CHAPTER - XI

CONCLUSIONS AND FUTURE OUTLOOK

In this thesis the sdgIBM which includes \( g (\ell = 4) \) bosons in addition to \( s (\ell = 0) \) and \( d (\ell = 2) \) bosons is explored and applied in detail to establish that it is a viable model for describing and predicting hexadecupole \( (E4) \) properties in nuclei. To this end several studies are carried out by exploiting the gIBM dynamical symmetries and their interpolations.

Geometric shapes corresponding to the various gIBM dynamical symmetries \( SU_{sdg}(3), SU_{sdg}(5), SU_{sdg}(6), SU_{sdg}(15), U_6(6) \oplus U_9(9), U_{dg}(14) \) and \( U_{dg}(5) \oplus U_{sg}(10) \) (they are subgroups of \( U_{sdg}(15) \)) are studied using Coherent State formalism. The \( U_{sdg}(15) \) Coherent States in \( sdg \) space are constructed; CS expectation value matrix elements of the general gIBM hamiltonian are derived and the equilibrium shape parameters \( (P_0, P_2, P_4) \) are determined in the symmetry limits. This exercise has given a good insight into the physical relevance of gIBM symmetries and based on these results successful model calculations are performed. The CS formalism developed in this thesis can be used to study dynamics and phase transitions associated with gIBM symmetries. This exercise is particularly useful in the case of \( SU_{sdg}(5) \) and \( SU_{sdg}(6) \), as it yields analytical results for band structures and \( E2 \) and \( E4 \) matrix elements in these limits.

Analytical expressions are derived for the \( \ell = 0, 2 \) and \( 4 \) two-nucleon transfer strengths in the \( SU_{sdg}(3) \) limit which is
applicable for deformed nuclei and in all the symmetry limits
large N limit expressions for a particular ratio $R$ involving $\ell = 0$
TNT strengths are obtained. In the $\text{SU}_{\text{nd}}(3)$ limit the known
Wigner-Racah algebra is employed while for the ratio $R$ the CS
formalism developed in the thesis is used. The selection rules and
structures seen in $^{166}\text{Er}(t,p)^{168}\text{Er}$ data are explained in a simple
fashion by analytical results in the $\text{SU}_{\text{nd}}(3)$ limit. Besides
these, the regions of applicability of gIBM dynamical symmetries
are determined by the analysis of data for $R$ in rare-earth region;
the peaks seen in the data are due to structure changes and in the
Sm region they are well reproduced by detailed numerical
calculations. In other regions it is plausible that one may have
to include two quasi-particle excitations. This calls for an
extension of gIBM to gIBFFM (two fermions coupled to gIBM core).

The low-lying bands built on a g boson excitation
(hexadecupole vibration) coupled to each of the three limiting
symmetries ($U_\omega(5), \text{SU}_{\text{nd}}(3)$ and $O_{\text{nd}}(6)$) of sdIBM are classified and
analytical expressions for (intra- and inter- band) $E2$ and $E4$
transition matrix elements are derived. These results are used in
describing hexadecupole vibrational $K^T = 3^+, 4^+$ bands in $^{156}\text{Gd},$
$^{178}\text{Hf}$ and $4^+$ levels in Cd isotopes. The $^{156}\text{Gd},^{178}\text{Hf}$ nuclei are
shown to serve as bench-mark examples for $\text{SU}_{\text{nd}}(3) \times 1g$ limit. It
will be interesting to analyze the data for $K^T = 3^+, 4^+$ bands in
various regions of the periodic table and they shed light on
possible hexadecupole nature of these bands.

The package SDGIBM1 which is a general computer code for
performing spectroscopic calculations (spectra, (E2,E4,M1)
transition strengths, static moments, occupation numbers and
two-nucleon transfer strengths for $l = 0$ transfer) in sdg space is
developed. All the relevant equations are derived using angular
momentum algebra. The codes employ the basis defined by boson
numbers $n_s$, $n_d$ and $n_g$ and the corresponding seniority quantum
numbers. The eigenvalues of group Casimir operators, selection
rules due to the tensorial nature of the group generators and
analytical results in the SU$_{sdg}(3)$ limit are used to test the
eigenvalues and eigenvectors given by the package. Similarly the
fixed-$(n_s, n_d, n_g)$ trace formulas for $H$ and $H^2$ and also the
fixed-$(n_s, n_d, v_s, n_g, v_g)$ trace formula for $H$ are given and they are
used to test the hamiltonian matrix. As a first application of the
package spectroscopic properties of $^{20}$Ne, $^{24}$Mg, $^{32}$S and $^{36}$Ar are
studied employing a model hamiltonian interpolating the gIBM
dynamical symmetries. Several levels lying outside sd space and
large number of E2 and E4 matrix elements are well reproduced.

