CHAPTER - X

SCISSORS STATES IN $^{185}$W

10.1 Introduction

The discovery of the M1 excited scissors 1$^+$ state in $^{156}$Gd by Bohle et al. [Bo-84] has lead to large number of studies, both theoretical [Lo-78, Lo-79, No-88, Fa-90a, Fa-90b, Sc-85, Va-86, De-91a] and experimental [Ri-88, Ha-87b], on collective M1 states in even-even nuclei all across the periodic table (in Fig. 1.1b a pictorial representation of the scissors mode is given). The scissors states are first predicted in the two-rotor model by Lo Iudice and Palumbo [Lo-78, Lo-79]. The simplest description of the structure of the scissors states and their M1 decay strength to the ground state is in terms of the proton-neutron interacting boson model (p-n IBM) [Sc-85]. In this model, for deformed nuclei the 1$^+$ states belong to the lowest mixed symmetry U(6) partition (N-1,1) and SU(3) representation (2N-2,1) with F-spin F = N/2-1, where N is the number of bosons. The geometric correspondence exhibited via Coherent States succinctly brings out [Ba-85a, Di-85] the scissors structure of the mixed symmetry states. With the growing data on several 1$^+$ states and the resulting M1 strength distributions [Ha-87b, Ri-88, De-91a], and the failure of the sdIBM to predict the same lead Wu et al [Wu-87] to study M1 states in the p-n IBM with the inclusion of g-bosons (p-n sdgIBM or simply p-n gIBM). The p-n gIBM accommodates much larger variety of 1$^+$ states; the first studies on 1$^+$ states in gIBM are reported in [Ba-85b, Pi-84, Mo-89].
Recently the question of scissors states in odd-A nuclei is being addressed and preliminary results are given in the geometric particle - core coupling model [Ra-89, Ra-90] and in the p-n interacting boson fermion model (p-n IBFM) [Va-89a, Fr-91]; this Chapter deals with the latter model. In the SU(3) ⊗ U(2) limit of IBFM the classification, energies and B(M1)'s for the scissors states are given [Va-89a] for $^{169}$Tm and these results are based on the fact that for this nucleus the SU(3) ⊗ U(2) limit is a good symmetry. It is shown in [Bi-88] that $^{169}$Tm (where the odd particle occupies $j = 1/2, 3/2, 5/2, 7/2$ orbits) and $^{185}$W (where the odd particle occupies $j = 1/2, 3/2, 5/2, 7/2, 9/2$ orbits) are the only nuclei where SU(3) ⊗ U(2) limit applies reasonably well. Therefore it is natural that one should study scissors states in $^{185}$W. Unlike in $^{169}$Tm, hexadecupole degree of freedom (hence g bosons) is important in W isotopes [Ca-74, Ne-86]. Moreover, there is adequate evidence for the applicability [Su-81] of the gIBM SU(3) limit to $^{184}$W (neighbouring nucleus for $^{185}$W). Therefore M1 states in $^{185}$W are studied (as an academic exercise; see however Sect. 10.4) in this Chapter with and without g bosons and the results add to the theoretical data base on scissors states in odd-A nuclei where no experimental data is available yet.

In Sect. 10.2 scissors states in the SU(3) ⊗ U(2) limit of p-n sdIBFM for $^{185}$W are classified and their energies and B(M1) strengths to ground state are given. With the inclusion of g-bosons two coupling schemes [Ko-86a; also Yu-88], SU(3) ⊗ U(2) and U(15) ⊗ U(2), are possible (unlike only the SU(3) ⊗ U(2) without g-bosons) and in addition the M1 operator will have an extra piece that is absent in the sd model. In Sect. 10.3 the M1
10.2 Scissors States in sd IBM for $^{185}\text{W}$

In the IBFM, for odd-$A$ nuclei, dynamical Bose-Fermi (BF) symmetries based on the SU(3) limit of IBM arise when the odd fermion occupies single particle orbits with angular momenta $j = 1/2, 3/2, ..., (2N_j + 1)/2$. In this situation the $j$-values can be split into pseudo-orbital part, $\tilde{\ell} = \eta, \eta - 2, ..., 0$ or 1 and pseudo spin part $\tilde{s} = 1/2$. Then the pseudo orbital angular momenta belong to the SU(3) irrep $(\eta \ell)$ and therefore it can be combined with the SU(3) group of the boson core to give rise to the SU(3) $\otimes$ U(2) BF symmetry; the U(2) group corresponds to the pseudo spin. For $^{185}\text{W}$ nucleus (and other nuclei with valence neutrons in the 82-126 shell) $\eta = 4$ with $j = 9/2, 7/2, 5/2, 3/2$ or 1/2. The boson number $N$ for $^{185}\text{W}$ is 11 with $N_x = 4$ and $N_y = 7$. In the SU(3) $\otimes$ U(2) limit of p-n IBFM, with $\eta = 4$, the group chain and the irrep labels are,

\[
\begin{align*}
U^B \langle 6 \rangle \otimes U^F \langle 30 \rangle & \supset U^B \langle 6 \rangle \otimes U^F \langle 15 \rangle \otimes U^F \langle 2 \rangle \supset SU^B \langle 3 \rangle \otimes SU^F \langle 3 \rangle \otimes \\langle f \rangle \quad M = 1 \\
U^F \langle 2 \rangle & \supset SU^{BF} \langle 3 \rangle \otimes U^F \langle 2 \rangle \supset O^{BF} \langle 3 \rangle \otimes SU^F \langle 2 \rangle \supset Spin^{BF} \langle 3 \rangle \\
& \begin{pmatrix} \lambda_{BF} : \mu_B \end{pmatrix} \begin{pmatrix} k_L, L, s = 1/2, J = L \pm 1/2 \end{pmatrix} 
\end{align*}
\]

In (10.1) $M$ is the fermion number. For the low-lying rotational
bands (with $F = N/2$) the $U_{sd}^B(6)$ irrep $\{f\} = \{N\}$ and the boson core $SU(3)$ irrep $(\lambda_B, \mu_B) = (2N,0)$. The $SU^B_{\text{BF}}(3)$ irreps $(\lambda_{\text{BF}}^B, \mu_{\text{BF}}^B)$ are given by

$$(2N,0) \otimes (40) \rightarrow (\lambda_{\text{BF}}^B, \mu_{\text{BF}}^B) = (2N+4,0) \oplus (2N+2,1) \oplus (2N,2) \oplus (2N-2,3) \oplus (2N-4,4).$$

(10.2)

The correspondence between the Nilsson labels and the $(\lambda_{\text{BF}}^B, \mu_{\text{BF}}^B)$ irreps established in [Bi-88] allows one to fix the $(\lambda_{\text{BF}}^B, \mu_{\text{BF}}^B)$ that correspond to the observed ground state (GS) rotational band; see Fig. 10.1. In $^{185}W$, the GS $K = 3/2$ band belongs to the Nilsson orbit $[512]3/2$ and there is a nearly degenerate (separated by $\sim 24$ keV) $K = 1/2$ band which belongs to $[510]1/2$ Nilsson orbit. They both belong (see Fig. 10.1) to the irrep $(\lambda_{\text{BF}}^B, \mu_{\text{BF}}^B) = (2N-2,3)$ with the band quantum number $K_L = 1$. Following Van Isacker and Frank [Va-89a], for the scissors states the $U_{sd}^B(6)$ irrep $\{f\} = \{N-1,1\}$ and the boson core $SU_{sd}^B(3)$ irrep is $(2N-2,1)$. The scissors $SU^B_{\text{BF}}(3)$ irreps $(2N-2,1) \otimes (40) \rightarrow (\lambda_{\text{BF}}^B, \mu_{\text{BF}}^B)$ are given in Table-10.1. Appendix-H gives the algorithm used in obtaining the reductions in analytical form.

