CHAPTER I

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Decision making is a cognitive process leading to the selection of a course of action among available alternatives. Numerous methods have been employed to assist Decision making in various aspects of human behavior. One such method is the Multi Criteria Decision Making (MCDM) and it has been an active area of research since 1970s. According to Bellman & Zadeh [8], “Much of the decision making in the real world takes place in an environment in which the goals, the constraints and the consequences of possible actions are known precisely”. Possibility theory (Carlsson & Fuller) along with fuzzy set theory proposed by Zadeh & Zimmermann provides a new avenue to deal with this kind of impreciseness in the problem.

1.1 LITERATURE REVIEW:
Multi Criteria Decision Making:

Multi Criteria Decision Making (MCDM) methods are used to take decisions when a number of multiple conflicting criteria are present in any decision making scenario. The MCDM procedure consists of generating alternatives, establishing criteria, evaluation of alternatives assessment of criteria weights and application of a ranking method. The alternative set may contain a finite number of elements or an infinite number of elements. The MCDM procedures are used to select the best one among the previously specified finite number of alternatives. MCDM may be broadly classified as:

- Multi-Attribute Decision Making (MADM), and
- Multi-Objective Decision Making (MODM).

MADM is best suited for selection or evaluation problems whereas MODM is best suited for operation design problems. The history of MCDM can be read extensively in the book authored by Zionts et al (1988) [44]. Several methods dealing with multi-criteria decision making problems are available in the literature of MCDM.
Some of them are

i. Multiplicative Exponential Weighting (MEW),

ii. Simple Additive Weighting (SAW),

iii. Technique for Ordering Preference by Similarity to Ideal Solution (TOPSIS),

iv. Analytic Hierarchy Process (AHP),

v. VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) method.

Decision makers often face problems whose characteristics possess incomplete and vague information. It is unrealistic to assign a crisp value for a subjective judgement, especially when the information is vague or imprecise.

**Fuzzy Multi Criteria Decision Making:**

It has been consistently pointed out in the literature that decision making models based on probability theory are hard to deal with, due to their complex stochastic structure. It is also difficult to justify them using deterministic model over a long period of time. Another drawback of decision making models based on probability theory is that these models are unable to capture features often exhibited by the various changing parameters in most decision making models. This is because decision making models exhibit some level of arbitrariness, vagueness and fuzziness.

Fuzzy set theory has the ability to deal with such vagueness and imprecision. Fuzzy models are attractive and have gained growing attention due to their simplicity in structure and interpretability. These features have been the subject of extensive studies ever since Zadeh [77] has initiated them. Although fuzzy set theory has been and still remains somewhat controversial, its successes are too clear to be denied. Ribeiro (1996) warns that “too much fuzzification does not imply better modelling of reality, it can be counterproductive”.

There are two approaches to MCDM in a fuzzy environment, “conventional” and “fuzzy”. The conventional approach is based on a non-fuzzy decision model, whereas the fuzziness dissolution (defuzzification) is performed at an early stage.
The fuzzy approach is based on processing fuzzy data for decision making, then dissolving the fuzziness at a later stage. In both cases, defuzzification is necessary since MCDM results must provide a crisp conclusion.

Defuzzification is a selection of a specific crisp element based on the output fuzzy set, and it also includes converting fuzzy numbers into crisp scores. There are several defuzzification methods, although the operation defuzzification cannot be defined uniquely. It even plays a vital role in ranking fuzzy numbers. Fuzzy ranking methods have been developed that can be used to compare fuzzy numbers.

Ordering of fuzzy quantities is based on extracting various features from fuzzy sets. These features may be a center of gravity, an area under the membership function, or various intersection points between fuzzy sets. A particular fuzzy set ranking method extracts a specific feature from fuzzy sets, and then ranks them based on that feature. As a result, it is reasonable to expect that different ranking methods can produce different ranking order for the same sample of fuzzy sets.

All fuzzy set ranking methods can be categorized into three classes:

1) First class aim to define a ranking function, mapping a fuzzy number into a real number, and then use a natural order for ranking purpose. Examples of this class are Yager (1981), Chang (1981).

2) Second class consists of methods that compare fuzzy numbers based on their relations to predefined reference set. Methods that follow this procedure are Baas and Kwakernaak (1977), Jain (1977) and Baldwin and Guild (1979), have followed this approach.

3) Methods of this class tend to construct a fuzzy binary relation on fuzzy numbers representing pair-wise comparisons between them and develop a procedure for obtaining the final ranking based on this comparison.

