CHAPTER-III

Nakagami-m Fading Channel with Noise and Interference for (NFSK) System

3.1 Introduction

The performance of digital communication systems are perturbed by Adoptive White Gaussian Noise (AWGN), multipath fading and co-channel interference. Effect are made to Analysis of the performance of digital communication on fading channels in presence of noise and interference is of considerable interest of theoretical as well as practical research. The diversity techniques for digital communication have been over Rayleigh fading and Nakagami fading channels in presence of a single as well as large number of interferers are described in chapter-2 and reference there in using optical combining technique. The effect of co-channel interference and noise on the bit error probabilities in amplitude shift keying (ASK) and binary phase shift keying (BPSK) have been explained through explicit expression of signal to interference plus noise ratio (SINR). The closed form expression for bit-error probability of BPSK for asynchronous interference in Rayleigh fading environment is presented. The outage probability in Nakagami fading channel under Rayleigh interference is studied by Paris and Jimenez [9]. In all the above studies, they have considered the interferences with the noise background. The derivation of closed form expression for probability of error is quite complicated and approximate.

In this chapter, the performance evaluation of frequency shift keying (FSK) and effect of noise and interference on Nakagami fading channel, FSK signal is considered independently at the receiver.

Finally, the combined decision variable due to noise and interference is achieved. i.e. the combined probability of error due to noise and interference is derived. The exact closed form expression for probability of error using
characteristics function method [10] for post detection combiner has been achieved. The probability of error depends on distribution of signal to noise ratio and distribution of the signal to interference ratio separately. Distribution of signal to interference plus noise has been neglected.

3.2 System Model

Let us consider a diversity reception system over flat fading channel having L-correlated branches. The receiver employs symbol by symbol detection. The signal received on \( k \)th diversity branch in the symbol interval \( T_b \) can be represented by

\[
r_{kN}(t) = \left[ \alpha_k e^{-j\phi_k} s_d(t) + n_k(t) \right] e^{-j2\pi f t}
\] (3.1)

And

\[
r_{kI}(t) = \left[ \alpha_k e^{-j\phi_k} s_d(t) + \alpha_k e^{-j\phi_k} s_i(t) \right] e^{-j2\pi f t}
\] (3.2)

Where \( r_{kN}(t) \) and \( r_{kI}(t) \) are the received signal containing noise and interference separately. \( s_d(t) \) is complex desired signal, \( s_i(t) \) is the complex interfering random signal having zero mean and variance \( \sigma_i^2 \). \( \alpha_k e^{-j\phi_k} = g_k \) is the channel gain, \( n_k(t) \) is additive Gaussian noise with zero mean and variance \( \sigma_n^2 \).

Instantaneous signal to noise ratio (SNR), \( \gamma_k \) of \( k \)th diversity branch is given as

\[
\gamma_k = \frac{E_d}{N_o} \alpha_k^2
\] (3.3)

Where, \( E_d \) is the energy of desired signal, \( N_o \) is the power of noise and \( \alpha_k \) is the channel gain.

Instantaneous signal to interference ratio(SIR) \( \left( \gamma'_k \right) \) can be written as

\[
\gamma'_k = \frac{E_d}{E_I} \alpha_k^2
\] (3.4)

\( E_I \) is the energy of interfering signal. Channel gain for the noise as well as interference is assumed to be the same.
Consider an order-L diversity system for non-coherent detection of FSK (NFSK) signals. As shown in figure 3.1, the signal components received at different antennas are jointly Nakagami distributed with an arbitrary covariance matrix, whereas the noise components at different branches are Gaussian and spatially independent. It is tried to first obtain the decision variable and then derive its error performance.

3.3 Decision Variable

The received signal at the kth antenna in the symbol duration 0<t<T can be written as

\[ r_k(t) = \alpha_k A \cos(\omega t + \phi_k) + n_k(t) \]  \hspace{1cm} (3.5)

Where i =1,2 and k =1, 2...L, and \( n_k(t) \) is zero-mean white Gaussian noise process with one-sided spectral density \( \sigma_n^2 \). The carrier frequency depends on the message with \( \omega_1 \) corresponding to symbol 1 and \( \omega_2 \) to symbol 0. The influence of the kth fading channel is characterized by the channel gain \( \alpha_k \) and phase delay \( \phi_k \) which remain unchanged over the symbol interval T. In a complex form, the influence of the fading channel can be compactly expressed as
Without loss of generality, suppose symbol 1 is transmitted, through branch \( k \). The upper band pass filters (BPF) is centered at the carrier frequency \( \omega_1 \), thereby allowing the desired signal to pass through while the lower BPF is centered at \( \omega_2 \), hence blocking the signal component. The noise output components from the upper and lower BPFs can also be represented in a complex form, denoted here by \( N_{k1} \) and \( N_{k2} \), respectively. They are mutually independent and each follows zero-mean complex Gaussian distribution with variance equal to twice that for \( n_k(t) \). The BPF outputs are further applied to the square-law detectors. The diversity system combines the outputs from all upper branches and accumulates information from all lower branches to enhance symbol detection. These two combined outputs, when expressed in terms of the complex channel gains and noise components, are given by, respectively

\[
u_1 = \sum_{k=1}^l |A g_k + N_{k1}|^2, \quad \nu_2 = \sum_{k=1}^l |N_{k2}|^2
\]

(3.7)

The decision variable is then taken as their difference.

\[D = \nu_1 - \nu_2\]

(3.8)

Since symbol 1 is transmitted, an erroneous decision is made when \( D < 0 \) and a correct event otherwise. The situation is reversed when symbol 0 is sent.

