CHAPTER II

FUZZY MANPOWER PLANNING MODELS

A few section of this chapter has been communicated to an International Journal for publications.
CHAPTER II
FUZZY MANPOWER PLANNING MODELS ON
VOLUNTARY RETIREMENT SCHEME

In the present global scenario, reduction of the excess manpower in an organization has become an important management strategy to revamp the operation and to improve the manpower efficiency by recruiting persons with latest knowledge. Voluntary Retirement Scheme (VRS) is one of the several organizational transitions having increased attention during the past few years.

The chief goal of VRS is to downsize the number of employees on a given payroll. Voluntary Retirement Scheme (VRS) was introduced in the early 1980’s in Central Public Sector Undertakings (PSU), India, to reduce the surplus and redundant workforce. Downsizing is an intentional and a deliberate decision made by the organization to reduce the workforce that is intended to restructuring of organizations, due to modernizing like introduction of applying new technology and new methods of operation. This may lead the organization to operate economically and to withstand the competition among organizations which have accepted foreign collaboration, innovative methods and technology upgradation.

Generally a business firm may opt for a voluntary retirement scheme under the following circumstances:-

- Recession in the business.
- Intense competition.
- Joint-ventures with foreign collaborations.
- Takeovers and mergers.
- Obsolescence of Product/Technology.

Though the eligibility criteria for VRS vary from company to company, generally, employees who have attained 40 years of age or completed 10 years of service are eligible for voluntary retirement. The scheme applies to all employees including workers and executives, except the directors of a company.
In production oriented organizations reduction of employees is necessary whenever there is surplus of staff strength. At that instant, attempts are made to reduce the staff strength to the required level. One way to reduce the staff strength is the announcement of VRS. This paves way for the young and skillful people to enter the organization and thereby reducing the unemployment problem to a certain extent. It is equally important for an organization to calculate the approximate cost involved in VRS.

It is also known that due to uncertainty and unpredictability of various factors, the cost spent in VRS by the organization can be assumed to be a fuzzy quantity. The first section of this chapter deals with models on VRS using fuzzy Poisson distribution to find the optimal number of exit of employees under the assumption that recruitment takes place or do not take place instantaneously in the organization.

After the announcement of VRS, there may be a chance for the organization to come across crisis situation due to the following reason:

During busy period, if the number of exit of employees exceeds the expected level, then they lack staff members to complete their jobs leading to crisis state.

The second section of this chapter deals with the stationary rate of crisis of the organization over a long period. Application of reliability theory with fuzzy parameters is dealt with to find out the stationary rate of crisis of an organization.

2.1 FUZZY MODELS ON VOLUNTARY RETIREMENT SCHEME:

Poisson Distribution:

Let X be a random variable having the Poisson mass function. If \( P(x) \) stands for the probability that \( X = x \), then

\[
P(x) = \frac{e^{-\lambda} \lambda^x}{x!}
\]  

(2.1)

for \( x = 0, 1, 2, \ldots \) and parameter \( \lambda > 0 \).
Fuzzy Poisson distribution:

If the parameter $\lambda$ is of imprecise information, then $\lambda$ can be substituted by the fuzzy parameter $\tilde{\lambda} > 0$ to produce fuzzy Poisson probability mass function. Let the fuzzy number $\tilde{P}(x)$ denote the fuzzy probability that $X = x$, then the $\alpha$-cut of this fuzzy probability is given by

$$\tilde{P}(x)[\alpha] = \left\{ e^{-\lambda} \frac{e^{\lambda x}}{x!} \middle| \lambda \in \tilde{\lambda} [\alpha] \right\}$$

(2.2)

for all $\alpha \in [0,1]$. We say that $X$ follows a Fuzzy Poisson distribution [22], and is denoted by $X \sim FP (\tilde{\lambda})$.

M. Jeeva and Rajalakshmi Rajagopal [39] have developed a model on finding the optimal number of exit of personnel under voluntary retirement scheme on the assumption that the number of employees who opt to retire under VRS is taken to be a random variable following Poisson distribution. Motivated by their work, we had developed a model on the assumption that, the number of exit of employees on announcement of VRS is a random variable that follows a fuzzy Poisson distribution with parameter $\tilde{\lambda}$. Using a fuzzy approach in this model, we are able to find the optimal number of exit of employees under VRS.

2.1.1 Derivation of Model I:

In this model we obtain the optimal number of exit of employees on VRS without recruitment in the place of excess exit.

Let us assume that the number of employees quitting the job on VRS be a random variable $X$ following Fuzzy Poisson Distribution with the parameter $\tilde{\lambda}$ being a fuzzy number. Then we denote it by $X \sim FP (\tilde{\lambda})$. The assumptions made in this model are as follows:
Assumptions and Notations:

1. \( X \) – A random variable denoting the number of employees quitting the job by VRS.

2. \( \tilde{f}(x) \) – Fuzzy probability mass function of the random variable \( X \) with the parameter \( \tilde{\lambda} > 0 \).

3. \( x_0 \) – The optimal number of exit of employees on VRS.

4. \( \tilde{c}_V \) – [\( c_{V1}, c_{V2}, c_{V3} \)], the fuzzy cost associated with the exit of an employee on VRS.