Literature review reveals that, the paper titled “A review of some of the ranking methods” by Bortolan and Degani (1985) [9] and the paper titled “Reasonable properties for the ordering of fuzzy quantities” by Wang and Kerre [72]...

Though many methods for ranking fuzzy numbers have been proposed in the last decades, none of them is a well accepted choice for all cases. Main drawbacks found in most of these methods are counterintuitive, nondiscrimination and inconsistency. Some ranking methods even assume that the membership function is either normal or non-normal and convex or non-convex, which is not possible in real life situations. That is a combination of either of them may also arise.

This motivated us to develop a new ranking method which is applicable to both the cases (Non-Normal and Non-Convex). Thus we extended Peneva-Popchev method, discussed in Chapter IV, so that it is available to rank non-normal and non-convex fuzzy numbers.

**Fuzzy Game Theory:**

Game Theory is a type of decision theory which is based on reasoning in which the choice of action is determined after considering the possible alternatives available to the opponents playing the same game. The aim is to choose the best course of action, because every player has got an alternative course of action. Systems that involve more than one decision maker are often optimized using the theory of games. Professor John Von Neumann and Oscar Morgenstern published their book entitled "The Theory of Games and Economic Behavior" provides new approach to many problems involving conflict situations. This approach is now widely used in Economics, Business Administration, Sociology, Psychology and Political Science as well as in Military Training. In games like chess, draught, pocker etc., where the players play the game with certain rules, victory of one side and the defeat of the other is dependent upon the decisions based in skillful evaluation of the alternatives of the opponent and also upon the selection of the right alternative.
Two person zero-sum game is the situation which involves two persons or players and gains made by one person is equals to the loss incurred by the other. For example, consider there are two companies Coca-Cola & Pepsi, struggling for a larger share in the Market. Now any share of the market gained by the Coca-Cola Company must be the lost share of Pepsi, and vice-versa and therefore, the sums of the gains and losses equals zero.

When we apply the game theory to model some practical problems which we encounter in real situations, we have to know the values of payoffs exactly. However, it is difficult to know the exact values of payoffs and we could only know the values of payoffs approximately, it is difficult to evaluate via probabilistic methods. In such situations, fuzzy set theory may offer an appropriate and powerful alternative, and make it possible to take human experiences, subjective judgment and intuitions into consideration. In such a framework, we may model the imprecise and vague payoffs as fuzzy variables, and indeed, many researchers have proposed games with fuzzy payoffs.

Based on the theory of fuzzy sets initiated by Zadeh [77], two-person zero sum matrix game with fuzzy payoffs was studied by Campos [13], who employed linear programming models to solve the fuzzy matrix game. J. J. Buckley and L.J. Jowers (2005) applied Monte-Carlo methods to solve and get approximate optimal fuzzy two person zero-sum games and programmed it in MATLAB. Fuzzy game theory is applied in a wide variety of area, where situations involving complex problems in economics, engineering political sciences, etc. Here we have developed a fuzzy two-person zero-sum game and solved it by applying fuzzy ranking method.

Fuzzy Allocation Problems:

Another important part of optimization theory is dealing with multiple objectives. Some of the methods can be identified in Coello Coello et al., (2000, 2002). The classical methods (also called traditional methods in the literature) include goal programming, constraint methods, the Tchebycheff method and others. For reviews on classical techniques for multi-objective optimization refer to Steuer, 1986; Belton et al., 2002; Goicoechea et al., 1982; Miettinen, 2001.
Generally, the resource allocation problems (transportation and assignment) in real life deal with many-objectives that are to be optimized. In the context of multi-objective decision making we cannot reach an ideal solution because of incommensurable objectives and constraints. Decision makers try to find out the solution which is closest to the ideal solution considering all objectives and constraints in the decision making situations.

Transportation problem (TP) is one of the most important applications of linear programming (LP) in the real life situations. TP is mainly used to find the resource allocations as well as route networks, so that the total cost would be minimum. Many models have been developed to solve multi-objective transportation problems. The basic transportation problem is developed by Hitchcock [32]. Efficient method from the simplex algorithm were developed by Dantzig (1947) and then by Charnes and Cooper.et.al [17] in 1953.

Koopmans studied TP extensively in the form of analysis of production and allocation in 1951. Charnes & Cooper presented the stepping stone method in order to explain LP calculation in TP in 1954. Haley (1963) proposed the solid TP model with single objective in 1963. The initial basic feasible solution for the transportation problem can be obtained by NWCR, LCM and VAM. But Multi-Objective Transportation Problems (MOTP) cannot be solved by applying these methods alone. Ammar & Youne [1] studied the efficient solutions and stability of multi-objective transportation problem.