### 3.4 Characteristic Function Method

Let \( \bar{R}_\gamma \) denotes the correlation matrix of SNR vector \( \bar{\gamma} = [\gamma_1, \gamma_2, ..., \gamma_L] \)

And \( \bar{R}_\gamma' \) the correlation matrix of SIR vector \( \bar{\gamma}' = [\gamma'_1, \gamma'_2, ..., \gamma'_L] \)

Joint characteristic functions of the instantaneous SNR and SIR can be written as

\[
\phi_p \left( j \tau_1, j \tau_2, ..., j \tau_L \right) = \det \left( I - j TT' \right)^{-m}
\]

(3.9)

and
\[ \varphi_D'(j^{\prime}_t, j^{\prime}_t, \ldots, j^{\prime}_t) = \det(I - j^{\prime}_T \Gamma^t)^{-m} \quad (3.10) \]

Where \( T = \text{diag}(t_1, t_2, \ldots, t_k) \) and \( T' = \text{diag}(t'_1, t'_2, \ldots, t'_k) \) \( m \) is fading parameter.

\[ \Gamma(k, l) = \sqrt{\frac{R_x(k, l)}{m}} \]

and

\[ \Gamma'(k, l) = \sqrt{\frac{R'_x(k, l)}{m}} \]

The characteristic functions can be written as

\[ \varphi_d(j^{\prime}_t) = \frac{\det \left( I - \frac{j^{\prime}_t \sigma^2}{1 - j^{\prime}_t \sigma^2} \Gamma \right)^{-m}}{(1 + j^{\prime}_t \sigma^2)^{\Gamma} \left( 1 - j^{\prime}_t \sigma^2 \right)^{\Gamma}} \quad (3.11) \]

and

\[ \varphi_d'(j^{\prime}_t) = \frac{\det \left( I - \frac{j^{\prime}_t \sigma^2}{1 - j^{\prime}_t \sigma^2} \Gamma \right)^{-m}}{(1 + j^{\prime}_t \sigma^2)^{\Gamma} \left( 1 - j^{\prime}_t \sigma^2 \right)^{\Gamma}} \quad (3.12) \]

The probability of error can be directly obtained by characteristic function

\[ P_r = \int_{-\infty}^{0} f(D < 0) dD \quad (3.13) \]

Here,
\[ f(D < 0) = \int_{-\infty}^{\infty} \varphi_D(jt)e^{-jDt}dt + \int_{-\infty}^{\infty} \varphi'_D(jt')e^{-jDt'}dt' \]

\[ P_e = \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} \varphi_D(jt)e^{-jDt}dt + \int_{-\infty}^{\infty} \varphi'_D(jt')e^{-jDt'}dt' \right]dD \]

\[ = \int_{-\infty}^{\infty} \varphi_D(jt)dt \left[ \int_{-\infty}^{\infty} e^{-jDt}dD \right] + \int_{-\infty}^{\infty} \varphi'_D(jt')dt' \left[ \int_{-\infty}^{0} e^{-jDt'}dD \right] \quad (3.14) \]

\[ P_e = \int_{-\infty}^{\infty} \left[ \pi\delta(t) - \frac{1}{jt} \right] \varphi_D(jt)dt + \int_{-\infty}^{\infty} \left[ \pi\delta(t') - \frac{1}{jt'} \right] \varphi'_D(jt')dt' \]

\[ = \frac{1}{2} - \frac{1}{2\pi j} \int_{-\infty}^{\infty} \varphi_D(jt)dt \frac{1}{t} + \frac{1}{2\pi j} \int_{-\infty}^{\infty} \varphi'_D(jt')dt' \left[ 1 \right] \quad (3.15) \]

The contour integration can be achieved by Residue theorem

\[ P_e = \left[ -\text{Res}\left\{ \frac{\varphi_D(s)}{s}, \text{poles in } \Re(s) < 0 \right\} \right] \]

\[ + \left[ -\text{Res}\left\{ \frac{\varphi'_D(s')}{s'}, \text{poles in } \Re(s') < 0 \right\} \right] \quad (3.16) \]

Where \( jt = s \) and \( jt' = s' \)

Using eqn. (3.11) and eqn. (3.12) in eqn. (3.16) we have

\[ P_e = \left[ -\text{Res}\left\{ \frac{\varphi_D(s)}{s}, s = \frac{-1}{\sigma_n^2} \right\} \right] + \left[ -\text{Res}\left\{ \frac{\varphi'_D(s')}{s'}, s' = \frac{-1}{\sigma^2} \right\} \right] \]

Or

\[ P_e = \left[ -\text{Res}\left\{ \frac{\varphi_D(z)}{z}, z = -1 \right\} \right] + \left[ -\text{Res}\left\{ \frac{\varphi'_D(z')}{z'}, z' = -1 \right\} \right] \]
\[
= \frac{1}{(L-1)!} \frac{d^{L-1}}{dz^{L-1}} \left[ \frac{1}{z(z-1)} \det \left( I - \frac{2z}{1-z} \Gamma \right)^{-m} \right]
\]
\[
+ \frac{1}{(L-1)!} \frac{d^{L-1}}{dz^{L-1}} \left[ \frac{1}{z'(z'-1)} \det \left( I - \frac{2z'}{1-z'} \Gamma' \right)^{-m} \right]
\]
\[(3.17)\]

Where, \( \sigma_n s^2 = z \) and \( \sigma_i s^{z^i} = z' \)

### 3.5 Closed Form Solution

Direct derivation of \((L-1)^{th}\) derivative in the expression is very difficult, so, we define

\[
G(z) = \frac{-1}{z(z-1)} \det \left( I - \frac{2z}{1-z} \Gamma \right)^{-m}
\]
\[(3.18)\]

and

\[
G(z') = \frac{-1}{z'(z'-1)} \det \left( I - \frac{2z'}{1-z'} \Gamma' \right)^{-m}
\]
\[(3.19)\]

and \( H(z) = \ln G(z) \), \( H(z') = \ln G(z') \)

Now we apply Faa di Bruno’s formula to obtain \((L-1)^{th}\) derivative of \( G(z) \) and \( G(z') \) which results in

\[
\frac{d^{L-1}}{dz^{L-1}} G(z) = (L-1)! G(z) \sum_{i=1}^{L-1} \sum_{i_{L-i}=1}^{L-i-1} \frac{1}{i_1! \cdots i_{L-1}!} \left( \frac{H^{(i)}(z)}{i!} \right)^{i_1 \cdots i_{L-1}}
\]
\[(3.20)\]

and
\[
\frac{d^{L-1}}{dz^{L-1}} G(z') = (L-1)! G(z') \sum_{i=1}^{L} \sum_{\pi(L-1,i)} \frac{1}{t_i!} \left( \frac{H^{(i)}(z')}{i!} \right)^t
\]  
(3.21)

Hence

\[
P_e = \left. G(z) \sum_{i=1}^{L} \sum_{\pi(L-1,i)} \frac{1}{t_i!} \left( \frac{H^{(i)}(z)}{i!} \right)^t \right|_{z=1} + \left. G(z') \sum_{i=1}^{L} \sum_{\pi(L-1,i)} \frac{1}{t_i!} \left( \frac{H^{(i)}(z')}{i!} \right)^t \right|_{z'=1}
\]  
(3.22)

Let \( \lambda_k \) and \( \lambda'_k \), \( k = 1, 2, \ldots, L \) are the Eigen values of \( \Gamma \) and \( \Gamma' \) respectively.