The scissors states that can be identified experimentally are usually the ones that can be excited by the $M1$ operator acting on the ground state. In the p-n IBFM the $M1$ operator, ignoring the single particle part (as assumed in [Va-89a]; see Sect. 10.4 ahead) is

$$T^{M1} = g_\pi L_\pi + g_\nu L_\nu$$

(10.3)

where $g_\pi$ and $g_\nu$ are the g-factors and $L_\pi$ and $L_\nu$ are the angular
Fig. 10.1 Classification of negative parity orbits in the 82-126 neutron shell (with the oscillator shell number $N = 5$). In the Nilsson model the orbits are labelled by $[\lambda n \Lambda] \Omega$, in the pseudo-Nilsson model they are labelled $= \pm \frac{1}{2}$, where $N > 1$, $\Lambda = 2\Omega - \Lambda$ and in the $SU(3) \otimes U(2)$ limit of IBFM by $(\lambda, \mu) \ K_l \ L = K_l \Omega = \Lambda$. The figure is taken from [Ko-91b; Bo-75, Bi-88]. Note that as $N \to \infty$, $\lambda \to \eta N \ (\eta = 2$ for sd and $4$ for sdg) and for finite $N$, $\lambda = \eta N + 4 - 2\mu$ for the orbits shown in the figure (see (10.10)). Note that $\delta$ is the deformation parameter [Bo-75].
TABLE 10.1

SU(3) irreps (λμ) in the kronecker product (λ₁μ₁) ⊗ (λ₂μ₂)

<table>
<thead>
<tr>
<th>(λ₁μ₁)</th>
<th>(λ₂μ₂)</th>
<th>(λμ)*</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3N-2,1) b</td>
<td>(40)</td>
<td>(3N+2,1) ⊗ (3N,2) ⊗ (3N-2,3) ⊗ (3N-1,1) ⊗ (3N+1,0) ⊗ (3N-4,4) ⊗ (3N-3,2) ⊗ (3N-6,5) ⊗ (3N-5,3)</td>
</tr>
<tr>
<td>(3N-2,3)</td>
<td>(11)</td>
<td>(3N-1,4) ⊗ (3N,2) ⊗ (3N-3,5) ⊗ (3N-2,3)² ⊗ (3N-1,1) ⊗ (3N-4,4) ⊗ (3N-3,2)</td>
</tr>
<tr>
<td>(4N-2,3)</td>
<td>(33)</td>
<td>(4N+1,6) ⊗ (4N+2,4) ⊗ (4N+3,2) ⊗ (4N+4,0) ⊗ (4N-1,7) ⊗ (4N,5)² ⊗ (4N+1,3)² ⊗ (4N+2,1)² ⊗ (4N-3,8) ⊗ (4N-2,6)² ⊗ (4N-1,4)³ ⊗ (4N,2)³ ⊗ (4N+1,0) ⊗ (4N-5,9) ⊗ (4N-4,7)² ⊗ (4N-3,5)³ ⊗ (4N-2,3)⁴ ⊗ (4N-1,1)² ⊗ (4N-6,8) ⊗ (4N-5,6)² ⊗ (4N-4,4)³ ⊗ (4N-3,2)³ ⊗ (4N-2,0) ⊗ (4N-7,7) ⊗ (4N-6,5)² ⊗ (4N-5,3)² ⊗ (4N-4,1)² ⊗ (4N-8,6) ⊗ (4N-7,4) ⊗ (4N-6,2) ⊗ (4N-5,0)</td>
</tr>
<tr>
<td>(4N+2,1)</td>
<td>(11)</td>
<td>(4N+3,2) ⊗ (4N+4,0) ⊗ (4N+1,3) ⊗ (4N+2,1)² ⊗ (4N,2) ⊗ (4N+1,0)</td>
</tr>
</tbody>
</table>

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\[
\begin{array}{ccc}
(\lambda_1 \mu_1) & (\lambda_2 \mu_2) & (\lambda \mu) \\
(4N+2,1) & (33) & (4N+5,4) \oplus (4N+6,2) \oplus (4N+3,5) \oplus (4N+4,3)^2 \oplus \\
& & (4N+5,1) \oplus (4N+1,6) \oplus (4N+2,4)^2 \oplus (4N+3,2)^2 \oplus \\
& & (4N+4,0) \oplus (4N-1,7) \oplus (4N,5)^2 \oplus (4N+1,3)^2 \oplus \\
& & (4N+2,1)^2 \oplus (4N-2,6) \oplus (4N-1,4)^2 \oplus (4N,2)^2 \oplus \\
& & (4N+1,0) \oplus (4N-3,5) \oplus (4N-2,3)^2 \oplus (4N-1,3) \oplus \\
& & (4N-4,4) \oplus (4N-3,2) \\
\end{array}
\]

In \((\lambda \mu)^r\), \(r\) denotes the number of occurrences (multiplicity) of \((\lambda \mu)\) in the product \((\lambda_1 \mu_1) \times (\lambda_2 \mu_2)\).

\(^a\)All the results are valid for \(N \geq 4\).

\(^b\)\(\eta = 2\) for sd and \(\eta = 4\) for sdg.
momentum operators for proton and neutron bosons respectively. The
M1 operator given in (10.3) behaves as \((\lambda, \mu) = (11)\) tensor
(irrep) with respect to the SU(3) group. With the GS SU(3) irrep
being (2N-2,3), the M1 operator can excite states with SU(3)
irreps \((2N-2,3) \otimes (11) \rightarrow (\lambda', \mu')\) and the \((\lambda', \mu')\) irreps are given
in Table-10.1. The SU(3) irreps \((\lambda, \mu)\) corresponding to the M1
excited scissors states are obtained by taking the intersection of
the SU(3) irreps in \((2N-2,1) \otimes (40)\) and \((2N-2,3) \otimes (11)\) and they
are

\[
(\lambda, \mu) = (2N,2) \oplus (2N-2,3) \oplus (2N-1,1) \oplus (2N-4,4) \oplus (2N-3,2)
\]

(10.4)

The expression for M1 matrix elements connecting the scissors
states to the ground state are derived using: (i) the \(U(6) \supset SU(3) \supset O(3)\)
tensorial form of \(L_\pi, L_\kappa = \sqrt{10} \tau^{(11,1)}\); (ii)\n\[\langle 1^n \vert L_\pi \vert 0^+ \rangle \vert^2 = 8 N_\pi N_\kappa (2N-1)\); given in eq (2.15b) of [Sc-85];
(iii) the \(U(6) \supset SU(3) \supset O(3)\) reduced matrix element \(<(N,1,1)
(2N-2,1) \vert \bigotimes \tau^{(11,1)} \bigotimes (N) (2N,0)\rangle = \{8N_\pi N_\kappa (N+1) / 10N(2N-1)\}\)\n\(1^2\),
given in eq(2.15b) of [Sc-85] and (4.11) of [Va-86]; (iv) Racah’s
factorization lemma [Wy-74] for \(U(6) \supset SU(3) \supset O(3)\) matrix
elements. The final result is

\[
\mathbf{B}^{(M1; (2N,0) \otimes (40) (\lambda, \mu) K_L L J \rightarrow (2N-2,1) \otimes (40) (\lambda', \mu') K_{L'} L' J')}
\]

\[
= \frac{3}{4\pi} (e_\pi e_\nu)^2 \left\{ \frac{8(N+1) \times N_\pi N_\nu}{N(2N-1)} \right\} \times
(2L'+1) (2J'+1) \left\{ \frac{L' J' 1/2}{J L 1} \right\}^2 \times
\]

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\[
\begin{align*}
\rho_{\text{max}} & \times \left[ \sum_{\rho=1}^{\rho_{\text{max}}} <(\lambda_{\mu})_{K_L} L (11)110|(\lambda_{\mu})_{K_{L'}} L'> \rho \times \right. \\
& \left. Z((11) (2N,0) (\lambda_{\mu}) (40); (2N-2,1) (\lambda_{\mu}) \rho)^2 \right] \\
\end{align*}
\]

(10.5)

