APPENDIX - G

Let us start with a one + two body hamiltonian

\[ H = h + V \]

(G.1)

where \( h = \sum_\ell \varepsilon_{\ell} \hat{n}_{\ell} \) is the one-body hamiltonian and the interaction \( V \) is defined in terms of the matrix elements \( V_{\ell_1, \ell_2; \ell_3, \ell_4}^L = \langle \ell_2, \ell_4 | V | \ell_1, \ell_3 \rangle \). In sdg boson space \( \ell \)'s stand for \( s, \, d \) and \( g \).

The hamiltonian \( H \) can be decomposed into tensors with respect to the \( U(\mathcal{N}) \) group [Ko-80, Ko-81] where \( \mathcal{N} = \sum_\ell \mathcal{N}_\ell \) and \( \mathcal{N}_\ell = 1, \, 5 \) and \( 9 \) for \( s, \, d \) and \( g \) respectively. Thus

\[ H = H^{\mathcal{N}=0} + H^{\mathcal{N}=1} + H^{\mathcal{N}=2}, \]

\[ H^{\mathcal{N}=0} = h^{\mathcal{N}=0} + V^{\mathcal{N}=0}, \quad H^{\mathcal{N}=1} = h^{\mathcal{N}=1} + V^{\mathcal{N}=1}, \quad H^{\mathcal{N}=2} = V^{\mathcal{N}=2} \]

(G.2)

where \( \mathcal{N} \) is the tensorial rank with respect to \( U(\mathcal{N}) \) group which is \( U(15) \) in sdg case. Using the results of [Ko-80, Ko-81] we have,

\[ h^{\mathcal{N}=0} = \hat{\varepsilon} \hat{n} ; \quad \tilde{\varepsilon} = \frac{1}{\mathcal{N}} \sum_\ell \mathcal{N}_\ell \varepsilon_\ell \]

(G.3)

\[ h^{\mathcal{N}=1} = \sum_\ell \bar{\varepsilon}_\ell \hat{n}_\ell ; \quad \bar{\varepsilon}_\ell = \varepsilon_\ell - \tilde{\varepsilon} \]

\[ V^{\mathcal{N}=0} = \overline{V} \left( \begin{array}{c} \mathcal{N} \\ 2 \end{array} \right) ; \quad \overline{V} = \left\{ \sum_{\ell_1 \geq \ell_2} V_{\ell_1, \ell_2; \ell_2, \ell_1}^L (2L+1) \left\{ \frac{\mathcal{N}(\mathcal{N}+1)}{2} \right\} \right\}^{\mathcal{N}=0} \]

(G.4)

\[ V^{\mathcal{N}=1} = \frac{\mathcal{N}+1}{\mathcal{N}+2} \sum_\ell \bar{\lambda}_\ell \hat{n}_\ell ; \quad \bar{\lambda}_\ell = \left[ \frac{1}{\mathcal{N}} \sum_{\ell'} V_{\ell, \ell'; \ell, \ell'}^L (2L+1)(1+\delta_{\ell \ell'}) \right] - (\mathcal{N}+1) \overline{V} \]

(G.5)

The \( \bar{\lambda}_\ell \)'s are known as traceless induced single particle energies.
Finally the $V_{0}^{*2}$ is defined by 

$$V_{1}^{L} \ell_{2} \ell_{3} \ell_{4} = V_{1}^{L} \ell_{2} \ell_{3} \ell_{4} - \left( \sqrt{V + \frac{1}{N+2} \left( \lambda_{\ell_{1}} + \lambda_{\ell_{2}} \right)} \right) \delta_{\ell_{1} \ell_{3}} \delta_{\ell_{2} \ell_{4}}$$

(G.7)

Norm $|O|_{m}$ of an operator $O$ in 'm' particle space is defined as

$$|O|_{m} = \left\{ \langle (O)^{2} \rangle_{m}^{\frac{1}{2}} \right\}$$

where the symbol $\langle \rangle$ stands for m-particle space average. The norms of various parts of the operator $H$ can be written down in the m-boson spaces using the following trace propagation equations [Ch-71, Ko-80, Ko-81],

$$\|H\|_{m} = \|H^{\mathrm{bos}}_{1}\|_{m}^{2} + \|H^{\mathrm{bos}}_{2}\|_{m}^{2}$$

$$\|H^{\mathrm{bos}}_{1}\|_{m}^{2} = \frac{m(N+m)}{N(N+1)} \langle (H^{\mathrm{bos}}_{1})^{2} \rangle_{m}^{1}$$

$$\|H^{\mathrm{bos}}_{2}\|_{m}^{2} = \|V^{\mathrm{bos}}_{2}\|_{m}^{2} = \frac{m(m-1)(N+m)(N+m+1)}{N(N+1)(N+2)(N+3)} \langle (V^{\mathrm{bos}}_{2})^{2} \rangle_{m}^{2}$$

In (G.9) $\xi_{\ell} = \varepsilon_{\ell} + \frac{m-1}{N+2} \lambda_{\ell}$ and the symbol $\langle \rangle$ stands for the m-particle trace.

297