Eqn. (3.18) and eqn. (3.19) can be written as

\[
G(z) = \frac{-1}{z(1-z)^\nu} \prod_{k=1}^{L} \left[ \frac{1-z}{1-(1+\lambda_k)z} \right]^m
\]  
(3.23)

and

\[
G(z') = \frac{-1}{z'(1-z')^\nu} \prod_{k=1}^{L} \left[ \frac{1-z'}{1-(1+\lambda'_k)z'} \right]^m
\]  
(3.24)

Now

\[
H(z) = -\ln z - L \ln(1-z) + \sum_{k=0}^{L} m \ln \left( \frac{1-z}{1-(1+\lambda_k)z} \right)
\]  
(3.25)

The first derivative of \( H \)

\[
H'(z) = -\frac{1}{z} - \frac{L}{z-1} + \sum_{k=0}^{L} m \left( \frac{1}{z-1} + \frac{(-1-2\lambda_k)}{(1+\lambda_k)z-1} \right)
\]  
(3.26)

So, \( i \)th derivative of \( H \) becomes

\[
H^{(i)}(z) = (-1)^i (i-1)! z^{i-1} + (-1)^i (i-1)! (z'-1)^{-i}
\]
\[ + \sum_{k=1}^{L} m \left[ (1)^{i-1} (i-1)! (-1)^i z^{-i} + (-1 + \lambda_k) (i-1)! (-1 + \lambda_k) z^{-i} \right] \]  
\[ (3.27) \]

Similarly

\[ H(i) (z') = (-1)^i (i-1)! z^{-i} + (-1)^i (i-1)! (z'-1)^{i} \]

\[ + \sum_{k=1}^{L} m \left[ (1)^{i-1} (i-1)! (Z' - 1)^i + (-1 + \lambda_k') (i-1)! ((1 + \lambda_k') z' - 1)^{i} \right] \]  
\[ (3.28) \]

Substituting eqn. (3.27) and eqn. (3.28) into eqn. (3.22), we have the probability error

\[ P_e = \left| G(z) \sum_{n=0}^{L} \sum_{l=1}^{L} \prod_{i=1}^{L} \frac{1}{l!} \left[ (1)^{i-1} (i-1)! z^{-i} + (-1)^i (i-1)! (z-1)^{i} + \sum_{k=1}^{L} m(-1)^{i-1} (i-1)! (-1 + \lambda_k) (i-1)! ((1 + \lambda_k) z - 1)^{i} \right] \right|_{z=1} \]

\[ + \left| G(z') \sum_{n=0}^{L} \sum_{l=1}^{L} \prod_{i=1}^{L} \frac{1}{l!} \left[ (1)^{i-1} (i-1)! z'^{-i} + (-1)^i (i-1)! (z' - 1)^{i} + \sum_{k=1}^{L} m(-1)^{i-1} (i-1)! (-1 + \lambda_k') (i-1)! ((1 + \lambda_k') z' - 1)^{i} \right] \right|_{z'=1} \]

\[ (3.29) \]

**Bit Error Probability (BEP) Characteristics**

The Bit Error Probability (BEP), \( P_e \) is dependent on values of Signal to Noise Ratio (SNR), Signal to Interference Ratio (SIR), Diversity (L) and Nakagami Fading Parameter (m). An effort is made to evaluate, plot and analyze the unreliability in terms of BEP characteristics by considering individual parameter at a time and others one constant.
Case-1: BEP vs SIR Characteristics with Different Values of Fading Parameter (m):

These Characteristics have been plotted for fixed value of L=3 and SNR = 10 dB for each value of fading parameter m. The value of m has been taken as 0.8, 1.6, 1.6, 2 and 4 and four Characteristics have been plotted.

Table-3.1, Variation of Pe with SIR for m = 0.8, 1.6, 2, 4 and L = 3, SNR =10 dB

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Average SIR (dB)</th>
<th>Unreliability in terms of Bit Error Probability (Pe) for L = 3 and SNR = 10dB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>m = 0.8</td>
</tr>
<tr>
<td>1</td>
<td>-10</td>
<td>1.00E -3.8</td>
</tr>
<tr>
<td>2</td>
<td>-5</td>
<td>1.00E -1.3</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1.00E -001</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>1.00E -01</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>1.00E -2.3</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>1.00E -04</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>1.00E -4.2</td>
</tr>
<tr>
<td>8</td>
<td>25</td>
<td>1.00E -4.4</td>
</tr>
<tr>
<td>9</td>
<td>30</td>
<td>1.00E -05</td>
</tr>
<tr>
<td>10</td>
<td>35</td>
<td>1.00E -4.8</td>
</tr>
</tbody>
</table>
Figure 3.2, Bit Error Probability $P_e$ with Average SIR (dB) at $L=3$, for $m=0.8, 1.6, 2$ and $4$. 

Graph showing the relationship between Bit error probability $P_e$ and Average SIR (dB) for different values of $m$. The graph has a grid with logarithmic scales for both axes. The x-axis represents Average SIR (dB), ranging from -10 to 35, and the y-axis represents Bit error probability $P_e$, ranging from $10^{-10}$ to $10^1$. The lines for different values of $m$ are marked with markers and colors: $m=0.8$ (green circles), $m=1.6$ (red squares), $m=2.0$ (blue triangles), and $m=4.0$ (black diamonds).
As expressed in the above graph of Figure 3.2, the variation of probability of bit error ($P_e$) with signal to interference ratio (SIR) for signal to noise ratio (SNR) =10 dB and L=3 for fading parameter $m = 0.8, 1.6, 2$ and $4$ has been presented. From the graph, it can be inferred that for negative value of SIR in dB where interference is large as compared to signal level essentially in code division multiple access (CDMA) digital mobile communication, probability of bit error increases for SIR from -10dB to -5 dB and then in the interval -5 dB to 0 dB, it decreases and finally it increases. Variation in $m=0.8$ Characteristics -10 dB to 35 dB of one SIR, $m=1.6$ and $m=2$ Characteristics started from Average SIR 0 dB and varied up to 35 dB of Average SIR. $M=4$ Characteristics varied between -10 dB to 10 dB of Average SIR. Peak value of $P_e$ in 1 for 0.8, 2 and 4 of m and this is 0.1 for $m=1.6$.

