SPECTROSCOPY OF SAMARIUM ISOTOPE

8.1 Introduction

Theoretical understanding (using models or microscopic theories) of experimental data on E4 matrix elements [Ic-87, Go-86, Go-87, To-85b, Se-90] and strength distributions [Wu-88, De-89a, Fu-89] is a challenging problem. The sdgIBM with hexadecuple degree of freedom (g-boson) is well suited for this purpose. With the development of the package SDGIBM1 described in Chapter-VI which allows one to construct and diagonalize gIBM matrices in spherical (n, l, m) basis and produces occupancies, two-nucleon transfer strengths and E2, M1 and E4 matrix elements, it is feasible to explore and apply this model in detail. This coupled with the fact that, as shown in Chapter-VII, a simple hamiltonian based on gIBM symmetries (with 6-8 free parameters) appears to reproduce the experimental data, makes the sdg model a powerful tool for analyzing E4 properties in nuclei. It is our purpose here to demonstrate the same by performing a systematic study of a series of isotopes and we choose Sm isotopes as they are sufficiently complex.

Recently Otsuka and Sugita (OS) analyzed [Ot-88] the energies and E2 properties of 0°, 2° and 4° levels of Sm isotopes in sdg framework employing variational (with projection) method. They demonstrated that the observed spherical-deformed shape phase transition (with respect to neutron number) in these isotopes is
due to boson number $N$. In the sdIBM calculations of Scholten et al [Sc-78] boson number dependent d-boson energy had to be used. However it is seen that the variational calculations do not reproduce [Ot-90] the $\beta$ and $\gamma$ band energies and other related properties with the same simple hamiltonian that reproduced the phase transition (see Sect. 8.3 ahead). Thus a good description of the excited bands calls for detailed gIBM calculations and with this one can predict reliably the E4 properties of Sm isotopes. It should be pointed out that sdIBM calculations necessarily fail in describing E4 properties (see for example [To-85b, Se-90, Wu-88]). Keeping these in mind, first we repeated OS calculations in the spherical basis to establish that the variational results are essentially kept intact in the matrix diagonalization approach and also extend the calculations to other observables such as $B(E4|\rangle\langle\gamma|)'s, g$-factors, nuclear radii and two-nucleon transfer strengths. The results are described in Sect. 8.2. Having shown that a truncated spherical basis is meaningful, we employ a hamiltonian based on gIBM dynamical symmetries (with six free parameters) to reproduce $\beta, \gamma$ energies and related properties. In Sect. 8.3 detailed calculations for $^{152,154}$Sm are reported and only for these isotopes $B(E4)$ data ($0^+_I \rightarrow 4^+_I$ and $0^+_I \rightarrow 4^+_I$) exists [Ic-87]. The effective charge that is used to reproduce the $B(E4; 0^+_I \rightarrow 4^+_I)$ predicts the $B(E4; 0^+_I \rightarrow 4^+_I)$ values close to the data. With this agreement, reliable predictions for E4 distributions in $^{152,154}$Sm are made. Moreover, a particular ratio $R$ (see Chapter-IV) involving two-nucleon transfer strengths (TNT) which shows a peak at neutron number 90 is well described by the
calculations. These results together with the predictions for the properties of $1^+_1$ levels in $^{152,154}$Sm are given in Sect. 8.3. Finally Sect. 8.4 gives a summary. The results given in this Chapter are first reported in [De-90c, De-91c].

8.2 Spectroscopy of Low-lying Levels in Sm Isotopes: Simple gIBM Calculations

The $0^+_1$, $2^+_1$ and $4^+_1$ levels of doubly even Sm isotopes $^{146-158}$Sm are studied employing a simple two parameter hamiltonian

$$H = F \{ \varepsilon_n n_d + \varepsilon_n n_s + \kappa Q^2(s).Q^2(s) \}$$

(8.1)