5. \( \tilde{c}_E \) – [\( c_{E1}, c_{E2}, c_{E3} \)], the excess cost per person(when the exit of employees > \( x_0 \))

6. \( \tilde{c}_F \) – [\( c_{F1}, c_{F2}, c_{F3} \)], the failure cost per person(when the exit of employees < \( x_0 \))

7. The failure cost is assumed to be less than the excess cost. That is \( \tilde{c}_F \leq \tilde{c}_E \).

8. \( \tilde{F}(x_0) \) – Distribution function of \( X \).

9. Recruitment is not instantaneous, in the place of excess exits on VRS.

Mathematical Formulation of the VRS model:

If the number of exit of employees by VRS is less than the optimal (or expected) level, then the scheme is considered to be a failure one and there is a cost associated with it known as the failure cost. Similarly when the number of exit of employees on VRS exceeds the expected level, the associated cost is known as the excess cost. Due to imprecise nature of costs involved in VRS the failure and excess cost are assumed to be fuzzy numbers. The fuzzy costs associated with this model are considered to be triangular or trapezoidal fuzzy numbers:

If \( x_0 \) is the optimal number of exit of employees and \( x \) is the expected number of exit of employees then,

Expected failure cost when \( x \leq x_0 = \tilde{c}_F \sum_{x=0}^{x_0}(x_0 - x) \tilde{f}(x) \)

Expected excess cost when \( x_0 \leq x = \tilde{c}_E \sum_{x=x_0+1}^{N}(x - x_0) \tilde{f}(x) \)

Expected voluntary retirement cost = \( \tilde{c}_V \sum_{x=0}^{N} x \tilde{f}(x) \)
Therefore the total expected cost $\tilde{C}$ for optimal number of exits is

$$[E (\tilde{C})]_{x_0} = \tilde{C}_F \sum_{x=0}^{x_0} (x_0 - x) \tilde{f}(x) + \tilde{C}_E \sum_{x=x_0+1}^{N} (x - x_0) \tilde{f}(x) + \tilde{C}_V \sum_{x=0}^{N} x \tilde{f}(x)$$

(2.3)

Hence

$$[E (\tilde{C})]_{x_0+1} = \tilde{C}_F \sum_{x=0}^{x_0+1} (x_0 + 1 - x) \tilde{f}(x) + \tilde{C}_E \sum_{x=x_0+2}^{N} (x - [x_0 + 1]) \tilde{f}(x)$$

$$+ \tilde{C}_V \sum_{x=0}^{N} x \tilde{f}(x)$$

$$= \tilde{C}_F \sum_{x=0}^{x_0} (x_0 + 1 - x) \tilde{f}(x) + \tilde{C}_F [(x_0 + 1) - (x_0 + 1)] \tilde{f}(x)$$

$$+ \tilde{C}_E \sum_{x=x_0+1}^{N} (x - [x_0 + 1]) \tilde{f}(x) + \tilde{C}_E [(x_0 + 1) - (x_0 + 1)] \tilde{f}(x)$$

$$+ \tilde{C}_V \sum_{x=0}^{N} x \tilde{f}(x)$$

$$= \tilde{C}_F \sum_{x=0}^{x_0} (x_0 - x) \tilde{f}(x) + \tilde{C}_F \sum_{x=0}^{x_0} \tilde{f}(x) + \tilde{C}_E \sum_{x=x_0+1}^{N} (x - x_0) \tilde{f}(x)$$

$$- \tilde{C}_E \sum_{x=x_0+1}^{N} \tilde{f}(x) + \tilde{C}_V \sum_{x=0}^{N} x \tilde{f}(x)$$

$$= [E (\tilde{C})]_{x_0} + \tilde{C}_F \sum_{x=0}^{x_0} \tilde{f}(x) - \tilde{C}_E \sum_{x=x_0+1}^{N} \tilde{f}(x)$$

$$= [E (\tilde{C})]_{x_0} + (\tilde{C}_F + \tilde{C}_E) \sum_{x=0}^{x_0} \tilde{f}(x) - \tilde{C}_E \sum_{x=x_0+1}^{N} \tilde{f}(x)$$

$$= [E (\tilde{C})]_{x_0} + [\tilde{C}_F + \tilde{C}_E] \sum_{x=0}^{x_0} \tilde{f}(x) - \tilde{C}_E$$

Thus

$$[E (\tilde{C})]_{x_0+1} = [E (\tilde{C})]_{x_0} + (\tilde{C}_F + \tilde{C}_E) \tilde{f}(x_0) - \tilde{C}_E$$

(2.4)

Similarly

$$[E (\tilde{C})]_{x_0-1} = \tilde{C}_F \sum_{x=0}^{x_0-1} (x_0 - 1 - x) \tilde{f}(x) + \tilde{C}_E \sum_{x=x_0}^{N} (x - x_0 + 1) \tilde{f}(x)$$

$$+ \tilde{C}_V \sum_{x=0}^{N} x \tilde{f}(x)$$

$$= \tilde{C}_F \sum_{x=0}^{x_0} (x_0 - 1 - x) \tilde{f}(x) - \tilde{C}_F \tilde{f}(x_0)$$