As we all know that the assignment problem is a particular type of transportation problem, where n tasks are to be assigned to an equal number of n machines in one to one basis such that the assignment cost is minimal. Different types of single-objective assignment problem in crisp environment are discussed in literature. Bao.et al studied the multi-objective assignment problem in crisp environment.

However, allocation problems representing real-world situations involve a set of parameters whose values are assigned by decision makers (DMs). But DMs do not know the exact value of those parameters. In such situations decision makers
utilize fuzzy set theory to represent the parameters. Thus fuzzy model was developed in multi-objective allocation problems.

Ringuest & Rinks [61] developed efficient interactive algorithm for solving multi-objective TP. Current & Min designed MOTP networks. Pramnik & Roy [57] studied fuzzy goal programming approach to MOTP with capacity restrictions. Pramanik and Roy [58] studied multi-objective transportation model with normal trapezoidal fuzzy numbers based on priority based fuzzy goal programming.

Lin and Wen proposed labelling algorithm to solve assignment problem with fuzzy interval cost. De and Yadev discussed Multi-Objective Fuzzy Assignment Problem (MOFAP) through interactive fuzzy goal programming approach. Kumar and Gupta developed a solution method for fuzzy assignment problems and fuzzy travelling salesman problems with different membership functions using Yager’s ranking index [74]. Emrouznejad et al. (2011) developed an alternative formulation for the fuzzy assignment problem with fuzzy costs or fuzzy profits for each possible assignment based on data envelopment analysis. Haddad et al. (2012) discussed two models for the generalized assignment problem in uncertain environment. The research is still an ongoing process in this field.

**Role Of Manpower Planning In Fuzzy Multi-Criteria Decision Making:**

In this modern world, it is well known that manpower is inevitable, inspite of the existence of advanced techniques. It is a device with which an attempt is made to match the supply of people with the demand in the form of jobs available in any organization so that the cost incurred is optimum. The management of a firm can make use of manpower models to find the best combination of workers of different categories with respect to cost, skill, nature of jobs, etc. A number of manpower models are generated in every field of departments, such as in Industry, Marketing, Education, Office Management, Transportation System etc. It is in fact an effective scientific tool to solve several managerial decision making problems.

Basic models on manpower planning have been developed and studied in the past by many well known researchers like Bartholomew [7], Smith [65], Grinold and Marshall [29], Forbes [25] and Vajda [69] and many others have rightly pointed out
the need and importance of manpower planning and utilization by stating that “Manpower planning refers to the rather complex task of forecasting and planning for the right number and the right kind of people at the right places at the right time to perform activities that will benefit both the organization and the individual in it”.

According to Grinold and Marshall [29], “There is interaction between the four ingredients of a manpower namely people, job, time and money. The growth of an organization depends on better management of money, time, job and the manpower”. Bennison and Casson (1984) stated that “organization have become more complex requiring a wider range of specialist”. A wider range of specialist means persons who have specialized in a particular skill such as operating special machineries, marketing skill, manufacturing skill, etc. any organization or company will have to recruit such skilled persons for its growth.

The failure to provide appropriate type of personnel (manpower) and money on right time may result of business getting into crisis. Manpower shortage will lead to recruitment process resulting in huge expenditure. Recruitment is done, whenever there is a need to expand or increase its efficiency or fill up vacancies due to wastage and promotion. Davies (1975, 1981) has obtained maintainability structures of manpower systems under recruitment control. The various costs associated with the policy making in manpower management are indicated by Poornachandra Rao. Several models on obtaining the optimal time to recruit people or optimal cost to be spent on recruiting people are developed in recent years by many authors.

On the other hand the supply of manpower more than the demand results in surplus manpower. In this case it leads the organization to announce voluntary retirement scheme. This made the researchers to develop models on voluntary retirement schemes. So manpower planning must be effectively handled in such a way that human resources are fully utilized in every possible way. Several authors have contributed models which are deterministic or stochastic or fuzzy, to the field of manpower systems, particularly in recruitment, wastage and promotion. The need for proper planning of manpower resources and proper utilization of the same had resulted in the emergence of manpower planning as a separate division of management sciences.
Bartholomew [7] is considered to be the Father of manpower planning. Bartholomew and Forbes (1979, 1999) have contributed lot of ideas to the manpower planning models. The work of researchers like Forbes [25], Bartholomew (1973), Morgan et al. (1974) on career patterns represents a different strand in the historical development of manpower planning. Young and Vassiliou (1974) have considered a non linear model for wastage and promotion in manpower system. Grinold and Marshall (1977) and Vajda (1975, 1978) studied in depth the mathematical aspects of manpower system and they were complimented by Bartholomew (1998) on statistical aspects.