In (10.5) \( \{-\} \) is 6-j symbol, < \( || \) > is SU(3) \( \supseteq \) O(3) reduced Wigner coefficient, \( \rho \) is the multiplicity of the coupling \( (\lambda_{\mu}) \otimes (11) \rightarrow (\lambda_{\mu})_{s} \) and \( Z(-) \) is Millner's [Mi-78] SU(3) Z-coefficient. The standard SU(3) U-coefficient coincides with the Z-coefficient for \( \rho_{\text{max}} = 1 \). Appendix I gives the definition of Z-coefficient and the relation between Z- and U-coefficients. Although the authors of [Va-89a] used [Va-91] (10.5), in their paper (eq(8) of [Va-89a]) they give U in place of Z-coefficient. The factor \( [\cdot] \) in (10.5), when written solely in terms of SU(3) \( \supseteq \) O(3) reduced Wigner coefficients, is given by

\[
\begin{align*}
\rho_{\text{max}} & \times \left[ \sum_{\rho=1}^{\rho_{\text{max}}} <(\lambda_{\mu})_{K_L} L (11)110|(\lambda_{\mu})_{K_{L'}} L'> \rho \times \right. \\
& \left. Z((11) (2N,0) (\lambda_{\mu}) (40); (2N-2,1) (\lambda_{\mu}) \rho)^2 \right] \\
& = \left[ \sum_{R,R',\ell,L} (-1)^{R+L+1} \sqrt{(2L+1)(2R'+1)} \{R' \ell \ell \}\right] \\
& \quad \times \left< (2N,0)_{R} (40)_{\ell}|(\lambda_{\mu})_{K_L} L > \times \right. \\
& \left. \left< (2N-2,1)_{R'} (40)_{\ell'}|(\lambda_{\mu})_{K_{L'}} L' > \times \right. \right. \\
& \left. \left< (2N,0)_{R} (11)110|(2N-2,1)_{R'} > \right. \right] \\
\end{align*}
\]

(10.6)
Using Akiyama and Draayer [Ak-73] codes the SU(3) recoupling coefficients appearing in (10.6) are evaluated and the B(M1) strengths, for $^{185}$W, are calculated via (10.5), (10.6) for the $(\lambda_\nu \mu_\nu^\prime)$ in (10.4). It is found that the strengths, from the GS $\ket{(2N-2,3)K = 1 \ L = 1 \ J = 3/2 \ or \ 1/2}$ to $\ket{(\lambda_\nu \mu_\nu^\prime)} = (2N,2), (2N-2,3)$ and $(2N-1,1)$ are smaller (similar to the situation [Va-89a] in $^{169}$Tm; see also Table-10.3 and Sect. 10.3 ahead) by a factor $\sim 10^2 - 10^3$ compared to the strengths for $(2N-4,4)$ and $(2N-3,2)$ irreps. Table-10.2 gives the calculated B(M1) strengths for the $(2N-4,4)$ and $(2N-3,2)$ irreps for all the allowed values of $(K', L')$.

The energies of the scissors states in $^{185}$W are calculated using the formula [Va-89a, Bi-88]

$$E \langle (\lambda_B \mu_B) \otimes (40) (\lambda_B^L \mu_B^L) K_L L J \rangle = 2N \left[ \frac{\Gamma}{4} (8-3\mu_B^L) + \frac{\Lambda}{120} (8-3\mu_B^L)^2 \right]$$

$$\quad + \gamma_1 L(L+1) + \gamma_2 J(J+1) + \alpha \left( \frac{N}{2} - F \right) \left( \frac{N}{2} + F + 1 \right)$$

(10.7)

The excitation energy of the scissors states is obtained by subtracting out the GS $\ket{(2N-2,3)K_L = 1 \ L = 1 \ J = 3/2}$ energy which is also calculated using (10.7). The energies of the scissors states are calculated employing the parameters $\Gamma = 50$ keV, $\Lambda = 500$ keV, $\gamma_1 = 16$ keV and $\gamma_2 = 1.5$ keV given in [Bi-88] and determining the value of $\alpha$ ($\alpha = 200$ keV) from the smoothed (with respect to $N_\pi \times N_\nu$) formula $\alpha = 4.3\delta(N_\pi N_\nu)^{-1/2}$ MeV, given in [Ha-87b, also Ri-88] where $\delta$ is the deformation parameter (we choose $\delta = 0.25$ for $^{185}$W). The results for energies of scissors states are given in Table-10.2. It is seen that the lowest M1 excited scissors states in $^{185}$W occur at an energy $\sim 2.8$ MeV and the next set of

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**TABLE - 10.2**
Predicted Energies and M1 Strengths of Mixed Symmetry States
in the SU(3) $\otimes$ U(2) limit of sd and sdg models for $^{185}$W

<table>
<thead>
<tr>
<th>$(\lambda', \mu')<em>{K'}</em>{L'J'}$</th>
<th>L' J'</th>
<th>E(MeV)</th>
<th>$B(M1; (0N-2, 3)<em>{11J} \rightarrow (\lambda', \mu')</em>{K'_{L'J'}}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$J=3/2$</td>
<td>$J=1/2$</td>
</tr>
<tr>
<td>(0N-4, 4) $^{0^+}$</td>
<td>0</td>
<td>1/2</td>
<td>2.71 0.43 0.84 0.43 0.84</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3/2</td>
<td>2.81 0.02 0.04 0.20 0.40</td>
</tr>
<tr>
<td></td>
<td>5/2</td>
<td></td>
<td>2.82 0.18 0.36 --</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3/2</td>
<td>2.81 0.09 0.19 0.93 1.85</td>
</tr>
<tr>
<td></td>
<td>5/2</td>
<td></td>
<td>2.82 0.84 1.67 --</td>
</tr>
<tr>
<td>(0N-3, 2) $^{0^+}$</td>
<td>1</td>
<td>1/2</td>
<td>3.29 0.11 0.21 0.42 0.83</td>
</tr>
<tr>
<td></td>
<td>3/2</td>
<td></td>
<td>3.30 0.52 1.03 0.21 0.41</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3/2</td>
<td>3.36 0.03 0.06 0.31 0.62</td>
</tr>
<tr>
<td></td>
<td>5/2</td>
<td></td>
<td>3.37 0.28 0.56 --</td>
</tr>
</tbody>
</table>

\text{a} The scissors states energies E are given relative to the energy of the experimental GS $|{(4N-2, 3)_{11J}1}\, 3/2\rangle$.

\text{b} The $B(M1)$ values are in units of $(g_\pi - g_\rho)^2$.

\text{c} $\eta = 2$ for sd and $\eta = 4$ for sdg.

\text{d} $N = 11$ for $^{185}$W.
states appear at about 600 keV higher. With $|g_\pi - g_V| \sim 0.6 \mu_N$ (given in Fig. 2 of [Wo-87]), there are scissors states in $^{185}$W (around $\sim 3$MeV) with sizeable B(M1) strengths to the GS. Fig. 10.2 gives a pictorial representation of the M1-decay pattern.

10.3 Scissors states in gIBM for $^{185}$W

In p-n gIBM, with the single particle states of the odd-particle transforming as the $(40)$ irrep (i.e. $j=1/2,3/2,5/2,7/2$ and 9/2) of SU(3) the group chain for the odd-particle is $U^{F}(30) \supset U^{F}(15) \otimes U^{F}(2) \supset SU^{F}(3) \otimes U^{F}(2) \supset O^{F}(3) \otimes SU^{F}(2) \supset Spin^{F}(3)$ and in this situation two BF coupling schemes, one involving the coupling at the SU(3) level (chain I) and the other at the U(15) level (chain II), are possible for deformed odd-A nuclei. The two chains (I and II) and the corresponding irreps are [Ko-86a],

\[
\begin{align*}
\text{(chain I)} & & U^{B}_{sdg}(30) \otimes U^{F}(30) \supset U^{B}_{sdg}(15) \otimes U^{F}(15) \otimes U^{F}(2) \supset \\
 & & \{f\} \{1\} \{f\} \{1\} \\
 & & SU^{B}_{sdg}(3) \otimes SU^{F}(3) \otimes U^{F}(2) \supset SU^{BF}(3) \otimes U^{F}(2) \supset \\
 & & (\lambda_{BF},\mu_{BF}) \quad (40) \\
 & & O^{BF}(3) \otimes SU^{F}(2) \supset Spin^{BF}(3) \\
 & & K_{L} \quad L = \frac{1}{2} \quad J
\end{align*}
\]

\[
\begin{align*}
\text{(chain II)} & & U^{B}_{sdg}(30) \otimes U^{F}(30) \supset U^{B}_{sdg}(15) \otimes U^{F}(15) \otimes U^{F}(2) \supset \\
 & & \{f\} \{1\} \{f\} \{1\} \\
 & & U^{BF}(15) \otimes U^{F}(2) \supset SU^{BF}(3) \otimes U^{F}(2) \supset \\
 & & \{f_{BF}\} \quad (\lambda_{BF},\mu_{BF}) \\
 & & O^{BF}(3) \otimes SU^{F}(2) \supset Spin^{BF}(3) \\
 & & K_{L} \quad L = \frac{1}{2} \quad J
\end{align*}
\]