In (8.1) $Q^2(s)$ is the SU$_{sdg}$(3) quadrupole generator given in (2.12a). For $^{146-158}$Sm, the boson number $N$ takes values 7-13 ($N_p = 6$ and $N_n = 1-7$ where $\pi$ and $\nu$ stand for proton and neutron bosons). In all the calculations the basis states are restricted such that $n \geq 2$ and $n \leq 2$. This truncation scheme is adopted based on the following facts: (i) in sdIBM vibrational (U$_d$(5)) limit the ground state (GS) s-boson occupancy $\langle n_s \rangle_{GS} = N$ and in the rotational (SU$_{sd}$(3)) limit $\langle n_s \rangle_{GS} = N/3 \approx 3-4$; (ii) microscopic theory (based on HFB intrinsic states) of Pannert et al [Pa-85] shows that even for a well deformed nucleus $^{154}$Sm $n_s$ takes a maximum value of 2 ($n_s \leq 2$). Moreover, calculations relaxing the above restrictions (allowing for $n_s \geq 0$ or 1, $n_s \leq 3$ or 4) showed negligible improvements in energies and B(E2) values; the maximum deviation for the levels below 2.5 MeV is $\approx 30$ keV and the quadrupole moments differ by 0.05 (eb). Table-8.1 gives the matrix dimensions for $L \leq 6$ and they follow from Table-6.2. The results of the calculations for energies, E2, M1 and E4 properties, isomer and
TABLE 8.1

Dimensionalities in sdg space with the restriction \( n_s \geq 2 \) and \( n_g \leq 2 \)

<table>
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<th>Nucleus</th>
<th>N/L</th>
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<th>2^+</th>
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<th>4^+</th>
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<td>235</td>
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isotope shifts and TNT strengths are described in Sects. 8.2a-8.2f and some comments on results in Sects. 8.2a-8.2f are given in Sect. 8.2g.

8.2a Energies

The parameters in the hamiltonian (8.1) are chosen to be $\epsilon_8 = 1.3$ MeV, $\epsilon_\delta = 1.8$ MeV, $F = 1.0$ and $\kappa = -0.005$ MeV respectively for $^{146-150}$Sm and the values of $(F, \kappa)$ are changed to $F = 0.7$ and $\kappa = -0.0065$ MeV respectively for $^{152-158}$Sm. With these two sets of parameters, properties of $^{146-158}$Sm are calculated. The set used for $^{146-150}$Sm was employed before by OS in their variational calculations (see also Sect. 8.2g ahead). Excitation energies ($E_x$) of $2^+_1$ and $4^+_1$ levels is shown in Fig. 8.1a and the ratio $R = E_x(2^+_1)/E_x(4^+_1)$ is shown in Fig. 8.1b. The vibration-rotation phase transition is clearly seen in theory and experiment both in the energies and the ratio $R$; the ratio $R$ changes from vibrational $R = 2$ to rotational $R = 10/3$ values as we move from $^{146-148}$Sm to $^{154-158}$Sm. In gIBM one can also calculate binding energies. For a fixed proton-boson number $N_p$, the two neutron separation energies are given by [Ot-88, Sc-78, Sc-80]

$$S_{2n} = A_0 + A_1N_p + E_{GS}(N_pN_p) - E_{GS}(N_pN_p+1),$$

(8.2)

where $E_{GS}$ is the ground state (GS) energy calculated with the hamiltonian (8.1) and $A$'s are free parameters. The calculated separation energies are shown in Fig. 8.1c. The values of the parameters obtained from a fit to data are $A_0 = 15.23$ MeV and $A_1 = -0.614$ MeV. The sudden discontinuity from the straight line behaviour in the phase-transition domain is well reproduced by the
Fig. 8.1 Comparison of properties of $2^*_1$, $4^*_1$, and $0^*_1$ states, between calculated (full-line) and experimental (points).

(a) excitation energies ($E_x$) of $2^*_1$ and $4^*_1$ states [Le-78].

(b) ratio $R = E_x(4^*_1)/E_x(2^*_1)$.

(c) Two-nucleon separation energies [Wa-77].

(d) Transition moments of $0^*_1 \rightarrow 2^*_1$ (closed circles) [Ra-87], and static moments of $2^*_1$ states (open circles) [Ra-87, Po-79].

(e) g-factors [Ku-72].

(f) $B(E4;0^*_1 \rightarrow 4^*_1)$ values [Co-76].

(g) isomer-shifts [Ka-74].

(h) isotope-shifts [Le-71].

(i) $0^*_1 \rightarrow 0^*_1$ (t,p) transfer cross-sections [Bj-66].

(j) $0^*_1 \rightarrow 0^*_2$ (t,p) transfer cross-sections [Bj-66].

(k) $0^*_1 \rightarrow 0^*_1$ (p,t) transfer cross-sections [De-72].

(l) $0^*_1 \rightarrow 0^*_2$ (p,t) transfer cross-sections [De-72].
calculations and the parameters A's are consistent with those of [Ot-88, Sc-78, Sc-80].

