In this chapter we will discuss a scheme to suppress the decoherence which is qualitatively different from the schemes we discussed in the previous chapters. The scheme presented here involves introduction of an ancilla in the bath close to the system of interest. The presence of the ancilla causes a shielding effect on the system and hence suppresses the decoherence. The ancilla and the system do not interact directly but by *exchange interactions* through the bath they interact [85].

11.1 Dynamics of the system plus ancilla in the presence of thermal bath

Consider a bipartite system $S_1 = A_1 + B_1$ initially in an entangled state. One of the subsystem (say $A_1$) is exposed to a thermal bath. We have already studied the evolution of entanglement in such systems in chapter 8. We have seen that in the case of thermal bath with non-zero temperature the entanglement in the system $S_1$ dies out in a short span of time. Let us introduce another bipartite system, namely ancilla, $S_2 = A_2 + B_2$ where $A_1$ and $A_2$ both interacts with the same bath (see Fig. (11.1)), i.e, the bath is acting on $A_1$ and $A_2$ together to generate a kind of bath induced interaction between $A_1$ and $A_2$. The Lindblad master equation for
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the system $A_1$ before we introduce $S_2$ (neglecting the unitary evolution part) is:

$$
\frac{d\rho_{A_1}}{dt} = \mathcal{L}_{1}\rho_{A_1}
$$

\begin{equation}
= \frac{(N + 1)\gamma}{2} [2\sigma_-\rho_{A_1}\sigma_+ - \sigma_+\sigma_-\rho_{A_1} - \rho_{A_1}\sigma_+\sigma_-] + \frac{N\gamma}{2} [2\sigma_+\rho_{A_1}\sigma_- - \sigma_-\sigma_+\rho_{A_1} - \rho_{A_1}\sigma_+\sigma_-],
\end{equation}

where $N$ is the mean occupation number of quanta in the reservoir, $\gamma$ is the spontaneous decay rate of the qubits, $\sigma_+ = |1\rangle\langle 0|$ and $\sigma_- = |0\rangle\langle 1|$.

Once we introduce $S_2$ the Lindblad master equation for $A_1$ will change. To see that consider the master equation for $A_1A_2$:

$$
\frac{d\rho}{dt} = \frac{(N + 1)\gamma}{2} [2J_-\rho J_+ - J_+ J_- - \rho J_+ J_-] + \frac{N\gamma}{2} [2J_+\rho J_- - J_- J_+ - \rho J_- J_+],
\end{equation}

where $J_\pm = \sigma_\pm^1 + \sigma_\pm^2 = \sigma_\pm \otimes I + I \otimes \sigma_\pm$.

To get the master equation for $A_1$ we need to trace out $A_2$ which will result in

$$
\dot{\rho}_{A_1} = \mathcal{L}_{1}\rho_{A_1} + \mathcal{L}_{eff}(t)\rho_{A_1},
\end{equation}

where

\begin{align*}
\mathcal{L}_{eff}\rho_{A_1} &= \text{Tr}_{A_2} \left\{ \frac{(N + 1)\gamma}{2} [2\sigma_-^1\rho\sigma_+^2 + 2\sigma_-^2\rho\sigma_+^1 - \sigma_+^1\sigma_-^2\rho - \sigma_+^2\sigma_-^1\rho - \rho\sigma_+^1\sigma_-^2 - \rho\sigma_+^2\sigma_-^1] \\
&\quad + \frac{N\gamma}{2} [2\sigma_+^1\rho\sigma_-^2 + 2\sigma_+^2\rho\sigma_-^1 - \sigma_-^1\sigma_+^2\rho - \sigma_-^2\sigma_+^1\rho - \rho\sigma_-^1\sigma_+^2 - \rho\sigma_-^2\sigma_+^1] \right\}.
\end{align*}

The presence of time dependent $\mathcal{L}_{eff}$ may change the dynamics of the system $S_1$. Note that even if the joint dynamics of $S_1 + S_2$ is of Markovian type, there is, in general, no guarantee that the dynamics of $S_1$ (or $S_2$) will be Markovian type (see for example, J. Piilo et al. [138], H.-P. Breuer et al. [36], H.-P. Breuer et al. [35], D. Chruściński et al. [51]).
11.2 Dynamics in terms of Kraus operators

To get a better understanding of the evolution of the system $S_1$ in the presence of ancilla $S_2$ and the bath, let us consider the four-partite initial density matrix $\rho_{A_1A_2B_1B_2}$. Suppose that $\{A_k\}$ are the two-qubit Kraus operators, the evolution of the state $\rho_{A_1A_2B_1B_2}$ when the bath is acting on the qubits $A_1$ and $A_2$ can be written as

$$
\rho(t)_{A_1A_2B_1B_2} = \sum_k (A_k \otimes I_{B_1B_2}) \rho_{A_1A_2B_1B_2} \left(A_k^\dagger \otimes I_{B_1B_2}\right) \tag{11.6}
$$