$$+ \tilde{C}_E \sum_{x=x_0+1}^{N} (x - x_0 + 1) \tilde{f}(x) + \tilde{C}_E \tilde{f}(x_0) + \tilde{C}_V \sum_{x=0}^{N} x \tilde{f}(x)$$
\[ = \hat{C}_F \sum_{x=0}^{x_0} (x_0 - x) \hat{f}(x) - \hat{C}_F \sum_{x=0}^{x_0} \hat{f}(x) + \hat{C}_F \hat{f}(x_0) + \hat{C}_E \hat{f}(x_0) \\
+ \hat{C}_E \sum_{x=x_0+1}^{N} (x - x_0) \hat{f}(x) + \hat{C}_E \sum_{x=x_0+1}^{N} \hat{f}(x) + \hat{C}_V \sum_{x=0}^{N} x \hat{f}(x) \]

\[ = \hat{C}_F \sum_{x=0}^{x_0}(x_0 - x) \hat{f}(x) + \hat{C}_E \sum_{x=x_0}^{N} (x - x_0) \hat{f}(x) + \hat{C}_V \sum_{x=0}^{N} x \hat{f}(x) \]

\[ + (\hat{C}_F + \hat{C}_E) \hat{f}(x_0) - \hat{C}_F \sum_{x=0}^{x_0} \hat{f}(x) + \hat{C}_E [ \sum_{x=0}^{N} \hat{f}(x) - \sum_{x=0}^{x_0} \hat{f}(x) ] \]

\[ = [E (\hat{C})]_{x_0} + (\hat{C}_F + \hat{C}_E) \hat{f}(x_0) - (\hat{C}_F + \hat{C}_E) \sum_{x=0}^{x_0} \hat{f}(x) + \hat{C}_E \]

\[ = [E (\hat{C})]_{x_0} + (\hat{C}_F + \hat{C}_E) \hat{f}(x_0) - (\hat{C}_F + \hat{C}_E) \hat{f}(x_0) \]

\[ - (\hat{C}_F + \hat{C}_E) \sum_{x=0}^{x_0-1} \hat{f}(x) + \hat{C}_E \]

Thus

\[ [E (\hat{C})]_{x_0-1} = [E (\hat{C})]_{x_0} - (\hat{C}_F + \hat{C}_E) \hat{f}(x_0 - 1) + \hat{C}_E \quad (2.5) \]

Using equations (2.3), (2.4) and (2.5), the condition for \( x_0 \) to be optimal is given by

\[ (\hat{C}_F + \hat{C}_E) \hat{f}(x_0) - \hat{C}_E \geq 0 \quad (2.6) \]

\[ - (\hat{C}_F + \hat{C}_E) \hat{f}(x_0 - 1) + \hat{C}_E \geq 0 \quad (2.7) \]

From equations (2.6) and (2.7), it follows

\[ \hat{f}(x_0 - 1) \leq \frac{\hat{C}_E}{(\hat{C}_F + \hat{C}_E)} \leq \hat{f}(x_0) \quad (2.8) \]

By the definition of Fuzzy Poisson distribution [22], equation (2.8) becomes

\[ \sum_{x=0}^{x_0-1} \hat{f}(x) [\alpha] \leq \left[ \frac{\hat{C}_E}{(\hat{C}_F + \hat{C}_E)} \right] [\alpha] \leq \sum_{x=0}^{x_0} \hat{f}(x) [\alpha] \quad (2.9) \]

for all \( \alpha \in [0,1] \).

The value of \( x_0 \), which satisfies equation (2.9) will be the optimal number of exit of employees on VRS.

This model is illustrated with a numerical example.
Numerical Illustration:

Consider an organization which announces VRS. The objective of the organization is to find the optimal number of exit of employees on announcement of VRS with a minimum cost. Let the number of exit of employees follow fuzzy Poisson distribution with the parameter \( \tilde{\lambda} = [3, 5, 7] \). Let the costs involved are assumed to be triangular fuzzy numbers and are given as

\[
\tilde{c}_F = [c_{F1}, c_{F2}, c_{F3}] = [1000, 2000, 3000], \quad \tilde{c}_E = [c_{E1}, c_{E2}, c_{E3}] = [3000, 5000, 7000]
\]

Then

\[
\left[ \frac{\tilde{c}_E}{(\tilde{c}_F + \tilde{c}_E)} \right] = \left[ \frac{[c_{E1}, c_{E2}, c_{E3}]}{[c_{E1} + c_{E1}, c_{E2} + c_{E2}, c_{E3} + c_{E3}]} \right]
\]

\[
= \left[ \frac{[3000, 5000, 7000]}{[4000, 5000, 7000]} \right]
\]

That is,

\[
\left[ \frac{\tilde{c}_E}{(\tilde{c}_F + \tilde{c}_E)} \right] = [0.3, 0.71, 1.75] \quad (2.10)
\]

\[
\left[ \frac{\tilde{c}_E}{(\tilde{c}_F + \tilde{c}_E)} \right] [\alpha] = [0.3 + 0.41 \alpha, 1.75 - 1.04 \alpha] \quad (2.11)
\]

and

\[
\tilde{\lambda}[\alpha] = [3 + 2 \alpha, 7 - 2 \alpha] \quad (2.12)
\]

for all \( \alpha \in [0, 1] \)

When \( \alpha = 1/2 \)

\[
\left[ \frac{\tilde{c}_E}{(\tilde{c}_F + \tilde{c}_E)} \right] [\alpha] = [0.505, 1.23] \quad (2.13)
\]

and

\[
\tilde{\lambda}[\alpha] = [4, 6] \quad (2.14)
\]