8.2b E2-Transitions and Quadrupole moments

Following OS, the E2 transition operator is chosen to be

\[ T^{E2} = e_2 Q^2_m(s) \]  

(8.3)

In (8.3) \( e_2 \) is the effective charge. Fig. 8.1d shows the transition moments \( Q(0^+ - 2^+) = [16\pi/5 B(E2; 0^+ - 2^+)]^{1/2} \) and the static moments \( Q(2^+) \). The value of the effective charge obtained from the fit to \( Q(2^+) \) is \( e_2 = 0.0414 \) eb. The phase-transition here manifests itself in a sharp increase in the value of the transition moment \( Q(0^+ - 2^+) \) which is reproduced by the calculations.

8.2c Magnetic g - factors

In gIBM, the general one-body M1 operator is,

\[ T^{M1} = \sqrt{\frac{30}{4\pi}} [\alpha(\hat{d}^\dagger \hat{d})^1 + \sqrt{6} \beta(\hat{g}^\dagger \hat{g})^1]. \]  

(8.4)

Note that the M1-operator (8.4) is a different form of the M1-operator in (6.13). In terms of the parameters \( \alpha \) and \( \beta \), the effective g-factor for the \( 2_1^+ \) level (\( g_{2_1^+}^{eff} \)) is,

\[ g_{2_1^+}^{eff} = \alpha + \sqrt{60} \left\{ \beta - \alpha \right\} \times \frac{G}{2}; \]

\[ G = \begin{pmatrix} 2 & 2 & 1 \\ 2 & -2 & 0 \end{pmatrix} <2_1^+ | (\hat{g}^\dagger \hat{g})^1 | 2_2^- >. \]  

(8.5)
In (8.5) \[ \left( \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \right) \] stands for the Wigner's 3-j symbol. The parameters \( \alpha \) and \( \beta \) are determined by fitting the g-factor data for the \( 2^+ \) states and the values obtained are \( \alpha = 0.21 \mu_N \) and \( \beta = 0.45 \mu_N \). The results for g-factors are shown in Fig. 8.1e. Here the data shows large deviations from the linear behaviour of the "standard" value of \( Z/A \) and the calculations reproduce the observed trend. In the sdIBM calculations a two-body M1 operator has to be employed [Sc-78] to obtain agreement with data.

8.2d \( E4 \)-Transitions

The general \( E4 \)-operator in gIBM can be written as,

\[
T_{E4}^{\text{osc}} = e_4 \left( s^+ \hat{g} + g^+ \hat{s} \right)^4 + \eta_1 \left( d^+ \hat{d} \right)^4 + \eta_2 \left( d^+ \hat{g} + g^+ \hat{d} \right)^4 + \eta_3 \left( g^+ \hat{g} \right)^4
\]

(8.6)

Note that the \( E4 \)-operator (8.6) is a different form of the \( E4 \)-operator in (6.13). In the present calculations the \( \eta \)'s in (8.6) are taken to be equal to the matrix elements of the \( r^4 \chi^{(4)} \) operator in the sdg harmonic oscillator basis [Wu-88] and the only free parameter is \( e_4 \). The resulting \( T_{E4}^{\text{osc}} \) operator is referred to as \( Q_{\text{osc}}^{e} \), thus

\[
T_{E4}^{\text{osc}} = e_4 Q_{\text{osc}}^{e}
\]

\[
Q_{\text{osc}}^{e} = \left[ (s^+ \hat{g} + g^+ \hat{s})^4 + 19\sqrt{5} \frac{(d^+ \hat{d})^4}{28} + 5\sqrt{11} \frac{(d^+ \hat{g} + g^+ \hat{d})^4}{14} + 3\sqrt{143} \frac{(g^+ \hat{g})^4}{28} \right]
\]

The \( B(E4) \) values for \( 0^+ \rightarrow 4^+ \) determine the value of the effective charge \( e_4 \) to be \( e_4 = 0.034 \) (eb\(^2\)); the results for \( B(E4) \) are shown in Fig. 8.1f. It can be seen that there is a marked rise in the \( B(E4) \) values in the phase-transition domain from vanishing values at the vibrational end to fairly large values for the rotational nuclei. The operator in (8.6) with the above value for
Effective charge predicts $B(E4; 0^+_1 \rightarrow 4p^+$) in close agreement with data [Ic-87]; see Sect. 8.3 ahead.