Thus, the variation of $P_e$ for negative SIR indicated that for large interference, the decision region of the received signal is changed from the decision region of small interference. The other reason for the variation of $P_e$ is that the effect of fading parameter $m$ is sharp and non-uniform in the negative region of SIR. For positive value of SIR, probability of bit error decreases with increase of fading parameter $m$.

**Case-2, BEP Vs SIR characteristics with different values of L**

<table>
<thead>
<tr>
<th>S. No</th>
<th>Average SIR (dB)</th>
<th>Unreliability in terms of Bit Error Probability ($P_e$) for $m = 1.6$ and SNR = 10dB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L = 1</td>
<td>L = 2</td>
</tr>
<tr>
<td>1</td>
<td>-10</td>
<td>1.00E -2.4</td>
</tr>
<tr>
<td>2</td>
<td>-5</td>
<td>1.00E -2.2</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1.00E -2.2</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>1.00E -2.8</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>1.00E -02</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>1.00E -2.4</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>1.00E -2.8</td>
</tr>
<tr>
<td>8</td>
<td>25</td>
<td>1.00E -03</td>
</tr>
</tbody>
</table>
Figure 3.3: Bit Error Probability ($P_e$) with Average SIR (dB) at SNR = 10 dB for $L = 1, 2, 3, 4$. 

[SNR=10 dB and $m=1.6$]
The above graph as shown in Figure 3.3, the variation of bit error probability with average SIR for L=1, 2, 3 and 4 with SNR =10 dB, $m = 1.5$. Also, the effect of L in the negative region of SIR shows random behavior. In the region -5 dB to 25dB, all the curves behave in similar fashion as there are decreasing with the increase of average SIR. The value of Pe is randomly varying in between -5 dB to 5 dB of average SIR the peak value of Pe is 1 for 2,3 and 4value of L. The variation of characteristics for 1,2 and 3 value of L are -10 dB to 25 dB of average SIR with their Pe varies from 1 to 0.0000001. For L = 4, the characteristics varies in between -10 dB to 15 dB of average SIR and Pe varies in between 1 and 1.00E-09. There is comparatively less variation in Pe for the L=1 characteristics. $P_e$ Decreases for certain value of SIR. For L=2, $P_e$ remains constant ($\approx 10^{-3}$) for the interval of SIR (-10dB to 0 dB) which indicates that there is no effect of interference. It is clear as the value of L increase, the range of SIR of the characteristics decreases.

**Case-3. BEP Vs SIR characteristics with different values of SNR, L = 3, m = 1.6**

**Table 3.3, Variation of Pe with SIR for SNR=-10dB, 0 dB, 5dB, 10dB and 15 dB at L=3**

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Avg SIR (dB)</th>
<th>Unreliability in terms of Bit Error Probability, $P_e$ For $m = 1.6$ and $L = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SNR = -10 dB</td>
<td>SNR = 0 dB</td>
</tr>
<tr>
<td>1</td>
<td>-10</td>
<td>----</td>
</tr>
<tr>
<td>2</td>
<td>-5</td>
<td>----</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.8</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>0.4</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>0.008</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>0.004</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>0.001</td>
</tr>
<tr>
<td>8</td>
<td>25</td>
<td>0.001</td>
</tr>
<tr>
<td>9</td>
<td>30</td>
<td>0.0004</td>
</tr>
<tr>
<td>10</td>
<td>35</td>
<td>0.0008</td>
</tr>
</tbody>
</table>
Figure 3.4, Bit Error Probability ($p_e$) with Average SNR (dB) at $L=3$, $m=1.5$, SNR = -10dB, 0dB, 5dB, 10dB and 15dB.
The graph shown in Figure 3.4, suggests that there is a change of bit error probability with SIR for SNR = -10dB, 0dB 5 dB, 10 dB and 15 dB with \( m = 1.5 \) and \( L = 3 \). From the figure it can also be inferred that all the curves show the approximate same value -1 dB of bit error probability at SIR = -5 dB. For a particular values of SIR, \( P_e \) decrease with the increase of SNR in the interval of SIR from -1 dB to 35 dB. \( P_e \) characteristic for 10 dB and 15 dB value of SNR are above available in graph for any SIR. The major and random variation of \( P_e \) available in between –5 dB to +5 dB average SIR. It is also observed as value of \( P_e \) conditionally decreases for 5 dB to 35 dB of average SIR. The value of \( P_e \) decrease as the value of \( L \) decrease.[167]

### 3.6 Performance Evaluation over Fading Channel

In this section, the performance of these frequency shift keying communication systems is analysed over generalized fading channels. Wherever possible, we shall again make use of the desired forms rather than the classical representations of the mathematical functions the treatment here will be quite brief since indeed the entire machinery that allows determining the results has been developed completely. Thus, for the most part we shall merely present the final results except for the few situations where further developed is warranted.

When fading is present, the received carrier amplitude, \( A_c \), is attenuated by the fading amplitude, \( \alpha \), which is a random variable (RV) with mean-square value \( \alpha^2 = \Omega \) and probability density function (PDF) dependent on the nature of the fading channel. Equivalently, the received instantaneous signal power is attenuated by \( \alpha^2 \), and thus it is appropriate to define the instantaneous SNR per bit by \( \gamma \equiv \alpha^2 E_b / N_0 \) and the average SNR per bit by \( \bar{\gamma} \equiv \alpha^2 E_b / N_0 = \Omega E_b / N_0 \). As such, conditioned on the fading, the Bit Error Probability (BEP) of FSK the modulation considered is obtained by replacing \( E_b / N_0 \) by \( \gamma \) in the expression for AWGN performance. Denoting this conditional BEP by \( P_b(E;\gamma) \) the average BEP in the presence of fading is obtained as

\[
P_b(E) = \int_0^\infty P_b(E;\gamma) p_\gamma(\gamma) d\gamma
\]  

(3.30)
Where \( p_\gamma(\gamma) \) is the PDF of the instantaneous SNR. On the other hand, if one is interested in the average SEP, the same relation applies using, instead, the conditional SEP in the integrand, which is obtained from the AWGN result with \( E_s/N_0 \) replaced by \( \gamma \log_2 M \). Our goal is to evaluate (3.30) for FSK modulation and detection schemes over fading channel model.

