket observations of Prakash et al. (1973), a two dimensional non-linear theory of cross-field instability was advanced by Rognlien and Weinstock (1974). It is shown by these workers that the non-linear term $\nabla \nabla \psi$ in the electron continuity equation is the most dominant non-linearity for the case of two dimensional perturbations. This is in contrast to earlier theories in which only one dimensional case was considered and wherein this non-linearity almost vanishes. Thus in a realistic calculation two dimensional perturbations should be considered as done by Rognlien and Weinstock (1974). This calculation shows that the linearly unstable mode generates a non-propagating mode in the vertical direction which is damped in the linear analysis. This serves to act as a saturation mechanism for the linear cross-field instability mechanism and these authors have calculated saturated amplitudes and spectra of density and electric field fluctuations in the resulting time steady situation. An agreement between the calculated shapes and spectra for the irregularities with our rocket observations has been claimed by these authors. The form of perturbation considered by Rognlien and Weinstock (1974) is

$$\frac{n_1}{n_0} \propto R_i \exp \left[ i(k_{yR} - \omega t) \right] \cos k_x x$$

which is equivalent to superposition of two plane waves propagating at equal but opposite angles to the $y$-axis i.e. the magnetic east. It has been shown by these authors that for such
a perturbation the non-linear term \( \nabla_{\epsilon_1} \cdot \nabla (n_1/n_0) \) is the most dominant non-linearity. For a case of single plane wave this term is vanishingly small because the two vectors are nearly perpendicular to each other. To illustrate the basic saturation process of this instability they consider interaction of only two modes. Density perturbation of the above mentioned form is assumed to be excited to a small level with an amplitude of \( A_{1,1}(y) \) and this mode can be written as following

\[
\begin{pmatrix}
\frac{n_1}{n_0}
\end{pmatrix}_{1,1} = A_{1,1} \sin (k_y y - \omega t) \cos k_x x
\]

The non-linear interaction of this mode with itself as manifested by the term \( \nabla_{\epsilon_1} \cdot \nabla (n_1/n_0) \) in the electron continuity equation produces a second mode. This becomes evident when the nonlinear term is written down,

\[
\nabla_{\epsilon_1} \cdot \nabla (n_1/n_0) = -\frac{\nu_i \omega}{2 \Omega_i k_i^2} A_{1,1}^2 \sin 2k_x x
\]

This non-linearity generates a new mode which is given by

\[
\begin{pmatrix}
\frac{n_1}{n_0}
\end{pmatrix}_{2,0} = A_{2,0} \sin 2k_x x
\]

The subscript 2,0 indicates that this mode varies as second harmonic of \( k_x \) in x direction (magnetic North) and as zeroth
Harmonic in the y direction (magnetic east) or time. This mode does not propagate and the density structures have variations only in x direction. This mode is damped in the linear analysis. The $A_{2,0}$ mode can couple with the $A_{1,1}$ mode through $\sqrt{\epsilon_1} \nabla^2 (n_1/n_0)$ term to produce a non-linear flux of electrons that will stabilize $A_{1,1}$ mode. Thus one finds that the stabilization mechanism in this theory is a wave enhanced flux or diffusion of electrons.

Following such an approach the saturated amplitudes of both the modes can be calculated. A rather surprising discovery of this analysis was that during the saturation process the linearly damped mode in the saturated steady state has a larger density amplitude than the linearly unstable mode. However, the electric field amplitude for the linearly damped mode remains smaller than the linearly unstable mode in the saturated steady state.

In order to determine the effect of cascading of energy from small wave numbers to high wave numbers many modes were considered in the analysis by these authors. The resulting coupled non-linear equations thus obtained were then numerically solved and the resulting shapes and spectra of the large scalelength modes were computed.

The above analysis pertains to only nighttime situation. The extension of the analysis for the daytime conditions was done later on (Rognlien and Weinstock, 1975). The daytime
situation in the equatorial electrojet is distinguished from that of nighttime by two factors. First the zero order density gradients are weaker during daytime and the second, owing to the large background density during daytime dissociative recombination becomes dominant linear wave damping mechanism rather than the ampipolar diffusion. It is found that irregularities attain large amplitudes $\sim 10\%$ and are found to break up into smaller dominant scale sizes in the horizontal westward direction than in the vertical direction. The density waveform in the horizontal direction shows a pronounced steepening in a direction opposite to zero order density gradient whereas in the horizontal direction no tendency towards wave steepening is observed in their calculations. The spectra and shape of the resulting modes were compared with the rocket observations. The observed saw tooth structure in the vertical direction was reproduced in this analysis. A detailed discussion of this theory in light of our results shall be given in chapter VI.