By the definition of \( \alpha - \text{cut} \), the probability mass function of Poisson distribution is

\[
\hat{f}(x) [\alpha] = \left\{ \frac{e^{-\tilde{\lambda}x}}{x!} / \tilde{\lambda} \in [4, 6] \right\} = \left\{ \begin{array}{l}
\left[ \frac{e^{-6}x}{x!}, \frac{e^{-4}x}{x!} \right] & \text{for} \ 0 \leq x \leq 4 \\
\left[ \frac{e^{-4}x}{x!}, \frac{e^{-6}x}{x!} \right] & \text{for} \ 4 \leq x \leq \infty
\end{array} \right.
\]

From (2.8), we have

\[
\sum_{x=0}^{4} \left[ \frac{e^{-6}x}{x!}, \frac{e^{-4}x}{x!} \right] + \sum_{x=5}^{\infty} \left[ \frac{e^{-4}x}{x!}, \frac{e^{-6}x}{x!} \right] \leq [0.505, 1.23] \leq \sum_{x=0}^{4} \left[ \frac{e^{-6}x}{x!}, \frac{e^{-4}x}{x!} \right] + \sum_{x=5}^{\infty} \left[ \frac{e^{-4}x}{x!}, \frac{e^{-6}x}{x!} \right] \quad (2.15)
\]
Using Table 2.1,
Midpoint of LHS of equation (2.15) ≤ Midpoint of [0.505, 1.23]
≤ Midpoint of RHS of equation (2.15)

On applying the interval ranking [3], we infer from the above inequality that,

\[ [0.6073, 1.0912] \leq [0.505, 1.23] \leq [0.637, 1.1953] \]  
\[
\text{TABLE 2.1} - \alpha\text{-CUTS OF PROBABILITY MASS FUNCTION AND DISTRIBUTION FUNCTION}
\]

<table>
<thead>
<tr>
<th>(X)</th>
<th>(\hat{f}(x)[\alpha])</th>
<th>(\bar{F}(x)[\alpha])</th>
<th>Midpoint of (\bar{F}(x)[\alpha])</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>[0.0025, 0.0183]</td>
<td>[0.0025, 0.0183]</td>
<td>0.0104</td>
</tr>
<tr>
<td>1</td>
<td>[0.0150, 0.0732]</td>
<td>[0.0175, 0.0915]</td>
<td>0.0545</td>
</tr>
<tr>
<td>2</td>
<td>[0.0450, 0.1464]</td>
<td>[0.0625, 0.2379]</td>
<td>0.1502</td>
</tr>
<tr>
<td>3</td>
<td>[0.0900, 0.1952]</td>
<td>[0.1525, 0.4331]</td>
<td>0.2928</td>
</tr>
<tr>
<td>4</td>
<td>[0.1350, 0.1952]</td>
<td>[0.2875, 0.6283]</td>
<td>0.4579</td>
</tr>
<tr>
<td>5</td>
<td>[0.1562, 0.1620]</td>
<td>[0.4437, 0.7903]</td>
<td>0.6170</td>
</tr>
<tr>
<td>6</td>
<td>[0.1041, 0.1620]</td>
<td>[0.5478, 0.9523]</td>
<td>0.7501</td>
</tr>
<tr>
<td>7</td>
<td>[0.0595, 0.1389]</td>
<td>[0.6073, 1.0912]</td>
<td>0.8493</td>
</tr>
<tr>
<td>8</td>
<td>[0.0297, 0.1041]</td>
<td>[0.6370, 1.1953]</td>
<td>0.9161</td>
</tr>
</tbody>
</table>

This implies that the solution to equation (2.15) is \(x_0 = 8\). Thus if \(\hat{\lambda} = [3, 5, 7]\), then the optimal number of exit of employees will be \(x_0 = 8\).

2.1.2 Derivation of Model II:

In Model II we obtain a model similar to that of Model I, but here the recruitment is made in the place of excess exit of employee and the associated recruitment cost is assumed to be a triangular fuzzy number.

If the number of employee exiting on VRS exceeds the optimal number, then recruitment takes place and the corresponding costs are taken into account in this model. Let us assume that the number of employee quitting the job on VRS be a random variable \(X\) following Fuzzy Poisson Distribution with the parameter \(\hat{\lambda}\) being
a fuzzy number. Then we denote it by $X \sim \text{FP} (\tilde{\lambda})$. The assumptions made in this model are as follows:

**Assumptions and Notations:**

1. $X$ – A random variable denoting the number of employees quitting the job by VRS.
2. $\tilde{f}(x)$ – Fuzzy probability mass function of the random variable $X$ with the parameter $\tilde{\lambda} > 0$.
3. $x_0$ – The optimal number of exit of employees on VRS.
4. $\tilde{c}_V = [c_{V1}, c_{V2}, c_{V3}]$ – the fuzzy cost associated with the exit of an employee on VRS.
5. $\tilde{c}_E = [c_{E1}, c_{E2}, c_{E3}]$ – the excess cost per person (when the exit of employees $> x_0$).
6. $\tilde{c}_F = [c_{F1}, c_{F2}, c_{F3}]$ – the failure cost per person (when the exit of employees $< x_0$).
7. $\tilde{c}_R = [c_{R1}, c_{R2}, c_{R3}]$ – The expected recruitment cost as the recruitment is instantaneous, in the place of excess exits on VRS.
8. The failure cost and the recruitment cost are assumed to be less than the excess cost. That is $\tilde{c}_F \leq \tilde{c}_E$.
9. $F(x_0)$ – Distribution function of $X$.

