APPENDIX-E

COSMIC RAY PRODUCED CHARGED ALBEDO
IN UPPER ATMOSPHERE:
CALCULATIONS:

The knock-on process predominates in the energies below 20 MeV; while \( W^-\mu^-e \) decay electrons produced in Nuclear interaction become dominant above this energy.

Using the production cross section given by Bhabha (1938) for this process, the production spectrum has been calculated by Abraham et al. (1966), the energy spectrum \( N(E) \) for vertical knock-on electrons can be written as:

\[
N(E) = \int_0^\infty \frac{k ( \rho_0 c^2 )^{(n-1)} d\ell (cm^2 \text{ Sec Ster, MeV}^{-1}) }{ E + \epsilon(1) } ^n
\]

\( \epsilon(1) \) is a factor which takes into consideration the primaries at different latitudes, \( h' \) is an integral which takes into account production of secondaries due to primaries arriving at various angles.

or \( N(E) = f h \int_0^\infty \frac{k (\rho_0 c^2)^{\alpha - 1} \exp - (1/H) d\ell (cm^2 \text{ Sec Ster, MeV}^{-1})}{[E + \epsilon(1)]^n} \) \ldots (1)

Here \( \rho \) is density at \( l \) depth and \( E(1) \) is Energy lost by the electrons of energy \( E \), in passing through \( l \), \( \rho_0 \) gives density of air at sea level and \( H \) is scale height; for \( E \) in low energy range, the energy loss \( \epsilon(1) \) can be written in terms of equivalent range \( R \) and a constant \( K \) as below:

\[
\epsilon^{1/\alpha}(1) = KR = K \frac{\rho_0}{g} \exp - \frac{1}{H}
\]

so \( \epsilon(1) = \left( \frac{K \rho_0}{g} \right)^\alpha \exp - \left( \frac{1}{H} \right) \) \ldots (2)
which when substituted in (1) gives

\[ N(E) = k(m_0 c^2)^{\gamma - 1} \int_0^{l_0} \exp \left( \frac{-1}{H} \right) \frac{dl}{E^+ \left( \frac{k_0}{g} \right) \exp \left( \frac{1}{H} \right)} \left( \text{cm}^2 \text{Sec Ster MeV}^{-1} \right) \]

where \( p_0 \) is atmospheric pressure at surface of earth.

Putting \( U = \frac{k_0}{g} \exp \left( \frac{-1}{H} \right) \); \( dU = \frac{k_0}{g} \exp \left( 1/H \right) \frac{dl}{H} \)

for \( l = l_0 \), \( U_m = \frac{k_0}{g} \exp \left( \frac{1}{H} \right) \) \& \( l = 0 \), \( U_0 = \frac{k_0}{g} \)

This when substituted in eqn. (3) gives

\[ N(E) = k(m_0 c^2)^{\gamma - 1} \frac{\int_0^{U_m} \frac{dl}{U_0}}{\int_0^{U_0} \left( \frac{E+U}{\gamma} \right)^{\gamma - 1} \left( \text{cm}^2 \text{Sec Ster MeV}^{-1} \right)} \]

On solving one gets

\[ N(E) = \frac{k(m_0 c^2)^{\gamma - 1} \int_0^{l_0} \exp \left( \frac{-1}{H} \right) \frac{dl}{E^+ \left( \frac{k_0}{g} \right) \exp \left( \frac{1}{H} \right)} \left( \text{cm}^2 \text{Sec Ster MeV}^{-1} \right)}{k_0 (f - 1) (\gamma - 2) \left( E^+ \frac{U_0}{\gamma} \right) \left( E^+ \frac{U_0}{\gamma - 1} \right)} \]

In this \( k \) is a constant whose value, given by Abraham et al., is 4.9, the contribution of primaries from 0° to 90° from Zenith can be taken into account by the factor \( h \), given below

\[ h = \int_0^{l_0} \exp \left( \frac{-X}{\cos \theta} \right) \]

in which \( X \) is absorption mean free path, \( X \) is depth and \( \theta \) is angle of arrival of primary from Zenith. The flux and energy spectrum of upward moving electrons are evaluated. All these electrons will appear as
re-entrant albedos in opposite hemisphere; thus, this also represents the flux and energy spectrum of re-entrant electrons. This spectrum is calculated for Ft. Churchill, where the cutoff rigidly is very low, for solar Minimum & Maximum periods, as shown in fig. This is obtained using single parameter ($\gamma$) fitting with the observed flux of Hovestadt & Meyer (1969).

Fluxes at Hyderabad, India & Palestein, Texas (U.S.A.), for Solar minimum period have been calculated from above spectrum and shown in fig. Flux above 20 MeV is predominantly due to $\pi^-$ decay electrons. This calculated spectrum by Verma (1979) is also shown in fig.
Flux & Energy spectrum of vertically moving Knock-on albedos electrons. Various lines are present calculated spectra, while Verma '79 (— — ) shows electron Spectrum due to \( \pi - \mu - e^- \) decay.

▲ Hovestadt & Meyer, 1969 (Ft. Churchill);
□ Israel, 1969 (Ft. Churchill);
○ Verma, 1967, (Texas)
●,◆ Schmoker & Earl, 1965 (Minneapolis & San Angelo respectively)