(10.8)
Fig. 10.2 M1-strengths from scissors states belonging to the
irreps $(\eta N-4,4)$ and $(\eta N-3,2)$ for $^{185}$W in the SU(3) $\otimes$
U(2) limit of sd and sdgIBM; $\eta = 2$ for sd and 4 for
sdg. The transition strength is proportional to the
width of the corresponding transition shown in the
figure. Note that the strongest strength is from the
state $|\eta N-4,4\rangle K = 2 L = 2 J = 5/2>$ and the
 corresponding value is $0.42 \times \eta (g_{\pi}^2 - g_{\nu}^2)$. The
$(\eta N-4,4)$ states appear at $\sim 2.75$ MeV and $(\eta N-3,2)$
states appear at $\sim 3.32$ MeV.
<table>
<thead>
<tr>
<th>(λ, µ)</th>
<th>K</th>
<th>L</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>(ηN-3, 2)</td>
<td>2</td>
<td>2 5/2</td>
<td>2 3/2</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1/2</td>
<td></td>
</tr>
<tr>
<td>(ηN-4, 4)</td>
<td>2</td>
<td>2 5/2</td>
<td>2 3/2</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1/2</td>
<td></td>
</tr>
</tbody>
</table>

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\(W\)
Note that chain-I above is similar to \((10.1)\). In addition to the complexity that arises because of two chains, the M1 operator in the sdgIBM is a mixture of \((11)\) and \((33)\) SU(3) tensors. Thus the problem of scissors states in the sdg model is both complex and rich. Let us begin with chain-I.

**10.3a CHAIN I: SU(3) \(\otimes\) U(2) LIMIT**

In chain-I the GS of \(^{185}\text{W}\) belongs to \(|N, (4N,0) (40); (4N-2,3) K = 1 L = 1 J = 3/2\rangle\). A simple bose-fermi (BF) force \(V_{\text{BF}}\) involving \(d\) and \(g\) bosons can be constructed such that the one-to-one correspondence that exists between the Nilsson orbits and SU\(^{\text{BF}}\)(3) irreps in sd case will extend to the sdg case with the replacement \(2N \rightarrow 4N\). One such \(V_{\text{BF}}\) is,

\[
V_{\text{BF}} = \Gamma \sum_{\ell \ell'}^{N_0} S_{\ell \ell'}^2 Q_{\text{sdg}}^2 \left( a_\ell^\dagger \tilde{a}_{\ell'} a_{\ell'} \right)^2 + \Lambda \sum_{\ell \ell'} \left\{ \sum_{\ell''} S_{\ell \ell''}^0 S_{\ell' \ell''}^{N_0} \frac{1}{\sqrt{2\ell''+1}} \left[ \left( a_\ell^\dagger d \right) \ell'' \left( d^\dagger a_{\ell'} \right) \ell' \right] : \right\}.
\]

\[
S_{\ell \ell'}^0 = \sqrt{\frac{2\pi}{5}} <N_0 \ell | \hat{r}^2 \hat{Y}^2(\theta, \phi) | N_0 \ell'>
\]

In \((10.9)\) \(Q_{\text{sdg}}^2\) is \(Q^2(s)\) defined in \((2.12)\), \(\cdot\cdot\cdot\) denotes normal ordering, \(S_{\ell \ell'}^0\) are the \(\hat{r}^2 \hat{Y}^2(\theta, \phi)\) matrix elements in the oscillator basis and the oscillator shell number \(N_0 = 4\) for us. Eq. \((10.9)\) follows from eq \((11.1)\) of \([\text{Bi-88}]\) with \(Q_{sd}^2\) (see Table-2.1) replaced by \(Q_{\text{sdg}}^2\). Note that in \((10.9)\) the first term is the quadrupole-quadrupole force and the second term is the exchange force. The correspondence between the Nilsson and SU\(^{\text{BF}}\)(3) descriptions is possible because
for any \( \eta \) (\( \eta = 2 \) for sd and \( \eta = 4 \) for sdg); see Fig. 10.1. The scissors states belong to \([N-1,1])(4N-2,1)\$ \{ \lambda_{\nu\mu} \}$ and the \( \lambda_{\nu\mu} \) irreps follow from Table-10.1. The SU(3) character of the M1 excited scissors states can be found by considering the general M1-operator in the sdg model. Ignoring the particle part (as in (10.3)), the general form of the M1 operator in \( \pi v \) space is,

\[
T^{M1} = \sum_{\rho = \pi, v} \sqrt{10} \left\{ g_{2,\rho}(d_{\rho}^{-\rho}) + \sqrt{6} g_{4,\rho}(g_{\rho}^{+\rho}) \right\}
\]

which has four g-factors unlike only two in sd case. With \( g_{2,\rho} = g_{4,\rho} \), the M1 - operator in (10.11) reduces to \( T^{M1} = g_{\pi}L_{\pi} + g_{v}L_{v} \) and it behaves as \( (\lambda_{\rho\mu}^{+}) = (11) \) SU(3) tensor. In this situation all the results (energies and B(M1)'s) can be derived using (10.5) and (10.7) and we will return to this a little later. In the case \( g_{2,\rho} \neq g_{4,\rho} \), in addition to the \( (\lambda_{\rho\mu}^{+}) = (11) \) tensor, there will be a piece that behaves as \( (\lambda_{\rho\mu}^{+}) = (33) \) irrep. Using \( (b_{\rho}^{P}p_{\rho}) = \sum_{\ell <(40)\ell(04)\ell} (\lambda_{\rho\rho}^{+}) \times T^{21} \), it can be shown that

\[
T^{M1} = \sum_{\rho = \pi, v} \left\{ A_{1,\rho} T^{21}^{(11)1} + A_{3,\rho} T^{21}^{(33)1} \right\};
\]

\[
A_{1,\rho} = \sqrt{10/7} (g_{2,\rho} + 6 g_{4,\rho}), \quad A_{3,\rho} = \sqrt{60/7} (g_{4,\rho} - g_{2,\rho})
\]

\[
T_{\rho}^{21}^{(11)1} = \sqrt{1/7} (d_{\rho}^{+\rho}) + \sqrt{6/7} (g_{\rho}^{+\rho})
\]

\[
T_{\rho}^{21}^{(33)1} = - \sqrt{6/7} (d_{\rho}^{+\rho}) + \sqrt{1/7} (g_{\rho}^{+\rho})
\]

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A tensorial decomposition that is somewhat different from (10.12) was given in [Wu-87] and [Li-86].