**Mathematical Formulation of Model II:**

As in model I, the total expected fuzzy cost $\tilde{C}$ for optimal number of exit of employees associated with the recruitment is given by:

$$
\left[ E (\tilde{C}) \right]_{x_0} = \tilde{c}_F \sum_{x=0}^{x_0} (x_0 - x) \tilde{f}(x) + \tilde{c}_E \sum_{x=x_0+1}^{N} (x - x_0) \tilde{f}(x) + \tilde{c}_V \sum_{x=0}^{N} x \tilde{f}(x) + \tilde{c}_R \sum_{x=x_0+1}^{N} (x - x_0) \tilde{f}(x) \quad (2.17)
$$

Using arguments similar to those of equations (2.4) and (2.5) we obtain the following equations

$$
\left[ E (\tilde{C}) \right]_{x_0+1} = \left[ E (\tilde{C}) \right]_{x_0} + (\tilde{c}_F + \tilde{c}_E + \tilde{c}_R) F(x_0) - (\tilde{c}_E + \tilde{c}_R) \quad (2.18)
$$
The optimal value $x_0$ of $x$ using equations (2.17), (2.18) and (2.19) is such that

$$(C_F + C_E + C_R) \tilde{F}(x_0) - (C_E + C_R) \geq 0$$

(2.20)

$$-(C_F + C_E + C_R) \tilde{F}(x_0 - 1) + (C_E + C_R) \geq 0$$

(2.21)

This implies

$$\tilde{F}(x_0 - 1) \leq \frac{(C_E + C_R)}{(C_F + C_E + C_R)} \leq \tilde{F}(x_0)$$

(2.22)

By the definition of Fuzzy Poisson distribution, equation (2.22) becomes

$$\sum_{x=0}^{x_0-1} f(x) [\alpha] \leq \left[ \frac{C_E}{(C_F + C_E + C_R)} \right] [\alpha] \leq \sum_{x=0}^{x_0} \tilde{f}(x) [\alpha]$$

(2.23)

for all $\alpha \in [0,1]$.

The value of $x_0$ that satisfies equation (2.23) is the optimal value and denotes the optimal number of exit of employees in this model.

This model is illustrated with a numerical example.

**Numerical Illustration:**

Consider an organization which recruits employees in the place of excess exit of employees on VRS. Assume that the recruitment is instantaneous. Let the number of exit of employees on announcement of VRS be a random variable that follows fuzzy Poisson distribution with fuzzy parameter $\lambda = (3, 5, 7)$. The objective of the organization is to find the optimal number of exit of employees at a minimum cost.

$$C_F = [c_{F1}, c_{F2}, c_{F3}] = [600, 800, 1200],$$

$$C_E = [c_{E1}, c_{E2}, c_{E3}] = [1000, 1500, 2000]$$

and

$$C_R = [c_{R1}, c_{R2}, c_{R3}] = [500, 800, 1000]$$

Then

$$\left[ \frac{C_F + C_R}{(C_F + C_E + C_R)} \right] = \left[ \frac{c_{E1}c_{E2}c_{E3}}{c_{E1}c_{E2}c_{E3}} + \frac{c_{R1}c_{R2}c_{R3}}{c_{R1}c_{R2}c_{R3}} + \frac{c_{F1}c_{F2}c_{F3}}{c_{F1}c_{F2}c_{F3}} \right]$$

$$= [0.357, 0.742, 1.429]$$
\[
\left[ \frac{\bar{C}_E + \bar{C}_R}{\bar{C}_E + \bar{C}_R} \right] [\alpha] = [0.357 + 0.385 \alpha, 1.429 - 0.687 \alpha]
\]

(2.24)

and

\[
\hat{\lambda}[\alpha] = [3 + 2\alpha, 7 - 2\alpha]
\]

(2.25)

for all \(\alpha \in [0, 1]\)

When \(\alpha = 1/2\)

\[
[\frac{(\bar{C}_E + \bar{C}_R)}{(\bar{C}_E + \bar{C}_R)}] [\alpha] = [0.5495, 1.0855]
\]

(2.26)

and

\[
\hat{\lambda}[\alpha] = [4, 6]
\]

(2.27)

\[
\bar{f}(x) [\alpha] = \left\{ \frac{e^{-\lambda x}}{x!} / \hat{\lambda} \in [4, 6] \right\}
\]

\[
= \left\{ \begin{array}{ll}
\frac{e^{-6x}}{x!}, & 0 \leq x \leq 4 \\
\frac{e^{-4x}}{x!}, & 4 \leq x \leq \infty 
\end{array} \right.
\]

\[
= \sum_{x=0}^{4} \left[ \frac{e^{-6x}}{x!}, \frac{e^{-4x}}{x!} \right] + \sum_{x=5}^{x_0-1} \left[ \frac{e^{-4x}}{x!}, \frac{e^{-6x}}{x!} \right]
\]

\[
[0.5495, 1.0855] \leq \sum_{x=0}^{4} \left[ \frac{e^{-6x}}{x!}, \frac{e^{-4x}}{x!} \right] + \sum_{x=5}^{x_0} \left[ \frac{e^{-4x}}{x!}, \frac{e^{-6x}}{x!} \right]
\]

(2.28)

The value of \(x_0\) which satisfies the condition (2.28) will be the optimal value of \(x\).