### 3.6.1 M-ary Frequency-Shift-Keying

Consider first the case of orthogonal signaling using M-FSK modulation described by the signal occurs when \( f(t) \) takes on equiprobable values \( \xi = (2i-1-M)\Delta f/2, \ i = 1,2,\ldots,M \), in each symbol interval \( T_s \) where the frequency spacing \( \Delta f \) is related to the frequency modulation index \( h \) by \( h = \Delta f T_s \). As such, \( f(t) \) is modeled as a random pulse stream, that is,

\[
f(t) = \sum_{n=-\infty}^{\infty} f_n p(t - n T_s) \quad (3.31)
\]

Where \( f_n \) is the information frequency in the \( n \)th symbol interval \( n T_s \leq t \leq (n+1)T_s \) ranging over the set of \( M \) possible values \( \xi \), as above, and \( p(t) \) is again a unit amplitude rectangular pulse of duration \( T_s \) seconds. Thus, the complex signal transmitted in the \( n \)th symbol interval is

\[
\tilde{s}(t) = A_c e^{j(2\pi(fn(t - n T_s)) + \theta)} \quad (3.32)
\]

The complex baseband modulation \( \hat{s}(t) = A_c e^{jfn(t-n T_s)} \) is not constant over this same interval but rather has a sinusoidal variation. After demodulating with the complex conjugate of \( \hat{C}_r(t) \) at the receiver, we obtain

\[
x(t) = A_c e^{j2\pi fn(t-n T_s)} + \tilde{N}(t) \quad (3.33)
\]

Multiplying (3.33) by the set of harmonics \( e^{-j2\pi \xi k(t-n T_s)}, \ k = 1, 2, \ldots, M \), and the passing each resulting signal through an I&D produces the decision variables (Figure 3.5)

\[
y_{nk} = A_c \int_{n T_s}^{(n+1)T_s} e^{j2\pi(fn-\xi k)(t-n T_s)} dt + \tilde{N}_{nk}, \ k = 1,2,\ldots,M,
\]
From which a decision corresponding to the largest \( \Re \{ \tilde{y}_{nk} \} \) is made on the information frequency transmitted in the \( n \)th signaling interval.

\[
\tilde{N}_{nk} = \int \frac{(n+1)T_s}{nT_s} e^{-j2\pi k(t-nT_s)} \tilde{N}(t) dt
\]

Figure 3.5 Generalized Block diagram of Optimum Receiver for Ideal Coherent Detection of M-FSK over the AWGN

For orthogonal signaling wherein the cross-correction \( \Re \{ \tilde{y}_{nk} \} \)

\[
\int \frac{(n+1)T_s}{nT_s} e^{-j2\pi k(t-nT_s)} \tilde{N}(t) dt = 0, \quad k \neq 1,
\]

the frequency spacing is chosen such that \( \Delta f = N/2T_s \) with \( N \) integer. If, for example, the transmitted frequency \( f_n \) is equal to \( \xi = (2l-1 - M)\Delta f/2 \), then (3.34) can be expressed as

\[
\tilde{y}_{nk} = A_x T_s e^{j\pi(l-k)nT_s} \frac{\sin[\pi(l-k)N/2]}{\pi(l-k)N/2} + \tilde{N}_{nk} k = 1, 2, \ldots, M,
\]

\[
\tilde{N}_{nk} = \int \frac{T_o}{o} e^{-j\pi(2k-1-M)N/2T_s} \tilde{N}(t+nT_s) dt
\]
Or, taking the real part,

\[ \text{Re} \{ \tilde{y}_{nk} \} = A_c T_s \frac{\sin[\pi (l-k)N]}{\pi (1-k)N} + \text{Re} \{ N_{nk} \} \quad k=1,2,...M \quad (3.36) \]

Thus, we observe that for orthogonal M-FSK, only one variable has a nonzero mean: the one corresponding to the transmitted frequency. That is,

\[ \text{Re} \{ y_{nl} \} = A_c T_s \quad \text{Re} \{ y_{nk} \} = 0, k \neq 1 \quad (3.37) \]

A popular special case of M-FSK modulation is binary FSK (BFSK), which corresponds to M=2. In addition to orthogonal signaling (zero cross-correlation), it is possible to choose the modulation index so as to achieve the minimum cross-correlation that results in the minimum error probability. Since for arbitrary \( \Delta f \) we have

\[ \text{Re} \left\{ \int \frac{(n+1)T_b}{nT_b} \tilde{s}_1(t) \tilde{s}_2(t) dt \right\} = \text{Re} \left\{ A_c \int_o^{2\pi/\Delta f} e^{-j2\pi t} dt \right\} \]

\[ = A_c^2 T_b \frac{\sin 2\pi \Delta f T_b}{2\pi \Delta f T_b} \quad (3.38) \]

The minimum of this cross-correlation is achieved when \( h = \Delta f T_b = 0.715 \) [13], which results in a minimum normalized cross-correlation value

\[ \rho = \frac{\text{Re} \left\{ \int \frac{(n+1)T_b}{nT_b} \tilde{s}_1(t) \tilde{s}_2(t) dt \right\}}{\sqrt{\int \frac{(n+1)T_b}{nT_b} | \tilde{s}_1(t) |^2 dt}} = \frac{\text{Re} \left\{ \int \frac{(n+1)T_b}{nT_b} \tilde{s}_1(t) \tilde{s}_2(t) dt \right\}}{\sqrt{\int \frac{(n+1)T_b}{nT_b} | \tilde{s}_2(t) |^2 dt}} \]

\[ = \frac{\sin 2\pi \Delta f T_b}{2\pi \Delta f T_b} |\Delta f T_b| = 0.715 = -0.217 - \frac{2}{3\pi} \quad (3.39) \]
Now, we consider first the case of orthogonal signaling using M-FSK modulation and the receiver of Figure 3.5. Assuming that the transmitted frequency in the \( n \)th symbol interval, \( f_n \), is equal to \( \bar{\xi}_n = (2l - 1 - M)\Delta f/2 \), the real parts of the integrate-and-dump outputs, \( \bar{y}_{nk}, k = 1, 2, ..., M \) as given by (3.39) are independent, identically disturbed. Gaussian random variable with means as in (3.39) and variance \( \sigma_n^2 = N_0 T_s / 2 \). The probability of a correct symbol decision is the probability that all

\[ \text{Re}\{ \bar{y}_{nk} \}, k \neq l, \text{ are less than } \text{Re}\{ \bar{y}_{nl} \}. \]

Thus, letting \( A_c = \sqrt{E_s / T_s} \) and denoting \( \text{Re}\{ \bar{y}_{nl} \} \) by \( z_{nk} \).