2.2 Neutral Turbulence

2.2.1 General Nature of the Turbulence

Another cause of generation of plasma irregularities is the neutral turbulence. In the following, an attempt would be made to discuss some basic properties of turbulence and the conditions under which turbulence can generate ionisation
irregularities. Some of the experimental observations which support the existence of turbulence would also be discussed.

Turbulence is a condition of fluid motion in which properties of the fluid, such as the heat content, water vapour content etc. are diffused at a rate large compared with the rate of diffusion by molecular motions. A fluid is said to be turbulent if (a) each component of the vorticity is distributed irregularly and aperiodically in time and space, if (b) the flow is characterized by transfer of energy from larger to smaller eddies and if (c) the mean separation of neighbouring fluid particles tends to increase with time. The term eddy denotes the volume of the fluid which moves more or less coherently with respect to the mean flow. The eddy motion need not be and usually is not of a rotating character. The term eddy is often replaced by a more familiar expression 'Scale of motion'. Any turbulent region contains eddies of varying sizes. The size of largest eddy is comparable to the boundaries which enclose the turbulent region whereas the size of smallest eddies would be decided by the point at which the turbulent structures are damped due to viscous forces. The size of smallest eddy is known as 'Kolmogorov microscale' and is given by

$$\lambda_k = \left( \frac{k^3}{\epsilon} \right)^{\frac{1}{4}}$$

where k is the kinematic viscosity and \(\epsilon\) is the rate at which energy is fed to large eddies (Batchelor 1956). The
energy is fed to larger eddies through wind shears. These larger eddies produce smaller and smaller eddies through momentum transfer processes. The damping due to viscosity is small for larger eddies and goes on becoming important for smaller eddies. The limit beyond which viscous damping becomes very enhanced is related to the mean free path of neutral particles.

Thus there is a range of scale sizes extending from very large eddies, which contain energy, down to smallest eddies which can remain turbulent. If one considers that large eddies are the structures much larger than Kolmogorov microscale then there exists a wave number range from $k_0$ (corresponding to large eddies) to $k_k$ (corresponding to Kolmogorov microscale). This range is known as 'Inertial subrange'. Wave numbers smaller than $k_0$ fall in energy containing subrange whereas the wavenumbers higher than $k_k$ fall in the viscous dissipation subrange. In the energy containing subrange it is not very clear so as how the energy is transferred from shearing forces. In the inertial subrange the energy transfer processes are independent of energy contained in the very large eddies and the viscous damping. In the inertial subrange the energy transfer depends only on the parameter $\varepsilon$ namely, the rate at which the energy is fed into very large eddies.

On the basis of dimensional analysis it has been shown that the energy contained in different wave numbers lying in
the inertial subrange can be expressed as following
(Kolmogorov 1941 and Batchelor 1956)

\[ E(k) = \alpha \varepsilon^{2/3} k^{-5/3} \]

where \( E(k) \) is the energy contained in the wave number \( k \) and \( \alpha \) is a constant of proportionality and is usually taken as unity. The spectral index is \(-5/3\) in the inertial subrange. At wave numbers larger than \( k^* \) i.e. scales smaller than Kolmogorov's microscale the energy \( E(k) \) drops off much rapidly than in inertial subrange because viscous damping goes on increasing with higher \( k \). The spectrum in the viscous dissipation range must, therefore, be steeper than that in the inertial subrange.

The turbulence can be generated when either the amount of energy fed into larger eddies is large or when the amount of energy extracted by viscous damping is very small. When the effect of wind shears is large in dumping energy to larger eddies one can use Richardson's criterion for knowing whether turbulence would occur or not. In cases where viscous damping is too small one can use Raynold's criterion to know the same.