Sathiyamoorthy and Elangovan [63], [64], studied an optimal recruitment policy for training, prior to placement. They also determined the expected time for recruitment through the shock model approach & discussed the optimum time interval between recruitment programmes. J. B. Esther Clara and A. Srinivasan (2011) presented a stochastic model for the expected time to recruitment in a single graded manpower system with two thresholds using univariate maximal policy.

The behavior of manpower system and the distribution of the vacancy level are studied by Yadavalli and Natarajan [76]. Nirmala and Jeeva [53] used optimization approach and developed a mathematical model which minimizes the manpower system cost during the recruitment and promotion period which are determined by the changes that takes place in the system. A. Srinivasan, P. Mariappan (2001, 2002, 2011) and S. Dhivy [22] presented Stochastic models on time to recruitment in a two grade manpower system using different polices of recruitment. S. Parthasarathy and R. Vinoth [55] have made an attempt to determine the expected time to recruitment and its variance when threshold follows Gamma distribution.

Overall, the field of manpower planning still offers considerable opportunity for operation research workers, especially those who are willing to operate in an environment with a qualitative rather than quantitative emphasis. Recruitment in fuzzy environment arises due to the uncertainty in human behavior. The recruitment problem can be modeled based on the principles of fuzzy set theory. The main
contribution by Bellman & Zadeh [8] to fuzzy and decision theory have led to many fuzzy models in various fields and particularly in manpower systems.

To mention a few, Guerry [30] used fuzzy set theory in manpower planning and Mutingi M. [52] applied fuzzy dynamics to manpower systems. Designing of Fuzzy Mathematical Model for Manpower Planning (Case: Twenty Millions Army) was done by Azar and Najafí [4]. Fuzzy input and output data were applied for manpower forecasting by Hong Tau Lee (2001). T De Feyter [21] and other co-authors evaluated Recruitment Strategies using Fuzzy Set Theory in Stochastic Manpower Planning.

The present world deals fully with uncertain and imprecise information, so it is better to introduce fuzzy concept wherever there is an uncertainty in the costs involved. Many fuzzy models have been developed in the area of Voluntary Retirement Scheme. Any organization which suffers from advanced technology will announce VRS to employ new technical experts. That is when the organization finds its employees lacking of advanced technology, they either send them for training or replace them with new employee of advanced technology, rather then keeping idle. This means, the concerned organization should be ready to shell out a huge amount of money for the employees exiting on VRS. When the number of employees who exit on VRS exceed the expected number (threshold level), the organization can then call off the scheme.

There are several articles in literature that discuss about voluntary retirement and its effects thereafter. To mention a few, Subramaniyan [68] discussed about the optimal time of with drawl of VRS. Sathiyamoorthy and Elangovan (1998) & developed a model which gives the optimal number of employees quitting the organization on VRS at minimum cost. Brockner [10] studied the relationship between downsizing and productivity. Appelbaum (Jan, 1991) [2] in his study discussed the issues and reasons that led to the decision to downsize, duration of the downsize process and assistance provided to terminated employees.

In 1993, Cameron [12] conducted a study on automobile industry in downsizing and redesigning organization. Mckinly et.al (1995) [51], in his study “Early retirement Schemes” observed that there are three main social forces that lead
to downsizing; Constraining, Cloning, and Learning. Rathan Khasnabis and Sudipti Banerjee (1996) [60] conducted a study on “political economy of voluntary Retirement”. The survey covered 629 rationalized workers out of which 602 reported to have opted for VRS and the reason for VRS are the compensation package offered.

Goyal [28] commented on the retirement scheme for Public Sector Banks, whether it will really make banks competitive. He views it on a positive note since it will bring down staff cost and improve productivity, to get an insight into their perceptions towards VRS. Shanbhag (2005), in his paper, “VRS- to take it or leave it” discusses the benefits of VRS. Models on uncertain situations are also developed. Fuzzy numbers have been widely used in many applications as they are suitable for representing uncertain values.

1.2 PRELIMINARIES:

**Fuzzy Set:**

A non-empty subset \( \tilde{A} \) of a universal set \( X \) is said to be a fuzzy set if for every \( x \) in \( \tilde{A} \), there exists a membership degree \( \mu_{\tilde{A}}(x) \) (or \( \mu(x) \) or \( \tilde{A}(x) \)) of \( x \), the degree of \( x \), for which \( x \) belongs to \( \tilde{A} \).