Let us assume that the single qubit bath be of full rank, i.e., one needs at least four Kraus operators for this bath. If $\{a_m\}_{m=1}^4$ are the single qubit Kraus operators then the operators $\{a_m \otimes a_n\}_{m,n=1}^4$ will form a basis for operators acting on two-qubit states. Thus, the two-qubit Kraus operators $\{A_k\}_{k=1}^{16}$ can be written as:

$$
A_k = \sum_{mn} \alpha_{mn}^k a_m \otimes a_n^*, \tag{11.7}
$$

where $\{\alpha_{mn}^k\}$ are the expansion coefficients. We can now write

$$
\rho_{A_1A_2B_1B_2}(t) = \sum_k \sum_{m,n,m',n'} \alpha_{mn}^k \alpha_{m'n'}^{k*} (a_m \otimes a_n^* \otimes I_{B_1B_2}) \rho_{A_1A_2B_1B_2}(a_m^\dagger \otimes a_n^{T} \otimes I_{B_1B_2}). \tag{11.8}
$$
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Let us take the initial density operator \( \rho_{A_1A_2B_1B_2} \) of the entire system to be of the form

\[
\rho_{A_1A_2B_1B_2} = \rho_{A_1B_1} \otimes |\chi\rangle_{A_2B_2} \langle \chi|,
\]

(11.9)

where \( \rho_{A_1B_1} \) is an arbitrary state of the two-qubit system \( A_1 + B_1 \) and \( |\chi\rangle_{A_2B_2} \) is a fixed state of ancilla. Therefore, we can write the Eq.(11.8) as

\[
\text{tr}_{A_2B_2} (\rho_{A_1A_2B_1B_2}(t)) = \sum_{mm'} Y_{mm'} (a_{m} \otimes I_{B_1}) \rho_{A_1B_1} \left(a_{m}^\dagger \otimes I_{B_1}\right),
\]

(11.10)

where

\[
Y_{mm'} = \text{tr}_{A_2B_2} \sum_k \sum_{mm'n'} \alpha_{mm'}^k \alpha_{mm'}^{k*} (a_{n}^\dagger \otimes I_{B_1}) (|\chi\rangle_{A_2B_2} \langle \chi|) \left(a_{n'}^{T} \otimes I_{B_1}\right).\]

(11.11)

Interesting thing to notice is that the matrix \( Y \) is positive semi-definite and hence we can write it as

\[
Y_{mm'} = \sum_{kl} U_{mk} d_{kl}(U_{lm})^\dagger
\]

(11.12)

where \( d_{kl} = d_{kl} \delta_{kl} \) are the eigenvalues of \( Y \) and are non-negative real numbers. Substituting this in Eq.(11.10) results in

\[
\tilde{\rho}_{A_1B_1}(t) = \sum_k d_k \left(b_k \otimes I_{B_1}\right) \left(b_k^\dagger \otimes I_{B_1}\right),
\]

(11.13)

where \( b_k = \sum_m U_{mk} a_m \) are new Kraus operators. Absorbing the \( d_k \) into the new Kraus operator give rise to \( \tilde{b}_k = \sqrt{d_k} b_k \). Our main task is to calculate the entanglement in the \( M \) matrix corresponds to Kraus operators \( a_i \) and \( \tilde{b}_k \) and compare the entanglement between these two states.

11.3 Results

Numerically it is straight forward to calculate the evolution of entanglement and coherence in the system \( S_1 \) (where the initial state of \( S_1 \) is chosen to be \( |\phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2} \) after introducing the ancilla \( S_2 \) with different initial states \( |\chi\rangle \).
Figure 11.2: Here we are plotting the coherence (real part of $\langle \sigma_+ \rangle$) for a single qubit. The blue curve shows the evolution of coherence in the absence of ancilla whereas the green curve is in the presence of ancilla. One can see that the green curve is always above the blue one and hence the presence of ancilla arrest the decay of coherence. Here the initial state of the system qubit is $(|0\rangle + |1\rangle)/\sqrt{2}$ and ancilla is in ground state ($|0\rangle$).

of $S_2$. In Fig. (11.2) and Fig. (11.3) we have shown the effect of ancilla on the evolution of the coherence $\langle \sigma_+ \rangle$ and $\langle \sigma_z \rangle$ of the system $S_1$. In Fig. (11.4) we can see the entanglement evolution for $S_1$ for different $|\chi\rangle$. We see that for $|\chi\rangle = |0\rangle \otimes |\psi\rangle$ where $|\psi\rangle$ can be arbitrary single-qubit state, the entanglement last for much longer time than for other $|\chi\rangle$’s and for $|\chi\rangle = |1\rangle \otimes |\psi\rangle$ the entanglement decays fastest. This shows the ‘shielding effect’ of ancilla on $S_1$. This effect is more prominent when the number of qubits in ancilla are more. In Fig. (11.5) we show the evolution of entanglement in $S_1$ for two-qubit, four-qubit and six-qubit ancilla. The improvement in the ESD time for $S_1$ is eminent.

Because of the exchange interactions, the presence of ancilla in the ground state may result in reducing the effective temperature of the bath. This might be a reason for the observed shielding effect.
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Figure 11.3: Here we are plotting the coefficient of $\sigma_z$ for a single qubit for the same setup as in Fig. (11.2). Again the improvement due to the presence of ancilla is obvious from the plot.

Figure 11.4: The effect of two-qubit ancilla on the evolution of entanglement when the initial state of the system $A_1 + B_1$ is maximally entangled state $(|00\rangle + |11\rangle)/\sqrt{2}$. The state of ancilla is $|\chi\rangle = \sqrt{p}|00\rangle + \sqrt{1-p}|11\rangle$. The average number of photons in the thermal cavity is $N = 0.3$. 

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Figure 11.5: The effect of the increasing number of qubits in ancilla on the evolution of entanglement when the initial state of the system $A_1 + B_1$ is maximally entangled state $(|00⟩ + |11⟩)/\sqrt{2}$. All the qubits in ancilla are in the ground state $|0⟩$. 