Synopsis

The standard model of particle physics which is based on the local gauge invariance of the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ has been extremely successful in describing the electromagnetic, weak and strong interactions between elementary particles. The theory has been verified to a high degree of accuracy in collider experiments such as the Large Electron-Positron Collider (LEP) at CERN in Europe and the Tevatron at Fermilab, USA. The other very profound characteristic of the standard model of particle physics is renormalizibility, which emerges because of its underlying quantum field theoretical description. However, although this theory gives a number of correct predictions, there are still certain issues which it is unable to explain, hence the standard model is considered a low energy description of some fundamental theory. The standard model of particle physics cannot explain the observed small neutrino mass and the peculiar mixing. It does not give any candidate for the dark matter of the Universe. It also fails to explain the observed matter-antimatter asymmetry of the Universe. In addition, one of its major theoretical drawback is the hierarchy or the naturalness problem. The Higgs particle which is an essential ingredient of the standard model is not stable under quantum corrections. There is no symmetry to protect its mass and hence the mass of the Higgs gets a quantum correction $\delta m_h \sim 10^{18}$ GeV, assuming the validity of the standard model up to the Planck scale. Beyond standard model physics such as supersymmetry gives a very natural solution to the hierarchy problem. In the minimal supersymmetric extension of the standard model, every standard model fermion/scalar is accompanied by its scalar/fermionic superpartner and hence the scalar and fermionic contributions mutually cancel each other, stabilizing the Higgs mass. Other than this, standard model does not give gauge coupling unification. It unifies electromagnetic and weak interaction, but fails to unify the electroweak and strong interactions. Also, it does not include gravity.

Apart from these drawbacks, the mass hierarchy between the standard model particles is itself a puzzle. In the standard model, the left and right handed fermions interact with the Higgs via the gauge invariant Yukawa Lagrangian and their masses are generated when the Higgs takes a non-zero vacuum expectation value. The nonzero vacuum expectation value of the Higgs field breaks the $SU(2)_L \times U(1)_Y$ symmetry of the standard model down to $U(1)_{em}$. The fermion masses in the standard model is determined by this nonzero vacuum expectation value and the Yukawa couplings. Although the mass generation mechanism is the same for all the standard model fermions, still there exist a $O(10^6)$ hierarchy between the top and electron masses. With the inclusion of neutrino mass, the hierarchy gets much enhanced. A series of outstanding experiments like solar and atmospheric neutrino experiments, KamLAND, K2K, MINOS provide information about the standard model neutrino mass splittings and its very peculiar mixing angles. Combined with cosmological bound specially from WMAP data, the sum of the light neu-
trino masses are bounded within 0.19 eV while the observed solar and atmospheric mass splitting are $\Delta m_{21} \sim 7.59 \times 10^{-5}$ eV$^2$ and $\Delta m_{23} \sim 10^{-3}$ eV$^2$ respectively. This extremely small neutrino masses ($< eV$) point towards a $10^{12}$ order mass hierarchy between the top quark and the neutrino. Unlike the mixing in the quark sector, in the leptonic sector two of the mixing angles $\theta_{12}$ and $\theta_{23}$ are quite large ($\sin^2 \theta_{12} \sim 0.32$, $\sin^2 \theta_{23} \sim 0.46$) while at present there is an upper bound on the third mixing angle $\theta_{13}$ as $\sin^2 \theta_{13} < 0.05$. The observed mixing angles are in very close agreement with the tribimaximal mixing pattern where the solar mixing angle is $\sin^2 \theta_{12} = 0.33$, reactor mixing angle $\sin^2 \theta_{13} = 0.0$ and the atmospheric mixing angle is maximal $\sin^2 \theta_{23} = 0.5$. The maximal mixing angle $\theta_{23}$ and $\theta_{13} = 0$ point towards a possible $\mu - \tau$ symmetry in the neutrino sector. As mentioned before, standard model of particle physics does not shed any light if there is any fundamental principle governing this extremely small neutrino masses as well as the peculiar mixing. It is possible to extend the standard model by introducing gauge singlet neutrinos and to explain the observed neutrino mass as a consequence of the Dirac type of Yukawa interaction between this gauge singlet neutrinos, lepton doublet and the Higgs. However to explain the eV-neutrino mass one eventually will need a Yukawa coupling which is $\mathcal{O}(10^{12})$ order of magnitude suppressed as compared to the top Yukawa coupling, thereby again leading to another fine-tuning problem. All of these above mentioned problems including the necessity for the "natural explanation" of the small neutrino masses and mixing set the motivation to look for beyond standard model physics scenario.