Richardson's Criterion

If \( \omega_q \) denotes the angular frequency of oscillation of an adiabatically displaced fluid element and if \( \omega_k \) represents the vertical gradient of the horizontal wind, these quantities
can be represented mathematically as following:

\[ \omega_g = \sqrt{\frac{g}{T} \left( \frac{\partial T}{\partial z} + \frac{g}{C_p} \right)} \]

and

\[ \omega_S = \left( \frac{\partial V_h}{\partial z} \right) \]

where \( g \) is the acceleration due to gravity,
\( T \) is the temperature,
\( V_h \) horizontal wind speed,
\( z \) is the displacement in the vertical direction and
\( C_p \) is the specific heat at constant pressure.

The Richardson number \( R_i \) is defined as the square of the ratio of \( \omega_g \) and \( \omega_S \)

\[ R_i = \left( \frac{\omega_g}{\omega_S} \right)^2 = \frac{g \left( \frac{\partial T}{\partial z} + \frac{g}{C_p} \right)}{T \left( \frac{\partial V_h}{\partial z} \right)^2} \]

Richardson's criterion tells that a medium is locally stable against the onset of the turbulence as long as \( R_i \) exceeds a certain critical value, \( R_{ic} \). There are many numbers suggested for \( R_{ic} \) from different workers such as

\[ R_{ic} = 1 \] (Richardson and Prandt)
\[ = 1/12 \] (Townscend)
\[ = 1/25 \] (David Layzer)
If $R_i < 1$, the medium would be turbulent otherwise it would be stable.

Reynold's Criterion

This criterion was obtained by Osborne Reynolds on the basis of his experiments on incompressible viscous fluids. According to this criterion for incompressible fluids, if a dimensionless quantity $R_e$ (known as Reynolds number) exceeds a critical value the turbulence would be observed. The Reynolds number is given by

$$R_e = \frac{LV}{\nu}$$

where $L$ is the typical scale length of the system; in case of a bounded system $L$ is the diameter of the pipe in which the fluid flows, whereas in unbounded system, it is the scale height, $V$ is the mean velocity of the fluids and $\nu$ is the kinematic viscosity.

The Reynolds number is in essence the ratio of inertial terms to the viscous terms in the Eulerian equations of fluid motion.

2.2.2 Regions of Turbulence in the Ionosphere

Regarding the existence of turbulence in the atmosphere it was theoretically predicted by de Jager (1952) and Steward (1959) that beyond 100 km altitude there should be no turbulence. In accordance with the theoretical predictions
the existence of neutral turbulence in the atmosphere upto about 100 km height has been confirmed experimentally by many workers (Greenhow and Neufeld, 1959, Blamont and de Jager 1962). Some of the ionospheric phenomena such as D-region scatter propagation have been explained on the basis of neutral turbulence.

If the electron-neutral and ion-neutral collision frequencies are greater than their respective gyrofrequencies the movement of neutral can be transmitted to the electrons, because the effect of magnetic field becomes unimportant and collisions play the major role. In the ionosphere as far as the ions are concerned the ion-neutral collision frequency is much higher than ion-gyrofrequency. But in case of electrons it can be seen from the collision frequency values obtained by Thrane and Piggott (1966) that upto about 75 km the electron-neutral collision frequency is greater than electron gyrofrequency. Thus in the region below 75 km or so the collisional effects dominate over magnetic field effects both for electrons and ions. It has been shown by Villars and Weisskopf (1955) that neutral turbulence can produce fluctuations in the electron density in the 80-90 km region. There are essentially two mechanisms which produce fluctuations in the electron density. The first one is due to fluctuations in the density of the carrier medium i.e. the neutrals. The second one is due to turbulent mixing which implies inhomogeneous average electron density, i.e. the presence of electron density gradient.
2.2.3 Electron Density Fluctuations Due to Fluctuations in the Neutral Density

For an eddy of scalesize $L$ one can define an associated eddy velocity $\Delta \mathbf{v}_L$ as a fluctuation that extends over a distance $L$. If $\langle \mathbf{v} \rangle_L$ is defined as velocity averaged over a spatial region of size $L^3$ the perturbed velocity can be defined as

$$ \left( \Delta \mathbf{v}_L \right)^2 = \left\langle \left( \mathbf{v} - \langle \mathbf{v} \rangle_L \right)^2 \right\rangle_L $$