### 3.6.2 Symbol Error Probability (SEP)

The probability of symbol error is given by

\[
P_s(E) = 1 - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left(-\frac{z_{nk}^2}{2\sigma_n^2}\right) dz_{nk} \left( -M^{-1} \int_{-\infty}^{\infty} \exp\left(-\frac{(z_{nk} - \sqrt{E_s/T_s})^2}{2\sigma_n^2}\right) dz_{nk} \right) \]

\[
\times \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left(-\frac{(z_{nk} - \sqrt{E_s/T_s})^2}{2\sigma_n^2}\right) dz_{nk} \quad (3.40)
\]

Or in term of the Gaussian Q-function,

\[
P_s(E) = 1 - \int_{-\infty}^{\infty} \sqrt{2\pi\sigma_n^2} Q\left(-q - \frac{2E_s}{\sqrt{N_o}}\right) \left[-M^{-1} \int_{-\infty}^{\infty} \exp\left(-\frac{q^2}{2}\right) dq \right] \quad (3.41)
\]

The corresponding bit error probability is given by

\[
P_b(E) = \frac{2^{k-1}}{2^{k-1}} P_s(E), k = \log_2 M \quad (3.42)
\]
Unfortunately, for arbitrary $M$, (3.41) cannot be put in the desired from by using the form of the Gaussian $Q$-function. The special case of binary orthogonal FSK ($M=2$), however, does have a simple from, namely,

$$P_b(E) = Q\left(\sqrt{\frac{E_b}{N_o}}\right)$$

(3.43)

This can put in the desired form,

$$P_b(E) = \frac{1}{\pi} \int_{0}^{\pi/2} \exp\left(-\frac{E_b}{2N_o} \frac{1}{\sin^2 \theta}\right) d\theta$$

(3.44)

Another $M$-FSK case whose error probability performance can be put into the desired from corresponds to binary non-orthogonal FSK with cross-correlation given by (3.37). In particular, the BEP for such a modulation is given,

$$P_b(E) = Q\left(\sqrt{\frac{E_b(1 - \sin 2\pi h / 2\pi h)}{N_o}}\right)$$

$$= \frac{1}{\pi} \int_{0}^{\pi/2} \exp\left(-\frac{E_b}{2N_o} \frac{1 - \sin 2\pi h / 2\pi h}{\sin^2 \theta}\right) d\theta$$

(3.45)

Where, as before, $h = \Delta f T_b$ is the frequency-modulation index. The minimum BEP is achieved when $h = 0.714$ (the value of $h$ that maximizes the argument of the Gaussian $Q$-function), resulting in

$$P_b(E) = Q\left(\sqrt{\frac{E_b(1.217)}{N_o}}\right) = \frac{1}{\pi} \int_{0}^{\pi/2} \exp\left(-\frac{E_b 1.217}{2N_o \sin^2 \theta}\right) d\theta$$

(3.46)

This is often approximately by

$$P_b(E) = Q\left(\sqrt{\frac{E_b(1 + 2/3\pi)}{N_o}}\right) = \frac{1}{\pi} \int_{0}^{\pi/2} \exp\left(-\frac{E_b(1 + 2/3\pi)}{2N_o \sin^2 \theta}\right) d\theta$$

(3.47)

In above, we observed that the expression (3.41) for the average SEP of orthogonal $M$-FSK involves the $(M-1)$ st power of the Gaussian $Q$-function. Since for
M arbitrary an alternative form is not available for $Q^{M-1}(x)$, (3.41) cannot be put in the desired form to allow simple evaluation of the average SEP on the generalized fading channel. Despite this consequence, however, it is nevertheless possible to obtain simple to evaluate, asymptotically tight upper bounds on the average error probability performance of 4-ary FSK on the Rayleigh and Nakagami-m fading channels. For the special case of binary FSK (M=2), we can use the desired from in (3.45) (from orthogonal signals) or (3.46) (for non orthogonal signals) to allow simple exact evaluation of average BEP on the generalized fading channel. Before moving on to the more difficult 4-ary FSK case, we first quickly dispense with the results for binary FSK since these follow immediately from the integrals developed or equivalently from the results obtained previously for binary AM and BPSK, replacing $\gamma$ by $\gamma / 2$ for orthogonal BFSK and by $(\gamma/2) [1-(\sin2\pi h)/2\pi h]$ for non-orthogonal BFSK. For example, for Rayleigh fading the average BEP of orthogonal BFSK is given by

$$p_b(E) = \frac{1}{2} \left( 1 - \sqrt{\frac{-\gamma/2}{1 + \gamma/2}} \right)$$  \hspace{1cm} (3.48)$$

Whereas for Nakagami-m fading the analogous results are

$$p_b(E) = 2 \left[ 1 - \mu \sum_{k=0}^{\infty} \left( \frac{2k}{k} \right) \left( \frac{1 - \mu^2}{4} \right)^k \right], \mu = \sqrt{\frac{-\gamma/2}{m + \gamma/2}}, m \text{ integer}$$  \hspace{1cm} (3.49)$$

and

$$P_b(E) = \frac{1}{2\sqrt{\pi}} \frac{\sqrt{\gamma/2m}}{(1 + \gamma/2m)^{m+1/2}} \frac{\Gamma(m + 1/2)}{\Gamma(m + 1)} \chi_2 F_1 \left( 1, m + \frac{1}{2}; m + 1; \frac{1}{1 + \gamma/2m} \right), m \text{ non integer}$$  \hspace{1cm} (3.50)$$

For M-ary orthogonal FSK, the Average Symbol Error Probability (SEP) on the AWGN can be obtained from (3.41) as

$$P_s(E) = 1 - \int_{-\infty}^{\infty} \left[ Q \left( -q - \frac{2E_s}{\sqrt{N_o}} \right) \right]^{M-1} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{q^2}{2} \right) dq$$
\[
\int_{-\infty}^{\infty} \left\{ 1 - \left[ 1 - Q \left( q + \sqrt{\frac{2E_s}{N_o}} \right) \right]^{M-1} \right\} \frac{1}{\sqrt{2\pi}} \exp \left( - \frac{q^2}{2} \right) dq \\
= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \left\{ 1 - \left[ 1 - Q \left( \sqrt{2} \left( u + \sqrt{\frac{E_s}{N_o}} \right) \right) \right]^{M-1} \right\} \frac{1}{\sqrt{2\pi}} \exp (-u^2) du \quad (3.51)
\]

