Abstract

The theory of semiring was first developed by H. S. Vandiver [50] (in 1934) in which he has obtained some important results of the objects. Semiring constitute a fairly natural generalization of rings, with broad applications in the mathematical foundation of computer science. Also, semiring theory has many applications to other branches. For example, automata theory, optimization theory, algebra of formal process, combinatorial optimization, Baysian networks and belief propagation. our approach here is purely algebraic.

In Chapter-1, we introduce the concept of partitioning ideal, maximal homomorphism and semiisomorphism for ternary semirings. We study some basic concepts of semirings, ternary semirings and review some of the background material that will be of value for our later pursuits. We refer [33, 34] and [42] for basic terminology and results in semirings and ternary semirings respectively.

In Chapter-2, we obtain characterizations of primary ideals, semiprime ideals and irreducible principal $T$-ideals in the semiring $\mathbb{Z}_0^+$. In third section of this chapter, we obtain some results about subtractive ideals, primary ideals, $Q$-ideal and semiprime ideals in the semiring $(\mathbb{Z}_0^+, \gcd, \text{lcm})$. In fourth section of this chapter, we introduce the concept of subtractive extension of ideals in ternary semirings which is a generalization of a subtractive ideal in ternary semirings and we characterize prime ideals, semiprime ideals, irreducible $k$-ideals, irreducible principal $T^{-}$-ideals, $Q$-ideals and subtractive extension of ideals in the ternary semiring $\mathbb{Z}_0^-$. Finally, we give a short and elementary proof of Lemma [43, Lemma 3.4] and Theorem [43, Theorem 3.12].

In Chapter-3, we prove results on a partitioning ideal of a ternary semiring which
are useful to develop the theory of quotient ternary semiring. Indeed we prove:
1) The quotient ternary semiring $S/I(Q)$ is essentially independent of choice of $Q$.
2) If $f : S \rightarrow S'$ is a maximal ternary semiring homomorphism, then $S/ker(f)(Q) \cong S'$. 3) Every partitioning ideal is subtractive. 4) Let $I$ be a $Q$-ideal of a ternary semiring $S$. Then $A$ is a subtractive ideal of $S$ with $I \subseteq A$ if and only if $A/I_{Q\cap A} = \{q+I : q \in Q \cap A\}$ is a subtractive ideal of $S/I(Q)$. In third section of this chapter, we introduce the concepts of i) closure of an ideal $A$ of a ternary semiring $S$ with respect to an ideal $I$ of $S$; ii) closure of an ideal $A$ of a ternary semiring $S$ with respect to a $Q$-ideal $I$ of $S$ and obtain the relationship between them. Also we obtain some equivalent conditions for subtractive extension of an ideal and hence characterize the ideals, prime ideals and semiprime ideals in quotient ternary semirings.

In Chapter-4, we introduce the concept of steady ternary semiring homomorphism and prove a relation between maximal and steady ternary semiring homomorphisms. In third section of this chapter, we prove equivalent conditions for an ideal $I$ of a ternary semiring $S$ to be a subtractive ideal. Also we prove the fundamental theorem of homomorphism for ternary semirings by using steady (or maximal or one-one) ternary semiring homomorphism.

In Chapter-5, we introduce the concept of $k$-regular ideal (lateral ideal) and hence we obtain some more characterizations of a commutative $k$-regular ternary semiring. In third section of this chapter, we prove if $S$ is an additively idempotent ternary semiring with identity element, then $S$ is $k$-regular if and only if the matrix ternary semiring $S_{n\times n}$ is $k$-regular.

In Chapter-6, we prove some results on $k$-ideals in a ternary semiring and hence extend prime avoidance theorem for $k$-ideals in a ternary semiring. In third section of this chapter, we introduce the concept of $k$-ternary semiring, strongly Euclidean ternary semiring and prove that: 1) Every principal ideal of a strongly Euclidean ternary semiring is a $Q$-ideal. 2) Every strongly Euclidean $k$-ternary semiring is a principal ideal ternary semiring.

In Chapter-7, $R$ denotes a commutative semiring with identity element $1 \neq 0$. In
second section of this chapter, we investigate some characterizations of 2-absorbing subtractive ideals of a semiring $R$. In third section of this chapter, we obtain the 2-absorbing avoidance theorem for $k$-ideals of a semiring $R$. 