(1)

where $\mathbf{v}$ is the magnitude of velocity change over a distance of $L$. Similarly the perturbed pressure, due to perturbation in velocity, can be defined as $\Delta p_L$. The generation of pressure fluctuations due to velocity fluctuations is given by Bernouilli theorem.

$$ p + \frac{1}{2} \rho \mathbf{v}^2 = \text{constant} $$

(2)

where $\rho$ is the density

from equation (2) it can be easily seen that

$$ |\Delta p_L| \sim \rho \left( \Delta \mathbf{v}_L \right)^2 $$

(3)

since the gas has to satisfy the equation of state, in presence of density fluctuations eqn. (3) will become

$$ \Delta p_L = \gamma \frac{\rho}{\rho_p} \Delta p_L \sim \rho \frac{\Delta \mathbf{v}_L^2}{\mathbf{v}_M^2} $$

(4)
where $Y$ is the adiabatic exponent and $\nu_M^2$ is the mean square velocity.

If one writes the perturbed velocity in terms of initial power supply, so, the following expression is obtained

$$\Delta V_L = \left( \frac{SoL}{\rho} \right)^{1/3}$$

By eliminating $\Delta V_L$ from equations (4) and (5) one gets

$$\left( \Delta \rho_L \right)^2 = \rho^2 \left( \frac{So}{\rho} \right)^{4/3} \frac{L^{4/3}}{\nu_M^4}$$

If the scalesize of the eddy is greater than Kolmogrov micro-scale the density fluctuations are so rapid that there is no substantial electron diffusion. These fluctuations would then be transmitted to electron density $N$ which will have a fluctuation $\delta N$ given by

$$\delta N = \frac{N}{\rho} \delta \rho$$

Substituting the value of $N$ in equation (6) one gets

$$\Delta N_L^2 = N^2 \left( \frac{So}{\rho} \right)^{4/3} \frac{L^{4/3}}{\nu_M^4}$$

2.2.4 Electron Density Fluctuations Due to Turbulent Mixing

This particular mode considered by Villars and Weisskopf (1955) is more applicable for present studies as in this case an average electron density gradient has been
assumed. In earlier case the density was assumed to be homogeneous one. If the medium is collisional and eddies of scale length \( L \) are present the fluctuations in the electron density would be produced and would be given by

\[
\Delta N_L \sim \left( \frac{dN}{dt} \right)_L
\]

The fluctuations in the electron density would be produced only if the life time of the eddy is much shorter than the time scales of the phenomena which tend to change the density. Some of these phenomena are recombination and attachment of electrons. But as the fluctuations are produced the effect of turbulence would be to smear out the gradients in electron density. In the inertial subrange the amplitude of fluctuations depends only on the density gradient and not on the intensity of excitation of turbulence.

Gallet (1955) has shown that in presence of electron density gradient, turbulence can give rise to fluctuations in electron density. He suggested that in presence of electron density gradient an upward moving air blob would carry electrons with it thus causing enhancement in the higher density region and a depletion in low density region. Thus the fluctuations in electron density are created. But since these fluctuations were an order of magnitude less than experimentally observed values, Gallet (1955) suggested an amplification mechanism for the fluctuations in neutral density. In
collision dominated regime these amplified neutral density fluctuations would be transmitted to electron density fluctuations. The amplification mechanism suggested by Gallet (1955) was the following. In presence of a positive background temperature gradient an air blob if displaced upwards loses energy adiabatically and its temperature decreases. As the air blob rises its potential energy increases and the kinetic energy decreases with the result that the temperature decreases. This results in temperature fluctuations and consequently in the neutral density fluctuations. Thus the large neutral density fluctuations are passed on to the electron density which then exhibits enhanced fluctuations.

Dungey (1959) has shown that in presence of a magnetic field a shear flow in neutral air can generate the irregularities in the electron density. He had used equations of motion for each constituent of the gas. As suggested by Dungey the neutral air tries to carry the ionized gas with its motion but the magnetic field resists the motion of the ionized gas across the lines of force. In presence of solenoidal velocity of neutrals he shows that irregularities can be generated.

Thus the existence of turbulence in D-region was theoretically predicted and was confirmed experimentally also. It has also been shown that the fluctuations in neutral density can be transmitted to electron densities when the collisional effects dominate over the magnetic field effects.