Using Table 2.1 and by the same procedure followed in the example of model I, the optimal number of exit of employees with instantaneous recruitment is \(x_0 = 7\).

\section*{2.2 STATIONARY RATE OF CRISIS:}

In any organization vacancies occur at any instant of time due to so many factors, such as retirement, resignation, announcement of VRS, illness, death, accidents, etc. This affects the normal routine work of the organization.

In continuation with the previous section, if the exit of employees on announcement of VRS reaches the optimal level then the organization has right number of employees to do the right number of jobs with the assumptions that no
employee is idle. Such a period is called a busy period. Otherwise it is called lean period.

Suppose there is excess exit of employees then there is a chance that the organization may run short of employees. If at this situation a busy period arises, then the organization has to face crisis, due to lack of employees.

The overall wastage process in manpower system so far has been dealt with stochastic process. But due to the prevalence of uncertainty in human behavior the manpower system can be dealt with fuzzy set theory. Here in this section we derive an expression for a busy manpower system under crisis situation in a fuzzy environment. The computation may help the decision maker to take necessary action to overcome the crisis.

Using the principle and application of M. Jeeva [40], the stationary rate of crisis under fuzzy environment can be determined. Earlier works on stationary rate of crisis in classical theory can be seen in Gaver [26], Yadavalli and Botha [75].

**Definitions:**

1. **Busy:** The system is said to be busy if it has considerably sufficient work to allot to its employees in such a way that there is no idle time for the employees.

2. **Lean:** The system is said to be lean if it not busy. That is there is a possibility that some of its employees may have idle time.

3. **Intermittent Busy Manpower System:** A manpower system is said to be Intermittently Busy Manpower System, if the busy and lean periods occur alternately in the system.

4. **Crisis:** If the number of exit of employees is more than the expected level of the organization and during this situation a busy period arises, then the organization is said to face a crisis situation.
(5) **Stationary Rate of Crisis:** Stationary rate of crisis of an organization is the annual frequency (that is the number of times the crisis occurs per unit point of time, usually taken as a year) of the occurrence of crisis in the long run (i.e. $t \to \infty$).

**Assumptions:**

1. The busy and lean periods occur alternately.
2. The time $T$, for which the staff strength remains up to the required (expected) level follows exponential distribution with parameter $\lambda$.
3. The time $R$, required for completing a recruitment drive for filling up vacancies follows exponential distribution with parameter $\mu$.
4. $T$ and $R$ are independent
5. The ‘Busy’ period is exponentially distributed with parameter $\alpha$, and the ‘Lean’ period is exponentially distributed with parameter $\beta$.
6. There is a recruitment board for the organization which starts its function as soon as a vacancy arises.
7. If an employee leaves the organization during the busy / lean period, the recruitment processes is immediately initiated and the recruitment is done regardless of whether the busy / lean period arises or not.

**2.2.1 Description of the Model:**

Let $(Z(t), t > 0)$ be the stochastic process depicting the state of the manpower system with the state space \{0, 1, 2, 3\} corresponding to various situations that arise in the organization and described in the following table.

**System States:**

Here State 2 represents the crisis state in the organization.
TABLE 2.2 - STATUS OF VARIOUS SYSTEM STATES

<table>
<thead>
<tr>
<th>State</th>
<th>Staff Strength After The Announcement of VRS</th>
<th>Busy / Lean Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Expected</td>
<td>Busy</td>
</tr>
<tr>
<td>1</td>
<td>Expected</td>
<td>Lean</td>
</tr>
<tr>
<td>2</td>
<td>Less than Expected</td>
<td>Busy</td>
</tr>
<tr>
<td>3</td>
<td>Less than Expected</td>
<td>Lean</td>
</tr>
</tbody>
</table>

Let \( p_i(t) = P[Z(t) = i] \), \( i = 0, 1, 2, 3 \)

Since our interest is in the stationary behavior of the system, let

\[
\lim_{t \to \infty} p_i(t) = p_i
\]

The transition diagram for the model is given below:

![Transition Diagram](image)

**Fig 2.1: Transition Diagram**

Derivation of differential equations for the fuzzy probabilities \( p_i(t) \) are given below

\[
p_i(t+h) = P[Z(t+h) = 0 \mid Z(t) = 1] p_1(t) + P[Z(t+h) = 0 \mid Z(t) = 2] p_2(t)
+ P[Z(t+h) = 0 \mid Z(t) = 0] p_0(t)
\]