Going beyond the standard model, seesaw mechanism can explain small neutrino masses very naturally, without fine tuning of Yukawa couplings to extremely small values. Considering the standard model as an effective low energy description, the only dimension-5 operator allowed by the standard model gauge symmetry is $y^2 \frac{L}{M} L H H$. The dimension-5 operator involving the lepton doublets and the Higgs field is generated when the heavy modes of the fundamental theory get integrated out. After the electroweak symmetry breaking, this dimension-5 operator gives rise to the Majorana mass term $\frac{y^2 v^2}{M}$ of the standard model neutrino. Since suppressed by the mass scale of the integrated-out heavy modes $M$, eV neutrino masses can be very naturally obtained even with large value of the Yukawa coupling $y$. Seesaw mechanism in its simplest version is of three types, depending on the heavy states which has been integrated out. type-I seesaw requires additional standard model gauge singlet Majorana neutrino, while type-II and type-III seesaw require SU(2) triplet Higgs and fermionic field (SU(3)$_\text{C}$ singlet) with hypercharge $Y = 2$ and $Y = 0$ respectively. While the neutrino mass generation mechanism are identical for type-I and type-III seesaw, in type-II seesaw the neutrino mass $\left(\frac{y v^2}{M}\right)$ has an additional suppression due to the small lepton number violating coupling $\mu$. The seesaw mechanism is well-fitted in the framework of grand unified theories. The other seesaw mechanisms such as inverse seesaw and double seesaw require additional particles as well as symmetries to justify the appropriate neutrino mass matrix.

In [1] we have build a model on type-III seesaw and have studied its detail phe-
nomenology. The triplet fermions which transform as an adjoint representation of SU(2), contain two charged fermionic states ($\Sigma^\pm$) and one charge neutral Majorana fermionic states ($\Sigma^0$). Since the type-I and type-III seesaw use different SU(2)$_L \times$ U(1)$_Y$ representations as the heavy modes, they offer distinct phenomenology. The gauge singlet right handed neutrino field of type-I seesaw interacts with the lepton and Higgs via the Yukawa Lagrangian, while its interaction with the gauge bosons is suppressed by the standard model neutrino-gauge singlet right handed neutrino mixing. Compared to this, the SU(2)$_L$ triplet fermion interacts directly with the standard model gauge bosons through their kinetic term, as well as with the leptons and the Higgs via the Yukawa Lagrangian. Hence for the 100 GeV mass range, the triplet fermions can be produced copiously at LHC, opening up the possibility to test the seesaw at LHC. This 100 GeV triplet fermions can be accommodated within the SU(5) grand unified framework, where their SU(5) origin could be identified with 24$_F$ representation of SU(5). Non-observation of proton decay and successful unification with this 24$_F$ demand that the SU(2) triplet component of this 24$_F$ should be of the order of few hundred GeV. Since the standard model neutrino masses are $M_\nu \simeq -Y_{\Sigma}^2 M^{-1}Y_{\Sigma}v^2$, hence for triplet fermion mass $M = \mathcal{O}(10^2)$ GeV, the Yukawa coupling $Y_{\Sigma}$ between the triplet fermions-Higgs doublet-leptonic doublet gets constrained as $Y_{\Sigma} \sim 10^{-6}$ by the eV neutrino mass. We show that the large Yukawa coupling and few hundred GeV triplet fermions are still possible with the addition of another SU(2)$_L \times$ U(1)$_Y$ Higgs doublet to this existing setup [1].

