Synopsis

The low energy limit of string theory is supergravity coupled to various matter fields. These theories have black hole solutions. Black holes are solutions of Einstein’s equation of motion with the properties that they are surrounded by a hypothetical surface - known as event horizon- such that no object inside the event horizon can escape the black hole. However in quantum theory black hole behaves as a black body with finite temperature and has entropy. In the Einstein’s gravity, this entropy known as the Bekenstein-Hawking entropy $S_{BH}$ is given by the expression

$$S_{BH} = \frac{A}{4G_N}$$

Here $A$ is the area of the event horizon and $G_N$ is Newton’s gravitational constant.

One of the success of string theory has been an explanation of the Bekenstein-Hawking entropy of a class of supersymmetric extremal black hole in terms of degeneracy of microscopic quantum states ($d_{micro}$). These extremal black holes are special class of black holes which have zero temperature and hence do not radiate and are usually stable. Technically extremal black holes are defined to be those whose near horizon geometries have the form $AdS_2 \times K$, where $K$ is some compact space. The initial comparison between $S_{BH}$ and $S_{micro}(= \ln d_{micro})$ was done in the limit of large charges. In the limit of large charges we can work with two derivative action in full string effective action. On the microscopic side we can use the asymptotic formula for $d_{micro}$ for large charges instead of having to compute it exactly. However it is clearly of interest to know if the correspondence between the black hole entropy and the microscopic entropy extends beyond the large charge limit. In order to address this problem we need to open two fronts. First of all we need to compute the degeneracy of states of black holes to greater accuracy so that we can compute corrections to $S_{micro}$. The other front involves understanding how higher derivative corrections/ string loop corrections affect the black hole entropy. Wald’s formalism gives a clear prescription for calculating the effect of tree level higher derivative corrections on the black hole entropy, and for extremal black holes this leads to the entropy function formalism. However in order to implement it we need to know the higher
derivative terms in the action. Inclusion of quantum corrections into the computation of black hole entropy is challenging both conceptually and technically.

My research projects are focussed on the macroscopic side.

For some extremal black holes in string theory the $AdS_2$ component of the near horizon geometry, together with an internal circle, describes a locally $AdS_3$ space. More accurately the near horizon geometry of these extremal black holes correspond to that of extremal BTZ black holes. The BTZ solution describes a rotating black hole in three dimensional theory of (super-)gravity with negative cosmological constant. A general three dimensional theory of pure (super-)gravity with arbitrary higher derivative terms without matter fields admits a field redefinition that reduces the action to the standard (super-)gravity action whose gravitational part contains sum of three terms, Einstein-Hilbert term, a cosmological constant term and the Chern-Simons term. We proved that this is true even in presence of matter fields. After field redefinition and consistent truncation (the scalar fields set to constant and the tensor fields set to zero) the action reduces to standard (super-)gravity action. The parameters labelling the truncated action are the cosmological constant and the coefficient of the Chern-Simons term. Of them the coefficient of the Chern-Simons term does not change under the field redefinition but the cosmological constant term is modified and can be calculated explicitly. The corrected central charge in the $CFT$ at the boundary of $AdS_3$ and hence the corrected entropy of the BTZ black hole is determined in terms of coefficients of Chern-Simons term and the modified cosmological constant.

Since extremal black holes have an $AdS_2$ factor in their near horizon geometry, one expects that the underlying quantum gravity theory in this background will have a dual description in terms of a conformal quantum mechanics ($CQM$) living at the boundary of $AdS_2$. This $CQM$ has only ground states parameterised by the charge of the black hole and there are no excited states. It has been conjectured that the logarithm of the ground state degeneracy of this dual $CQM$ in a fixed charge sector should be taken as the definition of the quantum corrected macroscopic entropy $S_{macro}$ of extremal black holes. Quantitatively this proposal states that quantum degeneracy associated with horizon degrees of freedom of the black hole is given as the finite part of the partition function of string theory on $AdS_2$. The relation is given as

$$e^{S_{macro} (\vec{q})} = d (\vec{q})_{macro} = \left\langle \exp \left[ -i \vec{q} \cdot \oint d \theta A^{(i)}_\theta \right] \right\rangle_{finite}^{AdS_2}$$  \hspace{1cm} (2)

Here $\vec{q}$ are the charges carried by the black hole, $A^{(i)}$ are the gauge field and the path integral is over all fields in string theory with the boundary condition that they asymptote to near horizon geometry of the black hole containing an $AdS_2$ factor.
We check this proposal in the case of supersymmetric extremal BTZ black hole where we have independent definition of the entropy. The BTZ black hole is identified with states in the dual CFT living at the boundary of the $AdS_3$. The entropy of this black hole is given as logarithm of degeneracy of the corresponding states in the CFT. In order to compare the proposal with the above definition of entropy, one has to find the relation between CQM dual to $AdS_2$ and the CFT dual to $AdS_3$. One finds that the CQM dual to gravity in $AdS_2$ is described by the chiral half of the (1+1) dimensional CFT dual to gravity in $AdS_3$. In other words the states of the CQM living on the boundary of $AdS_2$ are described by the $\bar{L}_0 = 0$ states of the (1+1) dimensional CFT living on the boundary of $AdS_3$. In fact the degeneracy of the ground states of CQM carrying a given charge $q$ is identified with the degeneracy of the states of the CFT which have $\bar{L}_0 = 0$ and $L_0 - \bar{L}_0 = q$. The later states appear in the definition of entropy of the black hole via $AdS_3/CFT_2$ correspondence. Hence the two definition of the entropy agree.

In order to compare macroscopic the quantum degeneracy computed from equa.(2) with the microscopic degeneracy, which is known for some of the supersymmetric black holes, one needs to perform the above path integral explicitly. However this is technically a very difficult problem. In the next project, instead of performing the path integral, we simplify this using the supersymmetry of the near horizon geometry. The isometry supergroup of the near horizon geometry has a factor $SU(1,1|2)$. Using supersymmetry and localization techniques we showed that the path integral could receives non-vanishing contribution only from a special class of field configurations which preserve a particular subgroup (we call it $H_1$) of $SU(1,1|2)$. We identify this subgroup and showed that path integral around such field configuration reduces to integration over $H_1$ invariant slice passing through the field configuration. This analysis is useful for finding the saddle points, i.e. classical string field configuration which could give non-perturbative corrections to the quantum entropy. We also find out some examples of such field configurations.