Summary

This thesis contains some theoretical studies on the various fluctuation effects on the low temperature dielectric properties of certain incipient ferroelectrics in the vicinity of a quantum phase transition. Studies are based on some experimental findings on the low temperature dielectric behavior of some incipient ferroelectrics such as SrTiO$_3$, KTaO$_3$, EuTiO$_3$ etc. under various external perturbations. We have studied a minimal model in each case of pure quantum paraelectrics, its coupling with anti-ferromagnetic fluctuations, strain and disorder. Studies based on self-consistent mean-field approximations and scaling arguments, are capable of explaining many experimental findings and making various interesting predictions about the dielectric behavior of these materials. These systems are of displacive type i.e. phase transition in these systems is associated with a softening of a transverse optic mode. More microscopic scenario could be, a set of dipoles sitting at the center of each unit cells of these perovskite materials are interacting via long range dipolar interactions. As a result of the long range nature of the dipolar interaction the fluctuations along the longitudinal direction with respect to the wave vector are gaped out and the transverse mode fluctuations become the most relevant to describe the low temperature dielectric properties of these materials.

To explore the possible consequences of a quantum fluctuations in the low temperature dielectric behavior of these materials a semi-phenomenological Landau-Ginzburg theory is used. We restrict ourselves to an one component model to make our analysis simpler. Moreover anisotropy induced by the dipolar interaction in the transverse optic modes is neglected and a justification for the same is given in chapter 2. With this minimal model we are able to describe the effects of quantum
fluctuations in case of pure SrTiO$_3$, a prototype quantum paraelectric material. The same analysis is extended to predict its dielectric behavior when it is tuned to a quantum critical point. A prediction about the $1/T^2$ behavior in contrast to the usual Curie-Weiss behavior is made and verified by a recent experiment[25]. A schematic phase diagram is proposed to to classify various dielectric materials in a quantum phase transition point of view.

Our first chapter was devoted to set up a basic theoretical ground for discussing low temperature properties of quantum paraelectrics along with some predictions about the quantum critical behavior of them. With this background we focus on understanding more detail experimental observations on various quantum paraelectrics. In this context some interesting behavior of quantum critical SrTiO$_3$ is revealed in a recent spectroscopic experiment which signals a weak first order nature of the quantum phase transition in SrTiO$_3$. We assume such a behavior is a result of the coupling of the paraelectric fluctuations to strain fluctuations. Strain fluctuations are integrated out and it results a long range interaction among paraelectric fluctuations. In a pure mean field scenario, a weak first order transition occurs when the effective quartic coupling of the paraelectric action is negative and close to zero. In this case one can add a higher order term with positive coefficient in the paraelectric action and make some mean field prediction about the transition. We emphasis that in such a case one should consider fluctuation effects in the quartic coupling, namely four point vertices and show that fluctuation effects can stabilize the system without invoking higher order terms. The crucial role played by long range interaction mediated by the strain fluctuations in this process is also explained. A self consistent parquet approximation is used to take care of leading order fluctuation effects. The fact that the presence of the finite temperature restores the second-order nature of the transition near a quantum phase transition is also captured in this theory.

Next, we extend our theory to an incipient ferroelectric EuTiO$_3$ where ferroelectric fluctuations are coupled to anti-ferromagnetic fluctuations. We write an action where paraelectric fluctuations are coupled to anti-ferromagnetic fluctuations in a bipartite lattice and and in presence of non-zero magnetic field. We consider the case of coupled quantum criticality and its effect on the dielectric behavior of this system. A new power law behavior of the static dielectric constant, namely $T^{-4}$ variation, in presence of small non-zero magnetic field is predicted. It
is in contrast of the $1/T^2$ behavior of the quantum critical paraelectric and already got attentions of the experimental community[67].

Next section is an account of the effects of quenched disorder in quantum critical paraelectrics using a replica formalism. In this case the coupling between random $T_c$ type disorder with energy density is considered. Near quantum criticality in these systems, a bare power counting scheme predicts such disorder effects to be marginally relevant. However a classical replica formalism with broken replica symmetry at the vector level predicts inhomogeneous solutions in these system. Gaussian fluctuations around such solutions in case of classical phase transitions were studied earlier. In their static limit the renormalization of the coefficient of the Gaussian fluctuations due to such inhomogeneous solutions are found to independent of their sizes and a single instability was predicted. We consider the tunneling of such solutions in the quantum limit and consider a quantum phase transition in terms of the instability of Gaussian fluctuations around them. A broad power law distribution of the quantum critical points is predicted. Its consequences of the static dielectric behavior at finite temperature is also emphasized.

In conclusion, in this work the physics of ferroelectrics is put in a broad perspective. The effects of quantum critical points on finite temperature properties of certain dielectric systems are studied. Possible exponents of the power law behavior of static dielectric constant at finite temperature are predicted. The effects of disorder induced inhomogeneity and their dynamics at low temperature is described in a replica formalism. Some results are in accord with experiments. Many aspects of these works are quite general in context of quantum phase transitions and deserve further experimental and theoretical studies.