In our model we have considered three sets of right handed triplet fermionic fields $\Sigma_i$, and one additional Higgs doublet $\Phi_2$. In addition, we also have introduced one discrete $Z_2$ symmetry, softly broken by the Higgs potential. The additional Higgs field $\Phi_2$ ($Z_2$ odd) has the same SU(2) and U(1)$_Y$ transformations as the standard model Higgs doublet $\Phi_1$ ($Z_2$ even), only differing in its $Z_2$ charge assignment. Hence in the Yukawa Lagrangian, the additional Higgs field $\Phi_2$ interacts only with the standard model leptons ($Z_2$ even) and the triplet fermions ($Z_2$ odd), whereas the standard model Higgs $\Phi_1$ interacts with all other standard model fermionic fields. Due to the very specific nature of the Yukawa Lagrangian, the standard model neutrino and the triplet fermionic neutral component mixing is governed by the vacuum expectation value $v'$ of the additional Higgs doublet. Hence small vacuum expectation value $v' \sim 10^{-4}$ GeV generates eV neutrino mass, even with large $\mathcal{O}(1)$ Yukawa coupling $Y_{\Sigma}$. In the charged lepton sector, the mixing between the standard model charged leptons and the triplet fermions is governed by the Yukawa coupling $Y_{\Sigma}$ and the VEV $v'$, however the standard model charged lepton masses are determined by the large vacuum expectation value $v \sim 100$ GeV of the standard model Higgs doublet. In this model the quark sector remains the same as the standard model and the quark masses are governed by the same vacuum expectation value $v$. The choice of the small vacuum expectation value of the additional Higgs field has a significant impact on determining the Higgs mass spectra and the mixing angle between the neutral Higgses. With two Higgs doublets the Higgs sector in our model is enriched with five physical degrees of freedom ($H^0, h^0, A^0, H^\pm$). Working within the framework of a
 softly broken $Z_2$ symmetry, the mass of the light Higgs $h^0$ is determined by the standard model Higgs vacuum expectation value ($v \sim 10^2$ GeV) as well as by the extent of the $Z_2$ symmetry breaking coupling $\lambda_5$, whereas all the other Higgs masses are governed by the standard model Higgs vacuum expectation value $v$. Hence, in our model it is possible to accommodate a light Higgs state $h^0$. However, the presence of the light Higgs does not violate the LEP bound, due to the vanishing $Z - Z - h^0$ coupling. Due to the order of magnitude difference between the two vacuum expectation values $v$ and $v'$, the mixing angle $\alpha$ between the two neutral Higgses $h^0$ and $H^0$ is proportional to the ratio of the two vacuum expectation values ($\tan \beta = \frac{v'}{v}$) and is extremely small $\tan 2\alpha \sim \tan \beta \sim 10^{-6}$.

As compared to the type-I seesaw, type-III seesaw offers much richer phenomenology due to its direct interactions with the leptons, Higgs and also with the gauge bosons. Triplet fermion production at the LHC is mostly governed by the gauge boson mediated partonic subprocesses. Once produced, the triplet fermion can decay to different final state particles such as to a lepton+Higgs or to a lepton+ gauge boson. In our model, due to the large Yukawa coupling $Y_S$ and small value of the mixing angle $\alpha$ as well as $\tan \beta$, the triplet fermions ($\Sigma^\pm, \Sigma^0$) decay predominantly into standard model leptons along with the neutral and charged Higgses $h^0, A^0, H^\pm$. The other decay modes where triplet fermions decay into a standard model lepton along with the neutral Higgs $H^0$ or the standard model gauge bosons is highly suppressed. The dominant decay of the triplet fermion into a standard model lepton and a Higgs $h^0, A^0, H^\pm$ is $10^{11}$ times larger compared to the one Higgs doublet type-III seesaw scenario. Another feature of our model is that it is possible to relate the neutrino phenomenology with the triplet fermions decay. In particular, the exact or approximate $\mu - \tau$ symmetry in the neutrino sector distinguishes among the different leptonic states when the triplet fermion decays into a standard model lepton and a Higgs. The $\mu - \tau$ symmetry in the neutrino sector provide equal opportunity to $\mu$ and $\tau$ states to be the leptonic final states, whereas it forbids the electron state $e$. In the Higgs sector, the different Higgs decay modes are governed by the Yukawa couplings and also by the small mixing angle $\alpha$ as well as $\tan \beta$. The neutral Higgs predominantly decays to $2b$ while the dominant decay mode for the charged Higgs $H^\pm$ is $H^\pm \rightarrow W^\pm h^0$. Other than this, a distinctive feature of our model is the displaced vertex of the Higgs $h^0$. Unlike the type-III seesaw with one Higgs doublet, in our model the triplet fermions do not have any displaced vertex. The type-III seesaw with two Higgs doublet can be verified at LHC via the different collider signatures which this model offers.