The scissors states \((\lambda^s_\mu^s_\nu)\) that can be excited by the \((\lambda^l_\mu^l_\nu)\) = (11) tensor piece are given by (10.4) with \(2N \rightarrow 4N\); as already stated the \(B(M1)'s\) in this case can be calculated using (10.5), (10.6) with the replacement \(2N \rightarrow 4N\), \(2N_\pi \rightarrow 4N_\pi\) and \(2N_\nu \rightarrow 4N_\nu\). This leads to an enhancement factor \(~2\) in the sdg case (for even-even nuclei a similar result is derived by Barrett and Halse [Ba-85b] using coherent state formalism). Similarly the scissors states energies can be calculated using (10.7) by changing \(2N \rightarrow 4N\), \(\Gamma \rightarrow \Gamma/2\) and \(A \rightarrow A/2\), then the energies will not be effected by going from sd to sdg case. The results for energies and \(B(M1)'s\) are given in Table-10.2. It is important to note that \((g^R_\mu - g^R_\nu)^2\) will remain essentially same as we go from sd to sdg case as the magnetic moments (which determine \(g_\mu\) and \(g_\nu\)) of \(2^+_\ell\) states will be same in sd and sdg models \((\Gamma^{M1} = g^R_\mu L\) for \(F = N/2\); \(g^R_\mu = (g_\mu^R N_\pi + g_\nu^R N_\nu)/N\)). In the presence of the \((\lambda^l_\mu^l_\nu) = (33)\) tensor there will be many more scissors states and \(A_{3,0}'s\) contribution to the \(B(M1)'s\) should be included. Taking the intersection of the SU(3) irreps in the products \((4N-2,3) \otimes (11)\) and \((4N-2,3) \otimes (33)\) with \((4N-2,1) \otimes (40)\), using Table-10.1, the (11) and (33) excited scissors states \((\lambda^s_\mu^s_\nu)\) are obtained,

\[
(\lambda^s_\mu^s_\nu) = (4N+2,1) \oplus (4N,2) \oplus (4N+1,0) \oplus (4N-2,3) \oplus (4N-1,1) \oplus (4N-4,4) \oplus (4N-3,2) \oplus (4N-6,5) \oplus (4N-5,3)
\]

(10.13)

In (10.13) the underlined irreps can be excited by both (11) and (33) tensor parts while the remaining ones can be excited only by
the (33) tensor part. The general expression for the reduced matrix elements of the M1 operator (10.11),(10.12) in sdg model is derived using (6.23c)

\[
\langle (\text{N}_L\text{N}_L-1,\text{L})(4N-2,1)\otimes(40); (\text{N}_L\text{N}_L-1,0)\rangle \sum_{\rho,\lambda_0} A_{\lambda_00} \times
\]

\[
T_p^{(21)^{13}}(\lambda_0\lambda_0)_{11} \langle (\text{N}_L)\otimes(\text{N}_L); (\text{N})(4N,0)\otimes(40); (\lambda_{\text{gs}}\mu_{\text{gs}})\rangle_{\text{KLJ}} =
\]

\[
\left[ (-1)^{L'+3/2} \sqrt{(2J+1)(2J'+1)} \begin{pmatrix} L' & J' & 1/2 \\ J & L & 1 \end{pmatrix} \right] \times \sum_{R,R',\ell} (-1)^{R'+L'+1} \times
\]

\[
\sqrt{(2L+1)(2L'+1)} \begin{pmatrix} R' & L' & \ell \\ L & R & 1 \end{pmatrix} \langle (4N-2,1)R' (40)\ell || (\lambda_0\lambda_0)\rangle_{\text{KLJ}} \times
\]

\[
\langle (4N,0)R (40)\ell || (\lambda_0\lambda_0)\rangle_{\text{KLJ}} \left[ \sum_{\lambda_0=1,3} \langle (4N,0)R (40)\ell || (4N-2,1)R' \rangle \right]
\]

\[
\times \sqrt{2R'+1} \times \left\{ A_{\lambda_00} \frac{N_\pi^2 D(4N,0) D(4N,0) D(4N,0)}{D(4N-4,0) D(4N,0) D(\lambda_0\lambda_0)} \times
\]

\[
U((4N,0)(0,4N,0):(4N-2,1)(4N,0);(4N,0)(\lambda_0\lambda_0)) \times
\]

\[
U((4N,0)(04)(4N,0)(40);(4N-4,0)(\lambda_0\lambda_0)) +
\]

\[
\left\{ A_{\lambda_00} \frac{N_\pi^2 D(4N,0) D(4N,0) D(4N,0)}{D(4N-4,0) D(4N,0) D(\lambda_0\lambda_0)} \times
\]

\[
U((4N,0)(0,4N,0):(4N-2,1)(4N,0);(4N,0)(\lambda_0\lambda_0)) \times
\]

\[
U((4N,0)(04)(4N,0)(40);(4N-4,0)(\lambda_0\lambda_0)) \right\} .
\]

(10.14)

In (10.14) U(---) are SU(3) U-coefficients and D(\lambda,\mu) = ((\lambda+1)(\mu+1)(\lambda+\mu+2))/2. The SU(3) recoupling coefficients appearing in (10.14) can be easily obtained using Akiyama and Draayer
Ak-73 codes. For $^{185}$W, as mentioned earlier, $(\lambda_{\text{GS}} | \mu_{\text{GS}}) = (4N-2,3)$ and the scissors SU(3) irreps $(\lambda_{\gamma} | \mu_{\gamma})$ are given in (10.13). In the special case when $A_{3\gamma} = 0$, (10.14) produces (10.5). Although it is not apparent from (10.14), it is seen that the expression for the M1 reduced matrix elements will be of the form

$$((A_{1\pi} - A_{1\nu})X^{(11)} + (A_{3\pi} - A_{3\nu})X^{(33)}) = \mathcal{A}_1X^{(11)} + \mathcal{A}_3X^{(33)}.$$  

(10.15)

and the values of $X^{(11)}$ and $X^{(33)}$ for $^{185}$W are given in Table-10.3 for all $(\lambda_{\gamma} | \mu_{\gamma})$ listed in (10.13). From Table-10.3 it is seen that for $(\lambda_{\gamma} | \mu_{\gamma}) = (4N-4,4)$ and $(4N-3,2)$, $|X^{(11)}|$ and $|X^{(33)}| \sim 0.4$, for $(4N-1,1)$ they are $\sim 0.02$ and for all the remaining irreps (see (10.13)) they are $\sim 0.001$. With $|(A_{1\pi} - A_{1\nu})| \sim 1$, only the $(4N-4,4)$ and $(4N-3,2)$ irreps are M1 excited with appreciable strength. Thus the $(\lambda_{\gamma} | \mu_{\gamma}) = (33)$ tensor cannot produce any further fragmentation. In Table-10.4 the results for $X^{(11)}$ and $X^{(33)}$ for the irreps $(4N-4,4)$ and $(4N-3,2)$ are collected and the resulting B(M1) strengths are

$$B(\text{M1}; J \rightarrow J') = \frac{3}{4\pi} (2J+1)^{-1} \left| \mathcal{A}_1X^{(11)} + \mathcal{A}_3X^{(33)} \right|^2.$$  

(10.16)

It is seen from Tables-10.3, 10.4 that the values of $|X^{(11)}|$ and $|X^{(33)}|$ are comparable and the sign of $X^{(11)}$ is always opposite to that of $X^{(33)}$. The values of $\mathcal{A}_1$ and $\mathcal{A}_3$ (they together with the results in Table-10.4 produce B(M1)'s) can be determined using the M1 data (magnetic moments of GS and $\gamma$ band members) of neighbouring even-even nuclei. The $g_\pi-g_\nu \approx 0.6 \mu_N$ for W isotopes as
<table>
<thead>
<tr>
<th>$\langle \lambda', \mu' \rangle L' J'$</th>
<th>$\langle \lambda, \mu \rangle L J \rangle T \rangle \langle (4N-2,3) 11 J^a \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J=1/2$</td>
<td>$J=3/2$</td>
</tr>
<tr>
<td></td>
<td>$(11)^b$</td>
</tr>
<tr>
<td></td>
<td>$(33)^c$</td>
</tr>
<tr>
<td>$(4N+2,1)^d$</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>1</td>
<td>3/2</td>
</tr>
<tr>
<td>2</td>
<td>3/2</td>
</tr>
<tr>
<td>2</td>
<td>5/2</td>
</tr>
<tr>
<td>$(4N,2)0$</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1/2</td>
</tr>
<tr>
<td>2</td>
<td>3/2</td>
</tr>
<tr>
<td>2</td>
<td>5/2</td>
</tr>
<tr>
<td>$(4N+1,0)^0$</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1/2</td>
</tr>
<tr>
<td>3</td>
<td>3/2</td>
</tr>
<tr>
<td>$(4N-2,3)^1$</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1/2</td>
</tr>
<tr>
<td>2</td>
<td>3/2</td>
</tr>
<tr>
<td>2</td>
<td>5/2</td>
</tr>
</tbody>
</table>