And the corresponding BEP is obtained from (3.51) using (3.42). The most straightforward way of numerically evaluating (3.51) (and therefore the BEP derived from it) is to apply Gauss-Hermite quadrature, resulting in

\[
P_s(E) = \frac{\Delta}{\sqrt{\pi}} \sum_{n=1}^{N_p} w_n \left\{ 1 - \left[ 1 - Q \left( \sqrt{E_s / N_o} \right) \right]^{M-1} \right\} 
\]

(3.52)

Where \( \{x_n; n=1, 2 \ldots N_p\} \) are the zeros of the Hermite polynomial of order \( N_p \) and \( w_n \) are the associated weight factors. A value of \( N_p=20 \) is typically sufficient for excellent accuracy. When slow fading is present, the average symbol error probability is obtained from (3.51) or (3.52) by first replacing \( E_s/N_o \) with \( \gamma=a^2 E_s/N_o \) and then averaging over the PDF of \( \gamma \), that is,

\[
P_s(E) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left\{ 1 - [1 - Q(\gamma)]^{M-1} \right\} \\
\times \int_{\gamma=0}^{\infty} \exp \left( - \frac{(\gamma-\sqrt{2}\gamma)^2}{2} \right) p_\gamma(\gamma) d\gamma \\
\]

(3.53)

Or approximately

\[
P_s(E) = \frac{\Delta}{\sqrt{\pi}} \sum_{n=1}^{N_p} w_n \left\{ 1 - \int_{0}^{\gamma} [1 - Q(\sqrt{2} \left( X_n + X_n \right) \right]^{M-1} p_\gamma(\gamma) d\gamma \right\} 
\]

(3.54)

Numerical evaluation of (3.53) and the associated bit error probability for Rayleigh and Nakagami-m fading channels is computationally intensive. Equation (3.54) does yield numerical value; however, its evaluation is very time consuming.
especially for large value of m. Thus, tight upper bounds on the result in (3.53) which are simple to use and evaluation numerically are highly desirable.

Using Jensen’s in quality [15], Hughes [16] derived a simple bound on the AWGN performance in (3.51). In particular, it was shown that

\[ P_y(E) \leq 1 - \left[ 1 - Q\left( \frac{E_s}{\sqrt{N_o}} \right) \right]^{M-1} \]  
(3.55)

This is tighter than the more common union upper bound.

\[ P_y(E) \leq (M - 1)Q\left( \frac{E_s}{\sqrt{N_o}} \right) \]  
(3.56)

Evaluation of an upper bound on average error probability for the fading channel by averaging the right-hand side of (3.55) (with \(E_s/N_o\) replaced by \(\gamma_s\)) over the PDF of \(\gamma_s\) and using the conventional form for the Gaussian probability integral is still computationally intensive. Using the alternative forms of the Gaussian Q-function and its square, it is possible to simplify the evaluation of this upper bound on performance. The details are as follows:

We begin by applying a binomial expansion to the Hughes bound of (3.55), which when averaged over the fading PDF results in

\[ P_y(E) \leq \sum_{k=1}^{M-1} (-1)^{k+1} \binom{M-1}{k} I_k \]  
(3.57)

Where

\[ I_k = \int_0^\infty Q^k(\sqrt{\gamma_s}) p_{\gamma_s}(\gamma_s) d\gamma_s, \quad k = 1, 2... M-1 \]  
(3.58)

The result based on the union upper bound would simply be the first term \(k=1\) of (3.57), and the integral in (3.58) can be evaluated for \(M=4\) \((k=1, 2, 3)\) either in closed form or in the form of a single integral with finite limits and an integrand composed of elementary functions. The results appear and are summarized here as follows:
\[ I_1 = [P(c)]^m \sum_{k=0}^{n-1} \binom{m-1+k}{k} [1 - P(c)]^k, \]

\[ P(c) = \frac{1}{2} \left( 1 - \frac{c}{\sqrt{1+c}} \right), c \approx \frac{\gamma}{2m}, \quad (3.59) \]

\[ I_2 = \frac{1}{4} - \frac{1}{\pi \sqrt{1+c}} \left\{ \left( \frac{\pi}{2} - \tan^{-1} \frac{c}{\sqrt{1+c}} \right) \right\} \sum_{i=1}^{m-1} \sum_{k=0}^{i} \frac{2k}{(1+c)^k} \frac{1}{[4(1+c)]^{k}} \]

\[-\sin \left\{ \tan^{-1} \frac{c}{\sqrt{1+c}} \right\} \sum_{i=1}^{m-1} \sum_{k=0}^{i} \frac{T_{2k}}{(1+c)^k} \left[ \cos \left( \tan^{-1} \frac{c}{\sqrt{1+c}} \right) \right]^{2(k-i)+1} \}

\[ T_{2k} = \frac{2k}{2(k-i)} \frac{(k+1)}{(k-i)} \frac{1}{4^i [2(k-i)+1]}, \quad c = \frac{\gamma}{2m}, \quad (3.60) \]

\[ \text{and } I_3 = \frac{1}{\pi} \int_{0}^{\pi/4} \left( \frac{2}{\gamma} \right)^m \left( P(c(\phi)) \right)^m \sum_{k=0}^{m-1} \binom{m-1+k}{k} [1 - P(c(\phi))]^k d\phi, \]

\[ c(\phi) = \frac{\gamma}{2} \left( \frac{\sin^2 \phi}{\sin^2 \phi + \gamma/2} \right) \quad (3.61) \]

Average Bit Rate ($P_E$) with Average SNR per bit (dB) characteristics depends on values Nakagami – $q$, $\rho$, $n$ and $m$. An effort is made to draw these characteristics by taking only on variables at a time and other two are taken constant. Here function characteristic may be plotted for the following condition.[165]
Case – 1, Average BEP with Average SNR per bit characteristics. Hence, q = 0, 0.3, 1 and (a) ρ=0; (b) ρ=0.2; (c) ρ=0.4; (d) ρ=0.6

The presented graph (figure 3.6) clearly shows the relation of average SNR per bit (dB) Average Bit Error Rate (E), where average BEP of correlated BFSK over Nakagami-q channel is q = 0, q =0.3 and q = 1 special relation to the case of total variance of ρ=0, ρ=0.2, ρ=0.4and ρ=0.6 shown in graph as a, b, c, and d. It is observed that as value of q increases the graph of average SNR (dB) decreases. This curve is plotted in semi log paper. The characteristic started from same value of \( P_b(E) \) at SNR 0 dB and decreases sharply as value of q increases.