The fuzzy set can also be represented as \( \tilde{A} = \{(x, \mu_{\tilde{A}}(x)) / x \in X\} \)

**Fuzzy Numbers:**

A fuzzy set \( \tilde{A} \) on the set of real numbers \( \mathbb{R} \), with the membership function \( \mu_{\tilde{A}}(x) \) is said to be a fuzzy number if it satisfies the following conditions:

1. \( \tilde{A} \) must be a normal set. i.e. \( \max_{x \in \mathbb{R}} \mu_{\tilde{A}}(x) = 1 \).
2. Its \( \alpha \)-cuts are closed intervals for all \( \alpha \in (0, 1] \)
3. Support of \( \tilde{A} \) is bounded. That is \( \text{Supp}(\tilde{A}) = \{x \in \mathbb{R} / \mu_{\tilde{A}}(x) > 0\} \) is bounded.

**\( \alpha \)-cut & Strong \( \alpha \)-cut of a fuzzy set:**

Let \( \tilde{A} \) be a fuzzy set, then the \( \alpha \)-cut of \( \tilde{A} \) is defined as,

\[ a^{\alpha} \tilde{A} = \tilde{A}[\alpha] = \{x \in X / \tilde{A}(x) \geq \alpha\} \]
and strong $\alpha$ -cut of $\tilde{A}$ is defined as, $^{\alpha}\tilde{A} = \{x \in X / \tilde{A}(x) > \alpha\}$

**Height of a Fuzzy set:**

The height, ht ($\tilde{A}$) of a fuzzy set $\tilde{A}$ on a universal set $X$ is defined as

$$ht(\tilde{A}) = \max \mu(x), \text{ for all } x \in X.$$ 

**Normal fuzzy set:**

A fuzzy set $\tilde{A}$ on $X$ is said to be normal if its maximal degree of membership is unity (That is there must exist at least one $x \in X$ for which $\mu_\tilde{A}(x) = 1$). Otherwise the fuzzy set is said to be non-normal.

**Convex fuzzy set:**

A fuzzy set $\tilde{A}$ is convex if and only if it satisfies the following property:

$$\mu(\lambda x_1 + (1 - \lambda) x_2) \geq \min(\mu(x_1), \mu(x_2))$$

where $\lambda$ is in the interval $[0, 1]$, and $x_1 < x_2$

**Positive Fuzzy Number:**

A fuzzy number $\tilde{A}$ is Positive if its membership function is such that $\mu_\tilde{A}(x) = 0$ for all $x < 0$.

**Triangular Fuzzy Number:**

A triangular fuzzy number $\tilde{A}$ represented as $[a_1, a_2, a_3]$ or $[a_1, a_2, a_3; h]$ (where $h$ is the height of $\tilde{A}$) accordingly whether it is normal or non-normal respectively is a fuzzy number whose membership function is defined as

$$
\mu(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1} & \text{if } a_1 \leq x \leq a_2 \\
1 & \text{if } x = a_2 \\
\frac{a_3-x}{a_3-a_2} & \text{if } a_2 \leq x \leq a_3 \\
0 & \text{otherwise}
\end{cases}
$$
Trapezoidal Fuzzy Number:

A trapezoidal fuzzy number $\tilde{A}$ represented as $[a_1, a_2, a_3, a_4]$ or $[a_1, a_2, a_3, a_4; h]$ (h=ht ($\tilde{A}$)) (accordingly whether it is normal or non-normal and its membership function is given by

$$
\mu(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1} & \text{if } a_1 \leq x \leq a_2 \\
1 & \text{if } a_2 \leq x \leq a_3 \\
\frac{a_4-x}{a_4-a_3} & \text{if } a_3 \leq x \leq a_4 \\
0 & \text{otherwise}
\end{cases}
$$

Non-Convex Fuzzy Number:

The trapezoidal non-convex fuzzy number $\tilde{A}$ is denoted as $\tilde{A} = [a_1, a_2, a_3, a_4; h_1] \oplus [a_1', a_2', a_3', a_4'; h_2]$ (here $a_4 < a_1'$) and is defined by the membership function as
\[ \mu(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1} & \text{if } a_1 \leq x \leq a_2 \\ h_1 \frac{x - a_2}{a_4 - a_2} & \text{if } a_2 \leq x \leq a_3 \\ h_2 \frac{a_4 - x}{a_4 - a_3} & \text{if } a_3 \leq x \leq a_4 \\ \frac{x - a_1'}{a_2' - a_1'} & \text{if } a_1' \leq x \leq a_2' \\ h_2' \frac{a_4' - x}{a_4' - a_3'} & \text{if } a_3' \leq x \leq a_4' \end{cases} 
\]

When \( a_2 = a_3 \) and \( a_2' = a_3' \), triangular non-convex fuzzy number is obtained.