\[
= \beta hp_1(t) + \mu hp_2(t) + [1 - (\alpha + \lambda) h] p_0(t) + o(h)
\]
Hence
\[ \lim_{h \to 0} \frac{p_0(t+h) - p_0(t)}{h} = \beta p_1(t) + \mu p_2(t) - (\alpha + \lambda) p_0(t) \]
Therefore
\[ p_0'(t) = \beta p_1(t) + \mu p_2(t) - (\alpha + \lambda) p_0(t) \]
Similarly
\[ p_1'(t) = \alpha p_0(t) + \mu p_3(t) - (\beta + \lambda) p_1(t) \]
\[ p_2'(t) = \lambda p_0(t) + \beta p_3(t) - (\alpha + \mu) p_2(t) \]
\[ p_3'(t) = \lambda p_1(t) + \alpha p_2(t) - (\beta + \mu) p_3(t) \]
Allowing \( t \to \infty \), we get the following equations
\[ \beta p_1(t) + \mu p_2(t) - (\alpha + \lambda) p_0(t) = 0 \quad (2.29) \]
\[ \alpha p_0(t) + \mu p_3(t) - (\beta + \lambda) p_1(t) = 0 \quad (2.30) \]
\[ \lambda p_0(t) + \beta p_3(t) - (\alpha + \mu) p_2(t) = 0 \quad (2.31) \]
\[ \lambda p_1(t) + \alpha p_2(t) - (\beta + \mu) p_3(t) = 0 \quad (2.32) \]
These equations can be solved uniquely by using the fact that \( \sum_{i=0}^{3} p_i = 1 \), and the solutions are as follows
\[
\begin{align*}
p_0 &= \frac{\beta \mu}{(\alpha + \beta)(\lambda + \mu)} \\
p_1 &= \frac{\alpha \mu}{(\alpha + \beta)(\lambda + \mu)} \\
p_2 &= \frac{\beta \lambda}{(\alpha + \beta)(\lambda + \mu)} \\
p_3 &= \frac{\alpha \lambda}{(\alpha + \beta)(\lambda + \mu)}
\end{align*}
\]
\( (2.33) \)

2.2.2 Derivation of Stationary Rate of Crisis:

\[ P[ \text{Crisis occurs in } (t, t + h)] = P[ \text{Crisis in } (t, t + h)/Z(t) = 0] P[Z(t) = 0] \]
\[ + P[ \text{Crisis occurs in } (t, t + h)/Z(t) = 3] P[Z(t) = 3] \]
\[ + o(h) \]
\[ = P[Z(t, t + h) = 2 / Z(t) = 0] P[Z(t) = 0] \]
\[ + P[Z(t, t + h) = 2 / Z(t) = 3] P[Z(t) = 3] + o(h) \]
\[= (\lambda h + o(h))p_0(t) + (\beta h + o(h)) p_3(t) + o(h)\]

\[= (\lambda p_0(t) +\beta p_3(t)) h + o(h)\]

So the Rate of Crisis in the organization at time ‘t’

\[C_t(1) = \lambda p_0(t) + \beta p_3(t)\]

Denoting the stationary rate of crisis by \(C_\infty(1)\), we have

\[C_\infty(1) = \lim_{t \to \infty} (\lambda p_0(t) + \beta p_3(t))\]

\[= \lambda p_0 + \beta p_3\]

\[= \frac{\beta \lambda (\alpha + \mu)}{(\alpha + \beta)(\lambda + \mu)} \quad (2.34)\]

**Particular Case:**

It should be observed that for an organization with same average busy time and with required number of staff strength time, which is \(\lambda = \alpha\) whatever may be the recruitment time, the stationary rate of crisis is given by

\[C_\infty(1) = \frac{\beta \lambda}{(\alpha + \beta)} = \frac{\beta \lambda}{(\lambda + \beta)} \quad (2.35)\]

and is dependent on \(\beta\) and \(\lambda\).

Clearly if \(\beta\) increases, the average lean period decreases. Hence the crisis rate increases. Similarly if \(\lambda\) decreases, the average duration of expected staff strength decreases with the increase in the crisis rate.

Now if the parameters \(\lambda, \alpha, \beta,\) and \(\mu\) are uncertain, then they all are replaced with the fuzzy parameters \(\tilde{\lambda}, \tilde{\alpha}, \tilde{\beta},\) and \(\tilde{\mu}\). Then equation (2.29) and (2.32) becomes

\[\tilde{p}_0 = \frac{\tilde{\beta} \tilde{\mu}}{(\tilde{\alpha} + \tilde{\beta})(\tilde{\lambda} + \tilde{\mu})} \quad (2.36)\]
The Model is illustrated with an example.

**Numerical Illustrations:**

1. Consider an organization where the average busy period lasts longer than the average lean period with the following parameters:

   \( \tilde{\lambda} = [1, 2, 3], \tilde{\mu} = [4, 6, 8], \bar{\alpha} = [0, 2, 4] \) and \( \bar{\beta} = [5, 6, 7] \)

   The values of \( \tilde{p}_i \) (i=0, 1, 2, 3) and the stationary rate of crisis \( C_\infty \) (1) are obtained as:

   \[
   \tilde{p}_0 = \frac{\bar{\beta} \tilde{\mu}}{(\bar{\alpha} + \bar{\beta})(\tilde{\lambda} + \tilde{\mu})} = [20, 36, 56] / [25, 64, 121]
   
   = [20/121, 36/64, 56/25]
   
   = [.165, 0.563, 2.24]
   
   \tilde{p}_1 = \frac{\bar{\alpha} \tilde{\mu}}{(\bar{\alpha} + \bar{\beta})(\tilde{\lambda} + \tilde{\mu})} = [0, 0.188, 1.28]
   
   \tilde{p}_2 = \frac{\bar{\beta} \tilde{\lambda}}{(\bar{\alpha} + \bar{\beta})(\tilde{\lambda} + \tilde{\mu})} = [0.041, 0.188, 0.84]
   
   \tilde{p}_3 = \frac{\bar{\alpha} \tilde{\lambda}}{(\bar{\alpha} + \bar{\beta})(\tilde{\lambda} + \tilde{\mu})} = [0, 0.063, 0.48]
This implies that the crisis rate of the organization is around 1.5.