### 8.2e Nuclear Radii: Isomer and Isotope shifts

In the sdg space the most general one-body rank-zero operator can be written as,

$$T(0) = a_0 + a_1 N + a_d \hat{n}_d + a_g \hat{n}_g \tag{8.7}$$

where $a_i$'s are free parameters. The structure of $T(0)$ can be used to calculate nuclear radii [Sc-78, Sc-80],

$$<\xi^2>_{L_i} = <\xi^2>_{0^+_I} + \gamma_{d} \hat{N} + \gamma_{d} <\xi^2>_{0^+_I} + \gamma_{g} <\xi^2>_{0^+_I} \tag{8.8}$$

where $<\xi^2>_{0^+_I}$ represents the mean square radius of a nucleus with $L_i^+$, $<\xi^2>_{0^+_I}$ is a constant term which represents the contribution due to the closed shell ($<\xi^2>$ and $<\xi^2>$ are the expectation values of the d and g boson number operators $\hat{n}_d$ and $\hat{n}_g$ respectively) and the $\gamma$'s are free parameters. The isomer shift $\delta(\xi^2)$ is defined to be the difference in the mean square radii of $2^+_1$ and the ground state $0^+_1$ states,

$$\delta(\xi^2) = <\xi^2>_{2^+_1} - <\xi^2>_{0^+_1} \tag{8.9}$$

$$= \gamma_d <\xi^2>_{2^+_1} - <\xi^2>_{0^+_1} + \gamma_g <\xi^2>_{2^+_1} - <\xi^2>_{0^+_1}$$

Similarly, the isotope shift $\Delta(\xi^2)$ is defined as the difference in the mean square radii of the ground states of the two neighbouring even-even nuclei.
The values of $\gamma_2$ and $\gamma_3$ in (8.9, 8.10) are determined by simultaneously fitting to the $\delta(\alpha^2)$ and $\Delta(\alpha^2)$ data and they are found to be $\gamma_1 = 0.3$ fm$^2$, $\gamma_2 = 0.025$ fm$^2$ and $\gamma_3 = 0$ fm$^2$. The results obtained with this parameter set for the isomer-shift $\delta(\alpha^2)$ and isotope-shift $\Delta(\alpha^2)$ are shown in Figs. 8.1g, 8.1h respectively. The present calculations reproduce the observed trends, (i.e) the sudden drop in isomer-shift and a rise followed by a drop of the isotope-shifts in the phase-transition domain.

8.2f Two-nucleon transfer intensities

In this Chapter, TNT intensities are calculated using the stripping and pickup operators (for $I = 0$ transfer) $P^{(0)}_{\pi\pi}$ defined in (4.1). In the present calculations the neutron pair degeneracy $\Omega_\nu$ is taken to be $(126-82)/2 = 22$ and the proton pair degeneracy $\Omega_\pi = (82-50)/2 = 16$. Following Scholten et al [Sc-78, Sc-80], the calculated two-nucleon transfer intensities are assumed to be proportional to the transfer cross-sections i.e

$$\sigma_{2\nu} = (\xi_{2\nu}^2) < (\xi_{2\nu})^\dagger P^{(0)}_{\pi\pi} \xi_{2\nu} >^2$$

(8.11)

The parameters $\xi_{2\nu}$ are defined in (4.1). The parameters $\xi_{2\nu}$ are determined to be $\xi_{2\nu} = 0.01$ mb/sr and $\xi_{2\nu} = 0.02$ mb/sr, by fitting to the data. These parameters are consistent with those of
Scholten et al [Sc-78, Sc-80]. It is to be noted that in general the knowledge of the kinematic factors is required to deduce the values of $E_{\pm V}$ (4.1) from $\xi_{\pm V}$ parameters in (8.11). The comparison with experimental (t,p) and (p,t) cross-sections for the transfer to ground state $0_1^+$ and the excited $0_2^+$ states are shown in Figs. 8.1.1-t. Both in theory and experiment the transition is manifested as a drop in the ground state transition and the corresponding increase in the $0_1^+ \rightarrow 0_2^+$ transition strength (see also Fig. 8.4 ahead).