Detailed spectroscopic calculations for Sm isotopes are
carried out using the package SDGIBM1 as a major test of the sdg
model. The observed spherical-deformed shape phase transition
manifested in the various spectroscopic properties of $0^+_1$, $2^+_1$ and
$4^+_1$ levels in $^{146-158}$Sm is well described using a simple two-
parameter hamiltonian in a truncated space ($n_s \geq 2$ and $n_g \leq 2$).
Simultaneous description of GS, $\beta$- and $\gamma$- bands in $^{152,154}$Sm is
obtained using a hamiltonian interpolating dynamical symmetries.
These calculations describe very well the observed E4 data;
$B(E4; 4^+_1 \rightarrow O_{GS}^+)$ in addition to the usual $B(E4; 4^+_1 \rightarrow O_{GS}^+)$. The
E4 transition strength distributions are also predicted for these

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The spectra, absolute values of E2 and E4 matrix elements and B(E2) values are consistently well reproduced in $^{198,194}$Pt and $^{192}$Os by employing an interpolating hamiltonian (the choice is determined by the structure of these nuclei and the results obtained in the study of geometric shapes, using CS, corresponding to gIBM symmetries) and two-parameter E2 and E4 transition operators. These 2g boson ($n_s \leq 2$) calculations give much better results (they could predict E4 matrix elements) compared to the 1g boson ($n_s \leq 1$) calculations available in literature.

Finally, the novel problem of scissors states in even-odd nuclei is studied with $^{185}$W as an example. Using the SU(3) limit of the proton-neutron Interacting Boson Fermion model (with and without g bosons), the M1 excited scissors states in $^{185}$W are classified, and the location of these states with the corresponding B(M1) values are predicted. These results easily extend to all even-odd nuclei with the odd-neutron in 82-126 shell and this is important in the search for scissors states in even-odd nuclei.

The number of free parameters used in the fits to 2s-1d shell nuclei, Sm isotopes and ($^{198,194}$Pt, $^{192}$Os)-nuclei reported in the thesis are not much more than what is used in sdIBM. This together with the other results reported in the thesis corroborate Casten and Warner's statement [Ca-88] "It is the clear need for g-bosons in this context, perhaps more than any other factor that suggests that the improved agreement obtained for observables is physically significant as not just an exercise in parameter proliferation".
The results obtained in the thesis provide a good understanding of the sdgIBM and establish that this model is a simple yet powerful tool in analyzing E4 properties of atomic nuclei. With this we have: (i) large number of analytical results that facilitate rapid analysis of data relating to hexadecupole degree of freedom; (ii) the package SDGIBM1 which allows for systematic spectroscopic calculations with g bosons. Therefore, one is now well-equipped to analyze: (i) the observed $\beta_4$ and $\alpha_{42}$ (which measures $\gamma_{42}$ deformation) systematics in rare-earth nuclei (also the available $\beta_4$ systematics in actinides); (ii) the E4 matrix elements and strength distributions in heavy nuclei where data is available; (iii) transition charge densities for the first three or four $4^+$ levels that are available for $^{188,190,192}$Os, $^{196}$Pt, $^{199}$Ru and $^{190}$Pd nuclei which indicate the possible hexadecupole nature of some of these levels; (iv) $K^\pi = 3^+,4^+$ bands systematics in rare-earth nuclei. From the studies that are carried out in microscopic models, it is clear that one has to extend gIBM to gIBFFM for a proper understanding of the hexadecupole content of the $K^\pi = 3^+,4^+$ bands (to distinguish them from the two-quasi particle or two-phonon structures). For a more realistic study of the systematics of the various observables mentioned above it is essential to extend the SDGIBM1 code to include proton-neutron degrees of freedom. In all these studies it is essential that one understands and estimates the free parameters in the model and a viable microscopic theory for the same should be worked out. These studies will provide a deeper insight into the role of hexadecupole degree of freedom in nuclei.