Reduced matrix elements of the M1 operator for mixed symmetry states in sdgIBM for $^{185}\text{W}$.
| $(\lambda', \mu') K'_L$ | $L'$ | $J'$ | $\langle (\lambda', \mu') K'_L | J' \rangle$ | $(\lambda_0, \mu_0)^{11}$ | $(4N-2, 3) \langle J | J \rangle$ |
|----------------|-----|-----|------------------|------------------|------------------|
| $(4N-1, 1) 1$ | 1   | 1/2 | 16.724           | -16.569          | -11.826          | 11.716           |
|               | 2   | 3/2 | 20.042           | -17.378          | -8.963           | 7.772            |
|               | 2   | 5/2 |               |                 | 26.890           | -23.315          |
| $(4N-4, 4) 0$ | 0   | 1/2 | 317.547          | -289.179         | 449.080          | -408.960         |
|               | 2   | 3/2 | 217.962          | -198.358         | -97.476          | 88.708           |
|               | 5/2 |     |                 | -292.427         | 192.378          |
| $(4N-3, 2) 0$ | 1   | 1/2 | 314.426          | -291.113         | -222.332         | 205.848          |
|               | 3/2 |     |                 | 497.150          | -460.290         |
|               | 2   | 3/2 | -272.301         | 253.754          | 121.777          | -113.482         |
|               | 5/2 |     |                 | -365.330         | 340.446          |
| $(4N-6, 5) 1$ | 1   | 1/2 |               | 1.025            | -0.725           |
|               | 3/2 |     |                 | 0.725            | 1.621            |
|               | 2   | 3/2 |               | -1.224           | 0.547            |
|               | 5/2 |     |                 | -1.642           |
| $(4N-5, 3) 1$ | 1   | 1/2 |               | 1.256            | -0.888           |
|               | 3/2 |     |                 | 0.888            | 1.985            |
|               | 2   | 3/2 |               | -1.503           | 0.672            |
|               | 5/2 |     |                 | -2.017           |
Foot Note to Table-10.3

aThe Ml matrix elements are in units of $\overline{A}_1 \times 10^{-3}$ for the (11) tensor and $\overline{A}_3 \times 10^{-3}$ for the (33) tensor. See (10.15) for the definition of $\overline{A}_1$ and $\overline{A}_3$.

bThe entries for $(\lambda_0^\mu_0^\eta_0)$ = (11) correspond to $X^{(11)} \times 10^3$; see (10.15) for the definition of $X^{(11)}$.

cThe entries for $(\lambda_0^\mu_0^\eta_0)$ = (33) correspond to $X^{(33)} \times 10^3$; see (10.15) for the definition of $X^{(33)}$.

d$N = 11$ for $^{185}$W.

Note that $B(M1; J \rightarrow J') = (2J + 1)^{-1} \left\{ \overline{A}_1 X^{(11)} + \overline{A}_3 X^{(33)} \right\}^2$. 

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Table 10.4

M1 reduced matrix elements for Scissors states in sdgIBM for $^{185}$W

| $(\lambda', \mu') K' L' J' E$ (MeV) | $\langle (\lambda', \mu') K'_L J' | T | (4N-2, 3)_{11J} \rangle$ |
|---------------------------------|---------------------------------|
|                                 | $J=3/2$                         | $J=1/2$                         |
| $(4N-4, 1)_{11} 0$ 1/2 2.71     | 449.080                         | -408.960                        |
| 2 3/2 2.81 97.476               | 88.708                          | 217.962                         |
| 5/2 2.82 292.427               | -266.125                        | -198.358                        |
| 2 3/2 2.81 -210.575            | 192.378                         | 470.859                         |
| 5/2 2.82 631.724               | -577.133                        | -430.170                        |
| $(4N-3, 2)_{11} 1$ 1/2 3.29     | -222.332                        | 205.848                         |
| 3/2 3.30 497.150               | -460.290                        | 222.332                         |
| 2 3/2 3.36 121.777             | -113.482                        | -272.301                        |
| 5/2 3.37 -365.330              | 340.446                         | -                        |

*The scissors states energies E are given relative to the energy of the GS \( (4N-2, 3)_{11} 3/2 \).

The M1 matrix elements are in units of $\bar{A}_1 \times 10^{-3}$ for the (11) tensor and $\bar{A}_3 \times 10^{-3}$ for the (33) tensor. See (10.15) for the definition of $\bar{A}_1$ and $\bar{A}_3$.

The entries for $(\lambda', \mu') = (11)$ correspond to $X^{(11)} \times 10^3$; see (10.15) for the definition of $X^{(11)}$.

The entries for $(\lambda', \mu') = (33)$ correspond to $X^{(33)} \times 10^3$; see (10.15) for the definition of $X^{(33)}$.

$N = 11$ for $^{185}$W.

Note that $B(M1; J \rightarrow J') = \frac{3}{4\hbar} (2J + 1)^{-1} \left( \frac{\bar{A}_1}{X^{(11)} } + \frac{\bar{A}_3}{X^{(33)} } \right)^2$. 

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determined by Wolf et al. [Wo-87] give $\alpha_1 = 5 \mu_N$. The g-factors for

the GS members and $2^+$ level in $^{186}_{\text{W}}$ measured by Stuchberry et

al. [St-85] together with eq (11b) of [Wu-87] determine $4h_\rho + 7h_\nu = -8.4 \mu_N$ where $h_\rho = -\sqrt{7/60} A_{sp}$ and $g_\rho = (N_k g_\rho + N_\nu g_\nu)/N$ as defined
earlier; $\rho = \pi$ or $\nu$. The g-factors for $^{156}_{\text{Gd}}$ given in [Wu-87], the
data for $^{168}_{\text{Er}}$ given in [Yo-88] and for Sm isotopes given in Sect.
8.2c determine $h_\pi$ to be $-0.1$, 0.4 and $-0.2 \mu_N$ respectively. Thus,
typically $-0.4 \mu_N \leq h_\pi \leq 0.4 \mu_N$ and this together with the above expression for $4h_\rho + 7h_\nu$ determine $-5 \mu_N \leq \alpha_3 \leq -2 \mu_N$. This leads
to the important result that, for $^{185}_{\text{W}}$, the $(33)$ tensor part produces an enhancement (as $\alpha_1$ and $\alpha_3$ are of opposite sign) by a
factor 2-4 in B(M1)'s compared to the results given in Table-10.2
(more data on g-factors for $\gamma$-band members in W isotopes will
unambiguously determine $\alpha_3$ and hence the enhancement factor).

10.3b CHAIN II: $U(15) \otimes U(2)$ LIMIT

In chain II, the GS of $^{185}_{\text{W}}$ belongs to the $U^{BF}(15)$ irreps $[N]_B \otimes [1]_F \rightarrow [f]_{BF} = [N+1] \oplus [N,1]$ and the scissors states belong to $[N-1,1]_B \otimes [1]_F \rightarrow [f]_{BF} = [N,1] \oplus [N-1,2] \oplus [N-1,1,1]$. The
$SU^{BF}(3)$ irreps contained in the above $U^{BF}(15)$ irreps are given in
Table-10.5 for any $N$ and they follow from $[N]_{U(15)} \rightarrow (\Lambda \mu)_{SU(3)}$
reductions given in Kota et al. [Ko-87a; see also Ko-86b and
Appendix-H] and the following elementary results,

$$
[N+1] \otimes [1] = [N+2] \oplus [N+1,1] \\
[N] \otimes [2] = [N+2] \oplus [N+1,1] \oplus [N,2] \\
[N,1] \otimes [1] = [N+1,1] \oplus [N,2] \oplus [N,1,1]
$$

(10.18)

Note that $[1]_{U(15)} \rightarrow (40)_{SU(3)}$ and $[2]_{U(15)} \rightarrow [(80) \oplus (42) \oplus (249)$. 