8.2g Summary and Comments

Some comments on the results given in Sect. 8.2a-f are in order:

(i) The agreements shown in Figs. 8.1a-t demonstrate that matrix diagonalization in a truncated sdg spherical basis is well suited for describing the structure of complex nuclei;

(ii) Scholten et al. [Sc-78, Sc-80] calculated in sd space, with neutron number dependent d-boson energies, all the observables given in Figs. 8.1a-t; the sdg results are obtained with comparatively less free parameters and the agreements are in general better;

(iii) The results show that the agreements obtained by OS in their sdg studies (with variational methods) for energies and E2's extend to all other observables. The spherical basis calculations of all the observables is equally easy which may not be the case with variational methods.
In all the calculations two sets of \((F,k)\) are employed while OS employed only one fixed set. This need not be considered to be a drawback of the spherical basis because of (i) above and the fact that neither calculation reproduces the \(\beta, \gamma\) band energies and other related properties; see below.

The simple Hamiltonian given in (8.1) produces \(\beta, \gamma\) band energies \(~1\) MeV higher compared to data as is the case with OS calculations [Ot-90]. This calls for a more elaborate Hamiltonian and with this reliable predictions for \(E_4\) distributions and other detailed properties can be made. The results of these calculations are discussed below.

8.3 Extended Calculations for \(^{152,154}\text{Sm}\)

In order to obtain a consistently good description of GS, \(\beta\) and \(\gamma\) bands in Sm isotopes, calculations in the spherical basis are carried out employing a Hamiltonian that interpolates various gIBM dynamical symmetries

\[
\mathbf{H} = \varepsilon_{\hat{n}_d} \hat{n}_d + \varepsilon_{\hat{n}_s} \hat{n}_s + \alpha_1 \left( H\left( SU_{sdg} (3) \right) \right) + \alpha_2 \left( H\left( SU_{sdg} (5) \right) \right) + \\
\alpha_3 \left( H\left( SU_{sdg} (6) \right) \right) + \alpha_4 \left( H\left( O_{sdg} (15) \right) \right) + \alpha_5 \mathbf{L} \cdot \mathbf{L}
\]

(8.12)

The \(H(G)\) with \(G = SU_{sdg} (3), SU_{sdg} (5), SU_{sdg} (6)\) and \(O_{sdg} (15)\) are defined in (7.1b). Note that the Hamiltonian \(H\) in (8.12) is same as the \(H\) in (7.1a), except for the fact that the full \(H(G)\) are employed in (8.12) unlike in the case of (7.1a) where only the two-body part of \(H(G)\) are considered. Based on the calculations of Chapter-III, where geometric shapes corresponding to the various
IBM dynamical symmetries are studied and as Sm isotopes exhibit spherical-deformed shape phase transition, the strength $\alpha_3$ of $\mathcal{H}(\text{SU}_{sdg}(6))$ is set to zero ($\text{SU}_{sdg}(6)$ is more appropriate for $\gamma$-unstable nuclei). Thus the hamiltonian given in (8.12) has six free parameters and they are determined by least square fit to the spectra. The results for, in $^{152,154}\text{Sm}$, spectra, E4 strength distributions, Burke’s ratio $R_\pm$ defined in (4.8) for TNT and $1^+$ states are given below.

The spectra for $^{152,154}\text{Sm}$ are shown in Fig. 2, the r.m.s deviation (from experiment) is 49 and 20 keV respectively. The parameters obtained from the fits are $\epsilon_d = 0.7$ MeV, $\epsilon_g = 1.2$ MeV, $\alpha_1 = 0.00344$ MeV, $\alpha_2 = 0.01668$ MeV, $\alpha_3 = 0$ MeV, $\alpha_4 = 0.03024$ MeV and $\alpha_5 = 0.01195$ MeV for $^{152}\text{Sm}$ and $\epsilon_d = 0.7$ MeV, $\epsilon_g = 1.2$ MeV, $\alpha_1 = 0.00447$ MeV, $\alpha_2 = 0.00846$ MeV, $\alpha_3 = 0$ MeV, $\alpha_4 = 0.03117$ MeV and $\alpha_5 = 0.00975$ MeV for $^{154}\text{Sm}$. As the $\mathcal{H}(G)$ in (8.12) are in multipole-multipole form (see (7.1b)) they contribute to the d and g boson energies. Adding this contribution, the $(\epsilon_d, \epsilon_g)$ values given above change to (0.48, 1.18) and (0.42, 1.10) MeV respectively for $^{152}\text{Sm}$ and $^{154}\text{Sm}$. The SPE and TBME of the above hamiltonians for $^{152,154}\text{Sm}$ are given in Table-8.2 and it can be seen that they are not very much different from each other.