Operations on Interval Numbers:

For any two intervals, \([a, b]\) and \([d, e]\) (where \(a, b, d, e\) are real numbers), the arithmetic operations are performed in the following way:
Addition:
\[ [a, b] + [d, e] = [a + d, b + e]; \]

Subtraction:
\[ [a, b] - [d, e] = [a-e, b-d]; \]

Multiplication:
\[ [a, b][d, e] = [\min (ad, ae, bd, be), \max (ad, ae, bd, be)]; \]
\[ [a, b][d, e] = [ad, be] \quad \text{if } a, b, d, e > 0 \]

Division:
\[ [a, b] / [d, e] = [\min (a/d, a/e, b/d, b/e), \max (a/d, a/e, b/d, b/e)], \]
\[ \text{provided that } 0 \notin [d, e]. \]
\[ [a, b] / [d, e] = [a/e, b/d] \quad \text{if } a, b, d, e > 0 \]

Midpoint of an interval:
If \([a, b]\) is any interval number, then the midpoint of \([a, b]\) = \(\frac{a + b}{2}\)

Ranking:
If \([a, b]\) and \([d, e]\) are interval numbers, then ranking of interval numbers is defined as
\[ [a, b] < [d, e], \text{ if midpoint of } [a, b] < \text{midpoint of } [d, e] \]

Since any fuzzy number can be represented by a series of crisp intervals (\(\alpha\) – cuts or strong \(\alpha\) – cuts), the interval arithmetic operations (such as addition, subtraction, multiplication, division) as defined above is an alternate way of performing fuzzy arithmetic.

Operations on Trapezoidal Fuzzy Numbers:
Suppose \(\bar{A} = [a_1, a_2, a_3, a_4]\) and \(\bar{B} = [b_1, b_2, b_3, b_4]\) (where \(a_i\)'s and \(b_j\)'s are real numbers, for \(i\) and \(j= 1, 2, 3, 4\) and are trapezoidal fuzzy numbers, then the operations on trapezoidal fuzzy numbers are defined as
Addition:
\[ \vec{A} + \vec{B} = [a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4] \]

Subtraction:
\[ \vec{A} - \vec{B} = [a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1] \]

Multiplication:
\[ \vec{A} \times \vec{B} = [a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4] \text{ for all } a_i \& b_j > 0 \]

Division:
\[ \vec{A} / \vec{B} = [a_1/b_1, a_2/b_2, a_3/b_3, a_4/b_4] \text{ for all } a_i \geq 0 \text{ and } b_j \neq 0 \]

Scalar Multiplication:
\[ k \vec{A} = [ka_1, ka_2, ka_3, ka_4] \text{ for } k \geq 0 \]
\[ k \vec{A} = [ka_4, ka_3, ka_2, ka_1] \text{ for } k \leq 0 \]

Scalar Division:
\[ \frac{\vec{A}}{k} = \left[ \frac{a_1}{k}, \frac{a_2}{k}, \frac{a_3}{k}, \frac{a_4}{k} \right] \text{ for } k > 0 \]
\[ \frac{\vec{A}}{k} = \left[ \frac{a_4}{k}, \frac{a_3}{k}, \frac{a_2}{k}, \frac{a_1}{k} \right] \text{ for } k < 0 \]

Operations on Triangular Fuzzy Numbers:

Suppose \( \vec{A} = [a_1, a_2, a_3] \) and \( \vec{B} = [b_1, b_2, b_3] \) (where \( a_i \)'s and \( b_j \)'s are real numbers, for \( i \) and \( j = 1, 2, 3, 4 \) are triangular fuzzy numbers, then the operations on triangular fuzzy numbers are defined as

Addition:
\[ \vec{A} + \vec{B} = [a_1 + b_1, a_2 + b_2, a_3 + b_3] \]

Subtraction:
\[ \vec{A} - \vec{B} = [a_1 - b_3, a_2 - b_2, a_3 - b_1] \]

Multiplication:
\[ \vec{A} \times \vec{B} = [a_1 b_1, a_2 b_2, a_3 b_3] \text{ for all } a_i \text{ and } b_j \geq 0 \]
Division:
\[ \tilde{A} / \tilde{B} = [a_1/b_1, a_2/b_2, a_3/b_3] \] for all \( a_i \geq 0 \) and \( b_j \neq 0 \)

Scalar Multiplication:
\[ k \tilde{A} = [ka_1, ka_2, ka_3] \quad \text{for} \quad k \geq 0 \]
\[ k \tilde{A} = [ka_3, ka_2, ka_1] \quad \text{for} \quad k \leq 0 \]

