Chapter 5

Conclusion

We have studied the correction to the entropy of extremal black hole in string theory. In the first part we discussed the higher derivative correction to the entropy of the BTZ black hole. In string theory one finds that the low energy effective action contains higher curvature terms. In fact at tree level it contains an Einstein-Hilbert term together with an infinite series of higher curvature terms that are suppressed by powers of $\alpha'$, so that they are subleading at low energy. We described field redefinition and consistent truncation in three dimensional general higher derivative theory of (super-)gravity coupled to arbitrary set of matter fields. After field redefinition and consistent truncation the action reduces to standard (super-)gravity action which is sum of three terms, Einstein-Hilbert term, a cosmological constant term and the Chern-Simons term. The effect of higher derivative corrections are encoded in the correction of the central charges. These will give classical correction to the entropy of extremal black hole in string theory whose near horizon geometry corresponds to that of extremal BTZ black hole.

In the second part we described the quantum entropy of the extremal black hole. The quantum degeneracy associated with horizon degrees of freedom of the extremal black hole is given as the finite part of the partition function of string theory on $AdS_2$. According to this proposal the macroscopic entropy in full quantum theory is equal to logarithm of degeneracy of the ground states of the CQM living on the boundary of $AdS_2$. We first check this proposal in case of extremal BTZ black hole where there exist an independent definition of entropy via $AdS_3/CFT_2$ correspondence. We found that both the definition of entropy agrees. We also simplify this path integral using the supersymmetry of the near horizon geometry. The isometry supergroup of the near horizon geometry has a factor $SU(1,1|2)$. Using supersymmetry and localization techniques we showed that the path integral could receives non-vanishing contribution only from a special class of field configurations which preserve a particular subgroup of $SU(1,1|2)$.

The next thing one would like to do is to compare this proposal with the known microscopic results. However in order to do this, one needs to know the full spacetime string effective action and perform the path integral explicitly.