From the parameter sets given above, it appears that the strength of $\mathcal{H}(\text{O}_{sdg}(15)) \gg \mathcal{H}(\text{SU}_{sdg}(5)) \gg \mathcal{H}(\text{SU}_{sdg}(3))$ and it may lead to a misleading conclusion that $\text{SU}_{sdg}(3)$ is not important for Sm isotopes and $\text{O}_{sdg}(15)$ is more relevant. The problem here is that the basic operators all do not have the same normalization.

In order to have a proper comparison, a measure for the norm...
Fig. 8.2 Comparison between calculated (theory) and experimental (Exp) low-lying energy levels [Le-78] in GS, β and γ bands in (a) $^{152}$Sm and (b) $^{154}$Sm. See text (Sect. 8.3) for the hamiltonian and the corresponding parameters. The matrix dimensions are given in Table-8.1.
Table 8.2

Interaction matrix elements for $^{152,154}_{\text{Sm}}$ isotopes

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<th>$\ell_3$</th>
<th>$\ell_4$</th>
<th>$L$</th>
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$\varepsilon_s$, $\varepsilon_d$, $\varepsilon_g$ are SPE and $v^\ell(\ell_1, \ell_2, \ell_3, \ell_4)$ are the TBME; See (2.1). $\ell_1 = 0$, 2 and 4 correspond to s, d and g respectively. The SPE and TBME are in units of MeV.
of the operator has to be used and one such measure is given by French [Fr-72] and it is employed in this Chapter. Appendix-G gives the definition and expressions for the norms $\| H \|_m$ in $m$-boson space. Eq (G.9) gives the following results:

$$
\| H(SU_{a_{1g}}(3)) \|_{N=10} \approx 4.2 \times \| H(SU_{a_{1g}}(5)) \|_{N=10}, \quad \| H(SU_{a_{1g}}(3)) \|_{N=10} \approx 10 \times \| H(O_{a_{1g}}(15)) \|_{N=10}, \quad \| H(SU_{a_{1g}}(3)) \|_{N=10} \approx 8 \times \| H(SU_{a_{1g}}(6)) \|_{N=10}
$$

They clearly demonstrate that the parameters $\alpha$'s are meaningful and all the interactions $\alpha_1 H(SU_{a_{1g}}(3))$, $\alpha_2 H(SU_{a_{1g}}(5))$, and $\alpha_4 H(O_{a_{1g}}(15))$ are, globally, roughly of the same size. Similar results are obtained for $N = 11$ (for $^{154}$Sm). In principle the energy dependence of the norms can be studied [Fr-72, Ch-71] and it is not pursued here.

The hamiltonian in (8.12) with the parameters given above, not only provides a good description of GS, $\beta$ and $\gamma$ bands but also keeps intact all the results reported in Sect. 8.2. For example the $B(E2; 2^+_1 \rightarrow 0^+_1)$ and $B(E4; 4^+_1 \rightarrow 2^+_1)$ obtained with the same $T^{22}$ operator and effective charge given in Sect. 8.2b are 0.64 $e^2 b^2$ and 0.91 $e^2 b^2$ while the experimental values are 0.66 $\pm$ 0.02 $e^2 b^2$ and 0.97 $\pm$ 0.04 $e^2 b^2$ respectively, for $^{152}$Sm. Similarly for $^{154}$Sm the calculated (experimental) values, in units of $e^2 b^2$ are 0.80 (0.82 $\pm$ 0.02) and 1.134 (1.17 $\pm$ 0.04) respectively. It should be mentioned here that the $B(E2)$'s from $\beta$, $\gamma$ band members to GS $0^+$ are smaller [Sc-78] by a factor of $10^2$-$10^3$ compared to $B(E2; 2^+_1 \rightarrow 0^+_1)$ and this trend is well reproduced by the calculations.