Scalar Division:
\[ \frac{\tilde{A}}{k} = \left[ \frac{a_1}{k}, \frac{a_2}{k}, \frac{a_3}{k} \right] \quad \text{for} \quad k > 0 \]
\[ \frac{\tilde{A}}{k} = \left[ \frac{a_3}{k}, \frac{a_2}{k}, \frac{a_1}{k} \right] \quad \text{for} \quad k < 0 \]

Poisson Distribution:
Let \( X \) be a random variable having the Poisson mass function. If \( P(x) \) stands for the probability that \( X = x \), then
\[ P(x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad (1.1) \]
for \( x = 0, 1, 2, \ldots \) and parameter \( \lambda > 0 \).

Fuzzy Poisson Distribution:

In the above definition, if the parameter \( \lambda \) is of imprecise information, then \( \lambda \) can be substituted as \( \bar{\lambda} \) (a fuzzy number \( \lambda > 0 \)), to produce fuzzy Poisson probability mass function [22]. Let the fuzzy number \( \tilde{P}(x) \) denote the fuzzy probability that \( X = x \), then the \( \alpha \)-cut of this fuzzy probability is given by
\[ \tilde{P}(x)[\alpha] = \left\{ \frac{e^{-\lambda} \lambda^x}{x!} \bigg| \lambda \in \bar{\lambda}[\alpha] \right\} \quad (1.2) \]
for all \( \alpha \in [0,1] \). We say that \( X \) follows a Fuzzy Poisson distribution, and is denoted by \( X \sim FP(\bar{\lambda}) \).
Exponential of a Fuzzy Number:

Let \( \tilde{A} = [a, b, c] \) be a fuzzy number. Then the \( \alpha \)-cut of \( \tilde{A} \) is

\[
\alpha \tilde{A} = [a + (b-a) \alpha, c - (c-b) \alpha]
\]

\[
\exp(\alpha \tilde{A}) = [\exp(a + (b-a) \alpha), \exp(c - (c-b) \alpha)]
\]

The exponential of a fuzzy number is a fuzzy number whose membership function is defined as

\[
\mu_{\exp(\tilde{A})}(x) = \begin{cases} 
\frac{\ln(x) - a}{b - a} & \exp(a) \leq x \leq \exp(b) \\
\frac{c - \ln(x)}{c - b} & \exp(b) \leq x \leq \exp(c)
\end{cases}
\]

Logarithm of a Fuzzy Number:

Let \( \tilde{A} = [a, b, c] \) be a fuzzy number. Then the \( \alpha \)-cut of \( \tilde{A} \) is

\[
\alpha \tilde{A} = [a + (b-a) \alpha, c - (c-b) \alpha]
\]

\[
\ln(\alpha \tilde{A}) = [\ln(a + (b-a) \alpha), \ln(c - (c-b) \alpha)]
\]

The logarithm of a fuzzy number is a fuzzy number whose membership function is defined as

\[
\mu_{\ln(\tilde{A})}(x) = \begin{cases} 
\frac{\exp(x) - a}{b - a} & \ln(a) \leq x \leq \ln(b) \\
\frac{c - \exp(x)}{c - b} & \ln(b) \leq x \leq \ln(c)
\end{cases}
\]

Fuzzy Power of a Real Number:

Let \( \tilde{A} = [a, b, c] \) be a fuzzy number and \( x > 0 \) be a real number. Then \( x^{[a,b,c]} \) is defined as

\[
x^{[a,b,c]} = [x^a, x^b, x^c]
\]
1.3 MOTIVATION AND SCOPE:

The problem of fuzzy ranking or ordering of fuzzy sets defined on R has been discussed in the literature quite extensively. However, an issue of choosing a proper ordering method in a given context is still an ongoing process. Fuzzy can also be applied to manpower planning, a branch of Decision Making, as it involves unpredictable quantities like cost, time, etc. This motivated us to find a ranking method for ranking fuzzy numbers and to propose models in fuzzy manpower planning.

Other than Triangular and Trapezoidal fuzzy numbers, the models can be applied to other types of fuzzy numbers, such as Gaussian fuzzy number, Interval fuzzy number etc. In chapter II the time T for the staff strength being up to the expected level and the time required to complete the recruitment are considered to be dependent. This work can be extended to independent case too. In this thesis we have dealt with recruitment and voluntary retirement scheme aspects of manpower planning in fuzzy environment. Other concepts such as promotion, wastage, training, etc., in manpower planning can also be studied in future.