2. For an organization, where the average lean period lasts longer than the average busy period with the following parameters:

\[ \tilde{\lambda} = [1, 2, 3], \quad \bar{\mu} = [4, 6, 8], \quad \tilde{\alpha} = [5, 6, 7] \quad \text{and} \quad \tilde{\beta} = [0, 2, 4] \]

Then the values of

\[ \hat{p}_0 = [0.000, 0.1875, 1.28] \]
\[ \hat{p}_1 = [0.165, 0.5625, 2.24] \]
\[ \hat{p}_2 = [0.000, 0.0625, 0.48] \]
\[ \hat{p}_3 = [0.041, 0.1875, 0.84] \]
\[ C_\infty (1) = [0.000, 0.75, 7.2] \]

This implies the crisis for this example is around 0.75.

From the above two examples, we note that the stationary rate of crisis is more in the first organization than in the second. That is, the first organization is busy with larger busy periods and shorter lean periods on the average and hence a higher crisis rate is natural.

3. Consider an organization with the following values of the parameters

\[ \bar{\mu} = [4, 6, 8] \]
\[ \tilde{\alpha} = [0, 2, 4] \]

and \[ \tilde{\beta} = [5, 6, 7] \]

With variations in \( \tilde{\lambda} \), the stationary rate of crisis is tabulated below using Excel sheet.
**TABLE 2.3 - VALUE OF THE CRISIS RATE WITH VARIATION IN $\tilde{\lambda}$**

<table>
<thead>
<tr>
<th>S.No</th>
<th>$\tilde{\lambda}$</th>
<th>Stationary rate of crisis $C_\infty$ (1)</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>[0, 50, 100]</td>
<td>[0, 5.4, 420]</td>
<td>Around 5.4</td>
</tr>
<tr>
<td>2.</td>
<td>[50, 100, 150]</td>
<td>[0.6, 5.7, 46.7]</td>
<td>Around 5.7</td>
</tr>
<tr>
<td>3.</td>
<td>[100, 150, 200]</td>
<td>[0.9, 5.8, 32.3]</td>
<td>Around 5.8</td>
</tr>
<tr>
<td>4.</td>
<td>[150, 200, 250]</td>
<td>[1.1, 5.8, 27.2]</td>
<td>Around 5.8</td>
</tr>
<tr>
<td>5.</td>
<td>[200, 250, 300]</td>
<td>[1.2, 5.9, 24.7]</td>
<td>Around 5.9</td>
</tr>
<tr>
<td>6.</td>
<td>[250, 300, 350]</td>
<td>[1.3, 5.9, 23.2]</td>
<td>Around 5.9</td>
</tr>
<tr>
<td>7.</td>
<td>[300, 350, 400]</td>
<td>[1.3, 5.9, 22.1]</td>
<td>Around 5.9</td>
</tr>
<tr>
<td>8.</td>
<td>[350, 400, 450]</td>
<td>[1.4, 5.9, 21.4]</td>
<td>Around 5.9</td>
</tr>
</tbody>
</table>

It is clear from the table as well as from the graph that the crisis rate $C_\infty$ increases as $\tilde{\lambda}$ increases (in crisp sense). The value of the crisis rate remains constant for higher values of $\tilde{\lambda}$. The minimum value of $\tilde{\lambda} (= [200, 250, 300])$ for which $C_\infty$ remains constant (= around 5.9) is called the threshold point for the crisis in this problem.

**Graph 2.1 Crisis Rate for Graph Variations in $\tilde{\lambda}$**
2.3 CONCLUSION:

The two models, one with recruitment and another without recruitment provide us the optimal number of exits on VRS with minimum fuzzy cost. These models suggest guidelines to the organization which opts for VRS, as when to call off the VRS scheme, based on certain assumptions. The general assumptions here in both models are the excess fuzzy cost is always greater than the other fuzzy costs (Failure & Recruitment). Every organization can use of these models for recruitment in the place of excess exit of employees on VRS, with suitable changes in their decisions either by simple substitutions or changes in the proper place.

The model discussed in section 2.2 highlights the fact that the stationary rate of crisis increases with the increase in VRS rates (the rate at which employee move out on VRS. i.e. The number of exit of employee > the optimal number of exit of employee) The computation helps the organization to initiate measures that would reduce the stationary rate of crisis. The parameter which could be controlled by the management is \( \mu \), the recruitment rate. By increasing this rate the crisis rate may be minimized to a desired level. The other parameters are not controlled by the management.

However, the departure rate of employees can be minimized (i.e. \( \lambda \) can be maximized) by providing attractive incentives to the employees. The entire focus of this section is not on minimizing the cost of running the organization, but on minimizing the long run crisis rate, so that by the timely and appropriate action, the functioning of the organization is made smooth. This would ultimately enhance the image of the organization as a whole among its customers in the society at large and also protects the goodwill of the organization.