<table>
<thead>
<tr>
<th>U(15) irrep</th>
<th>SU(3) irreps</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N+1)</td>
<td>(4N+4,0) Φ (4N,2) Φ (4N-2,3) Φ (4N-4,4)² Φ</td>
</tr>
<tr>
<td></td>
<td>(4N-2,0) Φ (4N-6,5) Φ (4N-5,3) Φ (4N-4,1) Φ</td>
</tr>
<tr>
<td></td>
<td>(4N-8,6)³ Φ (4N-7,4) Φ (4N-6,2)³ Φ -------</td>
</tr>
<tr>
<td>(N,1)</td>
<td>(4N+2,1) Φ (4N,2) Φ (4N-2,3)² Φ (4N-1,1) Φ</td>
</tr>
<tr>
<td></td>
<td>(4N-6,4)³ Φ (4N-3,2)² Φ (4N-2,0) Φ (4N-6,5)⁴ Φ</td>
</tr>
<tr>
<td></td>
<td>(4N-5,3)⁴ Φ (4N-4,1)³ Φ (4N-8,6)⁵ Φ (4N-7,4)⁶ Φ</td>
</tr>
<tr>
<td></td>
<td>(4N-6,2)⁷ Φ (4N-5,0)² Φ -----------</td>
</tr>
<tr>
<td>(N-1,2)</td>
<td>(4N,2) Φ (4N-2,3) Φ (4N-1,1) Φ (4N-4,4)³ Φ</td>
</tr>
<tr>
<td></td>
<td>(4N-3,2)² Φ (4N-2,0)² Φ (4N-6,5)⁴ Φ (4N-5,3)⁵ Φ</td>
</tr>
<tr>
<td></td>
<td>(4N-4,1)⁴ Φ (4N-8,6)⁷ Φ (4N-7,4)⁸ Φ</td>
</tr>
<tr>
<td></td>
<td>(4N-6,2)¹¹ Φ (4N-5,0)² Φ -----------</td>
</tr>
<tr>
<td>(N-1,1,1)</td>
<td>(4N+1,0) Φ (4N-2,3) Φ (4N-1,1) Φ (4N-4,4) Φ</td>
</tr>
<tr>
<td></td>
<td>(4N-3,2)³ Φ (4N-6,5)³ Φ (4N-5,3)⁵ Φ (4N-4,1)³ Φ</td>
</tr>
<tr>
<td></td>
<td>(4N-8,6)³ Φ (4N-7,4)⁹ Φ (4N-6,2)⁶ Φ</td>
</tr>
<tr>
<td></td>
<td>(4N-5,0)⁵ Φ -----------</td>
</tr>
</tbody>
</table>

All SU(3) irreps with \( c \geq 8N-10 \) are given; \( c = 2\lambda + \mu \). In \((\lambda, \mu)^r\), \( r \) denotes the number of occurrences (inner multiplicity) of \((\lambda, \mu)\) in the given U(15) irrep \( f \).

⁴All the results are valid for \( N \geq 5 \).
In the U(15) \( \otimes \) U(2) limit first the question of fixing the SU(3) structure of the GS of \( ^{186}W \) should be addressed. The choice \( H = \{ \alpha C_2(U_{BF}(15)) + \beta C_2(SU_{BF}(3)) + \gamma L.L + \gamma J.J \} \) with \( \beta < 0 \) and \( \alpha > 0 \) brings down the SU(3) irrep \((4N+2,1)\) of \( \{N,1\} \) to be lowest in energy and it has \( K_L = 1 \) and \( L = 1 \) as required; \( C_2(G) \) is the quadratic Casimir operator of the group G. Thus in this case (with \( C_2(U_{BF}(15)) \) which acts as an exchange force \([Sc-84]\)) the GS is given by

\[
\text{GS: } |\{N\}_B\{1\}_p; \{N,1\}(4N+2,1)K_L = 1 L = 1 J = 3/2> \tag{10.19}
\]

and this choice was adopted in \([Ko-86a]\). However if the correspondence between the Nilsson orbits and the \( SU_{BF}(3) \) irreps in chain-I (with core \( (\lambda^n_{BF},\mu_B) = (4N,0) \)) has to be maintained (as \( N \rightarrow \infty \)), then the GS \( SU_{BF}(3) \) irrep should be \((4N-2,3)\) as in chain-I. In chain-II \((4N-2,3)\) irrep belongs to \( \{N+1\} \) and \( \{N,1\} \); see Table-10.5. For the above correspondence to hold, the core \( SU(3) \) irrep \((\lambda^n_{BF},\mu_B)\) should be \((4N,0)\). The composition of the core irreps in the \( SU_{BF}(3) \) irrep \((4N-2,3)\) of \( U_{BF}(15) \) is given by,

\[
|\{f_{BF}\} \alpha (\lambda^n_{BF},\mu_{BF})K_LLJ> = \sum <\{N\}|(\lambda^n_{BF},\mu_{BF})\{1\}(40)|\{f_{BF}\} \alpha (\lambda^n_{BF},\mu_{BF})> (\lambda^n_{BF},\mu_{BF}) \times |(\lambda^n_{BF},\mu_{BF})(40); (\lambda^n_{BF},\mu_{BF})K_LLJ> \tag{10.20}
\]

The U(15) \( \rightarrow \) SU(3) reduced Wigner coefficients < \( || \) > in (10.20) for \( \{f_{BF}\} = \{N+1\} \) are related to the triple-bar matrix elements < \( |||| \) > defined in Chapter-IV and
where $< | | | | >$ is given in the last column of Table-4.2 (see also Table-4.1). They show that the intensity of $(4N,0) \otimes (40)$ in $(N+1)(4N-2,3)$ goes to zero as $N \rightarrow \infty$ (and that of $(4N-6,3) \otimes (40)$ goes to one). Thus the GS must belong to $(N,1)$ irrep of $\mathbb{U}^\text{BF}(15)$. However in $(N,1)$ irrep, the $SU^\text{BF}(3)$ irrep $(4N-2,3)$ appears twice (see Table-10.5) which we label as $\alpha = 0,1$. Thus the GS, with Nilsson vs $SU^\text{BF}(3)$ correspondence, takes the form

$$\text{GS} : |\{N\}_B^1; (N,1) \alpha = 0 \ (4N-2,3) K_L = 1 \ L = 1 \ J = 3/2>$$

(10.21)

The value of $\alpha$ in (10.21) is defined such that $<\{N\}(4N,0) \otimes (1) (40) | (N+1) \alpha (\lambda^\text{BF}_\alpha \mu^\text{BF}_\alpha) > = (N+1)^{-1} \times$

$$< (N+1) \alpha (\lambda^\text{BF}_\alpha \mu^\text{BF}_\alpha) | b^{\dagger}_{\{1\}(40)} | (N) \ (\lambda^\text{BF}_\alpha \mu^\text{BF}) >$$

The M1 excited scissors states have $\mathbb{U}^\text{BF}(15)$ character defined by $(N,1) \otimes \{211^3\} \rightarrow \{f\}^\text{BF}$ and $\{f\}$ takes all the allowed values; $\{f\}^\text{BF} = (N,1) \oplus (N-1,2) \oplus (N-1,1,1)$. Thus the $SU^\text{BF}(3)$ character of the scissors states is given by the intersection of the $SU(3)$ irreps given in Table-10.5 for $(N,1), (N-1,2)$ and $(N-1,1,1)$ with the $SU(3)$ irreps in the product $(\lambda^\text{GS}_\mu^\text{GS}) \otimes \{(11)\otimes(33)\}$
\( (\lambda_{\text{GS}}\mu_{\text{GS}}) = (4N+2,1) \) if the choice (10.19), or \((4N-2,3)\) if the choice (10.21), is adopted for the GS. Using the results of Tables-10.1 and 10.5, the scissors states with the choice (10.19) for the GS are

\[
\begin{align*}
\{N-1,1\} : (\lambda\mu) = (4N+1,0) \oplus (4N-2,3) \oplus (4N-1,1) \oplus (4N-4,4) \oplus (4N-3,2)^3 \\
\{N-1,2\} : (\lambda\mu) = (4N,2) \oplus (4N-2,3) \oplus (4N-1,1) \oplus (4N-4,4) \oplus (4N-3,2)^2 \\
\{N-1,1,1\} : (\lambda\mu) = (4N+1,0) \oplus (4N-2,3) \oplus (4N-1,1) \oplus (4N-4,4) \oplus (4N-3,2)^3
\end{align*}
\]

(10.22)