1.4 WORK PLAN OF THE THESIS:

This thesis consists of five chapters. The first chapter is in the form of introduction, which includes a survey of literature review, motivation and scope, basic notions relevant to the thesis and work plan of the thesis.

Chapter II discusses on the optimal number of exit of personnel under Voluntary Retirement Scheme (VRS) in a fuzzy environment. In production oriented organizations reduction of employees is necessary, whenever there is a surplus of staff members. At that instant, attempts are made to reduce staff strength with the announcement of VRS with suitable and attractive packages, which tempts the staff to leave the organization. Here the number of employees opting for VRS is assumed to be a random variable which follows a fuzzy Poisson distribution. The optimal
number of employees opting for VRS have been obtained using a suitable OR technique.

Unexpectedly, when excess number of employees exits on VRS and the organization lack even the minimum employee required to run the jobs or when a project due date is advanced, the organization faces a crisis period. In this chapter we calculate the stationary rate of crisis over a long period (under steady state) and thereby help the organization to make necessary arrangements to avoid the crisis.

Chapter III deals with recruitment in fuzzy manpower planning. In this chapter we are going to deal with fuzzy theory in a recruitment model discussed in crisp case by M. Jeeva [59]. Here we apply cluster analysis technique to group the applicants based on their specialization. On the assumptions that each applicant can perform any job offered by the management with varying efficiency, a table with fuzzy timings is framed. The solution to the problem is discussed and obtained by applying various ranking methods. After recruitment is over, the employee may undergo training period. During this period the experience what the employee gains is studied using Fuzzy learning index.

Chapter IV deals with ranking fuzzy numbers in Fuzzy Multi-Criteria Decision Making (FMCDM). The author Lee [46, 47] mentioned in his comparative study that Kim & Park [43] and Peneva-Popchev [56], had not applied their ranking methods for non-convex and non-normal fuzzy numbers. In this chapter we had applied Kim & Park’s method to non-convex and non-normal fuzzy numbers and have obtained a better result on comparison with some of the other existing methods.

We have also extended the Peneva-Popchev method in such way that it is applicable to non-convex and non-normal fuzzy numbers. The method then compared with some of the other existing ranking methods. Also when the fuzzy number is convex and normal, this method coincides with that of Peneva & Popchev. When the comparison is done, one more observation is noted. That is the ranking methods proposed by Yager [74], Liou-Wang [49] and Nakamori [33] were coinciding with that of the centroid ranking method under a particular case. The extended method is then applied to fuzzy game theory.
Chapter V is about solving the fuzzy allocation problem using the centroid ranking method and the extended Peneva-Popchev method. Fuzzy transportation problem solved by Stephen Dinagar [67] using fuzzy technique and fuzzy assignment problem solved by Sathi Mukherjee [62] using Yager’s method [74], both yielded same result when it is solved using extended Peneva-Popchev method and centroid ranking function. The study is done for both single-objective and multi-objective allocation problems. Algorithm based on Expected Monetary Value (EMV) to find the solution of the problem is discussed and illustrated with an example.

1.5 SALIENT FEATURES OF THE THESIS:

- Determination of the optimal number of exit of personnel under Voluntary Retirement Scheme (VRS) in a fuzzy environment.
- Calculation of the optimal time of completion of job and the optimal number of recruitments using Fuzzy Optimization Techniques.
- Extension of Peneva-Popchev’s Method for ranking Fuzzy numbers and an application of it to fuzzy game theory discussed.
- Comparison of the extended method with other ranking methods discussed.
- Coincidence of Yager’s, Liou-Wang and Lee’s ranking methods with centroid ranking method – A comparative study.
- Fuzzy allocation problems for single-objective and multi-objective problems solved using Expected Monetary Value method.
PUBLICATIONS


6. “Fuzzy Manpower Planning Models on Voluntary Retirement Scheme”, has been communicated to an International Journal.
SEMINARS / CONFERENCES ATTENDED


2. One Day Workshop on Development of Strategies for Quality Enhancement, organized by Internal Quality Assurance Cell, Ethiraj College for Women, Chennai-8, 12th July 2013.

3. Two Days National Level Workshop on Recent Advances in Mathematical Computing, organized by School of Advanced Sciences, Mathematics Division, VIT-Chennai on 30th November 2012 and 1st December 2012.


5. International Workshop on Decision Making & Optimization organized by the Operational Research Society of India, Chennai Chapter and supported by IEEE Computer Society and Computer Society of India on 7th September 2012.


7. State Level Seminar on Application of Mathematics in Advanced Fields, organized by Ethiraj College for Women, Chennai-8 on 19th December 2011.