**Figure 3.6 Average BEP of Correlated BFSK over a Nakagami-q Channel (a) ρ=0; (b) ρ=0.2; (c) ρ=0.4; (d) ρ=0.6**
Case – 2, Average BEP with Average SNR per bit characteristics. Hence, \( n = 0, 2, 4, \infty \), \( k = 6.02 \text{ dB}, 12.04 \text{ dB} \) and (a) \( \rho = 0 \); (b) \( \rho = 0.2 \); (c) \( \rho = 0.4 \); (d) \( \rho = 0.6 \)

Figure 3.7 demonstrates the co-relation between Average Signal to Noise Ratio per Bit (dB) to Average Bit Error Rate \( P_b(E) \), where Average BEP of co-related BFSK over a Nakagami-n channel (Rice) \( n = 0, n = 2, n = 4 \) and \( n > \infty \). It is observed that value of \( P_b(E) \) in between 1 and 0.1 for the value of average SNR per bit (dB) 0 dB. As average SNR per bit increases the characteristic sharply decrease as value of \( n \) increases. At \( n \rightarrow \infty \), the average bit error rate \( P_b(E) \) margin in between 15 dB and 20 dB of average SNR per bit (dB). Adoptive White Gaussian Noise (AWGN), in relation to special case of (a) \( \rho = 0 \); (b) \( \rho = 0.2 \); (c) \( \rho = 0.4 \); (d) \( \rho = 0.6 \) respectively in the presence of \( K = 6.02 \text{ dB} \) and \( K = 12.04 \text{ dB} \).
Case – 3, Average BEP with Average SNR per bit characteristics. Hence, \( m = 1/2, 1, 2, 4 \) and \( \geq \infty \) and (a) \( \rho=0 \); (b) \( \rho=0.2 \); (c) \( \rho=0.4 \); (d) \( \rho=0.6 \)

![Graph showing Average BEP of Correlated BFSK over a Nakagami-m Channel](image)

**Figure 3.8, Average BEP of Correlated BFSK over a Nakagami-m Channel** (a) \( \rho=0 \); (b) \( \rho=0.2 \); (c) \( \rho=0.4 \); (d) \( \rho=0.6 \)

The figure3.8, depicting the graph of Average Signal to Noise Ratio per Bit (dB) with Average Bit Error Rate \( P_b(E) \) highlights the correlation between them where the relation of Average BEP of correlated BFSK over a Nakagami-m channel and Average Bit Error Rate \( P_b(E) \) is depicted. The value of \( m \) is taken as \( 1/2, 1, 2, 4 \) and \( \geq \infty \), Adopting White Gaussian Noise (AWGN), in relation to (a) \( \rho=0 \); (b) \( \rho=0.2 \); (c) \( \rho=0.4 \); (d) \( \rho=0.6 \) respectively. It observed the all characteristics started at \( P_b(E) \) in between 1 and 0.1. The value of \( P_b(E) \) decreases. It is also observed that the decrements of \( P_b(E) \) become sharper with the avg. SNR per bit on value of \( m \) increases. For AWGN, the value of \( P_b(E) \) is lowest i.e as \( 10^{-9} \) in between 15 dB to 20 dB of SNR.
3.7 Conclusion

With all the previous discussion made, and as shown in above figure, the variation of bit error probability with SIR for L = 1, 2, 3 and 4 with SNR = 10 dB, m = 1.5 the effect of L in the negative region of SIR shows random behavior. In the region -1 dB to 25 dB, all the curves behave in similar fashion and with the increase of L, whereas decreases for certain value of SIR. For L = 2, remains constant for the interval of SIR (-10 dB to -3 dB) which indicates that there is no effect of interference. The Figure 3.4 indicates the change of bit error probability with SIR for SNR = -10 dB, 0 dB, 5 dB, 10 dB and 15 dB with m = 1.5 and L = 3. From the figure we infer that all the curves show the same value -1 dB of bit error probability at SIR = -5 dB. For a particular values of SIR, decrease with the increase of SNR in the interval of SIR from -1 dB to 25 dB.

In graphs show the curves for average bit error probability versus average bit SNR for 4-ary orthogonal signaling over the Nakagami-m channel, the special case of m = 1 corresponding to the Rayleigh channel. For each value of m, three curves are calculated. The first is the exact obtained (with much computational power and time) by averaging (3.54) over the PDF in

\[
P_m(\gamma) = \frac{m^m \gamma^{m-1} \exp(-m\gamma / \gamma)}{m \gamma \gamma^m}
\]

where \( \gamma \geq 0 \).

The second is the Hughes upper bound obtained from (3.57) together with (3.59), (3.60), and (3.61). Finally, the third is the union upper bound obtained from the first term of (3.54) together with (3.61). The curves labeled m = \( \infty \) correspond to the non fading (AWGN only) results. We observe, not surprisingly, that as m increases (the amount of fading decreases) the three results are asymptotically equal to each other. For Rayleigh fading (the smallest integer value of m) we see the most disparity between the three, with the Hughes bound falling approximately midway between the exact result and the union upper bound. More specifically, the “averaged” Hughes bound is 1 dB tighter than the union bound for high-average bit SNR values. As m increases, the differences between the Hughes bound and the exact results is at worst less than a few tenths of 1 dB over a wide range of average bit SNR’s. Hence, for high values of m, we can conclude that it is accurate to use the former as a prediction of true system performance, with the advantage that the numerical results can be obtained instantaneously.
Hence, it has been tried to examine that the relation of bit error performance of non-coherent detection FSK over Nakagami fading channel with the total effect of noise and co-channel interference treating independently with arbitrary covariance matrices. Here, the problems have been discussed using characteristic functions of noise and interference and closed form solution of differentiation. In receiver, when SIR is small, the probability of bit error is lower as compared to that conventional match filter and optimal receiver. At lower values of SIR, the proposed receiver shows much better performance. Also, it is observe, not surprisingly from all the figures and graphs that as m increases the amount of fading decreases and the three results are asymptotically equal to each other. For Rayleigh fading (the smallest integer value of m) we see the most disparity between the three, with the Hughes bound falling approximately midway between the exact result and the union upper bound. More specifically, the “averaged” Hughes bound is 1 dB tighter than the union bound for high-average bit SNR values. As m increases, the differences between the Hughes bound and the exact results is at worst less than a few tenths of 1 dB over a wide range of average bit SNR’s. Hence, for high values of m, we can conclude that it is accurate to use the former as a prediction of true system performance, with the advantage that the numerical results can be obtained instantaneously.