In (10.22) the underlined irreps can be excited by both (11) and (33) tensor parts of the M1 operator while the remaining ones are excited only by the (33) part; note that some of the irreps \((\lambda\mu)\) occur more than once. The scissors states that are excited by the (11) tensor part alone with the choice (10.21) for the GS are,

\[
\begin{align*}
\{N-1,1\} : (\lambda\mu) = (4N+1,0) \oplus (4N-2,3) \oplus (4N-1,1) \oplus (4N-4,4) \oplus (4N-3,2)^3 \\
\{N-1,2\} : (\lambda\mu) = (4N,2) \oplus (4N-2,3) \oplus (4N-1,1) \oplus (4N-4,4) \oplus (4N-3,2)^2 \\
\{N-1,1,1\} : (\lambda\mu) = (4N-2,3) \oplus (4N-1,1) \oplus (4N-4,4) \oplus (4N-3,2)^3
\end{align*}
\]

(10.23)

while all the SU(3) irreps that are listed in Table-10.5 for
\( \{N,1\}, \{N-1,2\} \) and \( \{N-1,1,1\} \) can be excited by the (33) tensor. Comparing (10.13) with (10.22), (10.23) and Table-10.5 it is seen that there will be more fragmentation in \( U(15) \otimes U(2) \) limit compared to the \( SU(3) \otimes U(2) \) limit. Note that the (4N-4,4) and (4N-3,2) irreps that are strongly excited in the \( SU(3) \otimes U(2) \) limit (see Tables-10.2, 10.4) appear in all the three scissors \( U(15) \) irreps (see (10.22), (10.23) and Table-10.5). Expressions for B(M1)'s involve \( U(15) \rightarrow SU(3) \) reduced Wigner coefficients \( \langle f | \lambda_{f} \mu_{f} | 1 \rangle (40) \rightarrow \langle f | \lambda_{f} \mu_{f} | f' \rangle \). As the core structure (both for GS and scissor states is complicated (see (10.20)) in the \( U(15) \otimes U(2) \) limit, without adequate knowledge of \( U(15 \rightarrow SU(3) \) Wigner coefficients it is not possible to produce numerical numbers or analytical expressions and this calls for a large scale extension of the results given in [Wu-86]; the group theory given in [Ch-89b] may be useful here. Thus in the \( U(15) \otimes U(2) \) limit there are two important unsolved problems: (i) construction of a hamiltonian (say from some microscopic input) so that a choice between (10.19) and (10.21) can be made for the GS. (ii) \( U(15) \rightarrow SU(3) \) reduced Wigner coefficients for \( | f \rangle \otimes | 1 \rangle \rightarrow | f' \rangle \) where \( | f \rangle = | N \rangle \) or \( | N-1,1 \rangle \) and \( | f' \rangle = | N,1 \rangle \), \( | N-1,2 \rangle \) or \( | N-1,1,1 \rangle \). In the absence of any experimental data, these are not dealt with in the thesis.

10.4 Summary

In this Chapter M1 excited scissors states in \( ^{185}W \) are discussed in detail in the sd and sdg models employing the \( SU(3) \otimes U(2) \) limit of the IBFM. Explicit results for energies and B(M1)'s are given. It is shown that, as in the case of \( ^{169}Tm \) [Va-89a] that there are scissors states with sizeable M1 strength concentrated
in two SU(3) irreps \((\eta N-4,4)\) and \((\eta N-3,2)\) which appear around 3 MeV excitation; \(\eta = 2\) for sd and \(\eta = 4\) for sdg. The role of g-bosons depend on the factors \(\mathcal{A}_1\) and \(\mathcal{A}_3\) defined in Sect. 10.3 and their determination from magnetic moments data is clearly called for. With the inclusion of g bosons, besides the SU(3) \(\otimes\) U(2) limit, a new coupling scheme U(15) \(\otimes\) U(2) is possible. The structures involved in the U(15) \(\otimes\) U(2) limit of gIBM are made clear and the problem of classification of scissors states in this limit, for \(^{185}\text{W}\), is solved; the predictions for energies and B(M1)'s are not worked out. To complete this exercise, the mathematical problem of deriving general U(15) \(\otimes\) SU(3) reduced Wigner coefficients need to be addressed.

It should be remarked that \(^{185}\text{W}\) is unstable (\(T_{1/2} = 75.1\) d) and therefore the M1-states that are predicted in this Chapter may hardly be populated by conventional methods \((e,e'), (p,p')\) and \((\gamma,\gamma')\). Therefore they should be investigated via particle-transfer experiments or by some other novel technique. However, the formalism given for \(^{185}\text{W}\) in this Chapter extends easily to all deformed even-odd rare-earth nuclei with valence neutrons in 82-126 shell. As reported in [Wa-90] and also seen from Fig. 10.1, the GS of the nuclei \({^{187,189}\text{Os}}\), \({^{179,177}\text{W, Hf}}\), \({^{172}\text{Yb}}\) and \({^{161,163}\text{Er, 159Dy, 159Gd}}\) belong to \((\eta N-2,3)\) \(K_L = 1\), \((\eta N-2,3)\) \(K_L = 3\) and \((\eta N,2)\) \(K_L = 2\) respectively. Especially for \(^{187}\text{Os}\) nucleus the results derived for \(^{185}\text{W}\) apply directly as both of them have same number of neutrons (\(N_Y = 7, N = 10\)) and the odd neutron occupies the same pseudo-spin doublet giving rise to nearly degenerate \(K = 1/2^-, 3/2^+\) bands; goodness of pseudo-spin for \(^{187}\text{Os}\) is established recently [Wa-90]. The GS of \(^{187}\text{Os}\) is \(|(\eta N-2,3)1 1/2>\) where \(\eta = 2\)
for sd and \( \eta_1 = 4 \) for sdg. Here the results for M1 strengths given in Tables-10.2, 10.3, 10.4 for \( J = 1/2 \) apply (the change from \( N = 11 \) of \(^{185}\text{W} \) to \( N = 10 \) of \(^{187}\text{Os} \) produces negligible changes in M1 values) and the energies of the scissors states do not alter (they should be increased by a negligible 5 keV). Similarly the results extend to \(^{189}\text{Os} \) whose GS is \( |\eta N-2,3\rangle \) 11 3/2\rangle with \( N = 9 \).

The single-particle M1-operator \( T^\dagger (M1) = \sqrt{3/4\pi} \left( g_e \hat{J} + g_s \hat{\delta} \right) \) when mapped into the pseudo \( \ell-s \) space takes the simple form

\[
T^\dagger (M1) = \sqrt{3/4\pi} \left[ \left( g_e \hat{J} - \frac{1}{3} \left( g_s - E_\ell \right) \right) \right. \\
+ \left. \left( g_s - E_\ell \right) \right] \left\{ \sum_{\ell} \frac{2}{3} \frac{(\ell+1)(2\ell+1)}{(2\ell-1)(2\ell+3)} \left( a_\ell^+ \tilde{a}_\ell - a_{\ell+1} \tilde{a}_{\ell+1} \right) \right\} \\
- \sum_{\ell=\ell_{\min}}^{\ell_{\max}} \frac{(\ell+1)(\ell+2)}{6(2\ell+3)} \left[ a_\ell^+ \tilde{a}_\ell + a_{\ell+1} \tilde{a}_{\ell+1} \right] \\
(10.24)
\]

Numerical results for the factor multiplying \( T^{211} \) in (10.24) are given in [Ra-73] for \( \eta_1 = 3,4 \); \( \ell = \eta_1, \eta_1-2, \cdots, 0 \) or 1. Using (10.24) the single particle contribution to the M1-strength can be calculated and it will be non-zero only when there is F-spin mixing (\( \{N\} \) and \( \{N-1\} \) admixture). With the known result that F-spin mixing is usually small \( \sim 5\% \) [Ha-87a, Le-90] the single-particle contributions can be ignored. However, they might become significant when there is coupling to non-collective states. Finally, as Raduta and Delion [Ra-90] state "Unfortunately there is no data available yet for the even-odd nuclei concerning the M1 states. However we should not forget that for the even-even
systems the theory of the M1 states preceded the measurements by several years .......". It is our hope that the present exercise together with the ones in [Ra-89, Ra-90, Va-89a, Fr-91] will stimulate future experiments looking for scissors states in odd-A nuclei.