More strikingly, using the $E4$ operator given in (8.6) with the effective charge $e_4 = 0.034 eb^2$ as determined by fitting $B(E4; 0^+_1$
The ratio of sum of the TNT cross sections to excited $0^+$ states to that of the GS $0^+$ as discussed in Chapter-IV and shown in Figs. 4.1 and 4.2 exhibits peaks at neutron numbers 90 and 88 (Sm region) in (t,p) and (p,t) data respectively. The cross sections' ratio can be approximated by the corresponding ratio involving TNT strengths and is given by $R_\pm$ defined in (4.8). Dynamical symmetries of gIBM explain, as discussed in Sect. 4.6b, the average $R_\pm$ value reasonably well in the rare-earth region (valence neutrons in 82-126 shell) but they fail to predict the peaks. In Sm isotopes the peak at neutron number 90 is due to spherical-deformed phase transition; the value of $R_+$ changes from vibrational ($U_4^d(5)$) $R = 0$ to rotational ($SU_{sdg}(3)$) $R_+ = 4/N = 0.3$ value while $R_-$ is zero in both the limits. The hamiltonian in (8.1) predicts the peaks at neutron numbers 90 for $R_+$ and 88 for $R_-$ very well (as it interpolates $U_4^d(5)$ and $SU_{sdg}(3)$) but the values for $^{152}$Sm $\rightarrow^{154}$Sm are much smaller than the experimental values. For these transitions, the value of $R_\pm$ is recalculated using the wave functions given by the hamiltonian (8.12) and the results are shown in Fig. 8.4. In using (4.8), the cut-off (square
Fig. 8.3  E4 strength distributions in (a) $^{152}\text{Sm}$ and (b) $^{154}\text{Sm}$.

The $B(E4|) = B(E4; 0^+_1 \rightarrow 4^+_1)$ values are shown in the figure as a function of the energy of $4^+_1$ levels. For the $4^+_1$ level $B(E4|)$ is to be multiplied by a factor six for $^{152}\text{Sm}$ and factor eight for $^{154}\text{Sm}$. The E4 transition operator and the corresponding effective charges are given in Sect. 8.2d.
Fig. 8.4 Comparison of the ratio \( R \) of the sum of the cross-sections to the excited \( 0^+ \) states to that of the ground state \( 0^+ \); (a) for \((t,p)\), (b) for \((p,t)\). The circles \((\circ)\) are calculated values and boxes \((\Box)\) are from experimental data (see Fig. 4.1a, 4.1b). The ratio \( R_{\pm} \) is given for the final nucleus \( A \). The data compilation is due to Burke \([Bu-90, Bu-91]\). All the gIBM results (except for \(^{152}\text{Sm} \rightarrow ^{154}\text{Sm}\)) are obtained using the simple two parameter interpolating Hamiltonian (8.1) of Sect. 8.2. For \(^{152}\text{Sm} \rightarrow ^{154}\text{Sm}\) the wavefunctions obtained by diagonalizing a more general gIBM Hamiltonian (see Sect. 8.3) are used.
factors appearing in $p_{\pm}^{(4)}$ (see (4.1)) are ignored as we are dealing with a ratio. One sees that the detailed numerical calculations reproduce the observed variation in $R_\pm$ very well; for $(p,t)$ the predicted peak value is somewhat smaller.

In gIBM one can generate $1^+$ levels which is not possible with s and d bosons alone. With the hamiltonian (8.12), the $1^+_1$ levels appear at 1.91 and 2.01 MeV in $^{152}\text{Sm}$ and $^{154}\text{Sm}$ respectively and the $B(M1; 0^-_1 \rightarrow 1^+_1) = B(M1\uparrow)$ value, with the M1 operator and the corresponding g-factors given in Sect. 8.2c, are predicted to be 0.10 and 0.15 $\mu_N^2$ respectively. In $^{154}\text{Sm}$ the scissors [Ha-87b] $1^+$ level is observed at 3.2 MeV with $B(M1\uparrow) \approx 0.8 \mu_N^2$ and obviously it cannot be the gIBM $1^+_1$ level. Therefore it would be interesting to look for $1^+_1$ levels around 2 MeV in $^{152,154}\text{Sm}$.

8.4 Summary

First systematic calculations for a series of isotopes in sdgIBM framework are presented, in this Chapter with Sm isotopes, which exhibit spherical-deformed phase transition, as the example. It is clearly demonstrated that the spherical basis with a symmetry defined hamiltonian with few free parameters (not much more than what one has in sdIBM) describes the spectroscopic data rather well, the former is well confirmed in Sect. 8.2 and the latter (together with the former) by the results in Sect. 8.3. These studies establish that gIBM is a simple yet powerful tool in analyzing and predicting simultaneously the E2 and E4 properties in nuclei. In the following Chapter gIBM calculations for some nuclei in Os-Pt region (where good E4 data is available [To-85b, Se-90]) are reported.