The observed data on solar and atmospheric neutrino mass splitting constraint the number of triplet fermion generation to be minimally two. However the R-parity violating supersymmetric framework enables a viable description of the neutrino mass and mixing even with one generation of triplet matter chiral superfield which has R-parity $-1$ [2]. R-parity which is a discrete symmetry is defined as $R_p = (-1)^{3(B-L)+2S}$ and has been implemented in the minimal supersymmetric extension of the standard model to forbid the baryon number $(\tilde{U}^c \tilde{D}^c \tilde{D}^c)$ and the lepton number violating $(\tilde{L} \tilde{L} \tilde{E}^c, \tilde{L} \tilde{Q} \tilde{D}^c)$ operators.
Non-observation of proton decay constraints the simultaneous presence of lepton and baryon number violation, however leaving some space for the individual presence of either of these two. To accommodate the Majorana mass term of the standard model neutrino, lepton number violation is required. Spontaneous violation of R-parity meets both ends, it generates neutrino mass and satisfies the proton decay constraint, as in this scheme, the R-parity violating operators are generated very selectively. In our model R-parity is spontaneously broken by the vacuum expectation value of the different sneutrino fields. As a consequence, only the lepton number violating bilinear operators are generated while working in the weak basis. Sticking to the framework of the perturbative renormalizable field theory, the baryon number violating operators \((\bar{U}c \bar{D}c \bar{D}c)\) would never be generated, hence naturally satisfying the proton decay constraint. Because of the R-parity violation, the standard model neutrinos \(\nu_i\) mix with the triplet fermion \(\Sigma^0\), as well as with the Higgsino \(\tilde{h}^0_{u,d}\) and gauginos \(\tilde{\lambda}^0_{3,6}\). Hence, in our model we have a \(8 \times 8\) color and charge neutral fermionic mass matrix. With one generation of the triplet matter chiral superfield and the R-parity violation, two of the standard model neutrino masses can be generated as a consequence of the conventional seesaw along with the gaugino seesaw, while the third neutrino still remains massless. Hence, in this scenario viable neutrino masses and mixings are possible to achieve. In addition, the standard model charged leptons \((l^\pm)\), triplet fermions \((\Sigma^\pm)\) and the charginos \((\tilde{\lambda}^\pm, \tilde{h}^\pm_{u,d})\) mixing is also determined by the different R-parity violating vacuum expectation values, as well as the different couplings of the superpotential. Hence, the charged lepton mass matrix is an extended \(6 \times 6\) matrix. In our model the spontaneous violation of R-parity is not associated with any global \(U(1)\) lepton number breaking. Hence, the spontaneous R-parity violation does not bring any problem of Majoron.

While the smallness of the neutrino mass can be explained via the seesaw mechanism, the very particular mixing of the standard model neutrinos can be well explained by invoking a suitable flavor symmetry. Among the widely used flavor symmetry groups, \(A_4\) and \(S_3\) are very promising ones. \(A_4\) is an alternating group where the group elements correspond to even permutation of four objects. This symmetry group has three different one dimensional \((1,1',1'')\) and two three dimensional irreducible representations, and has one \(Z_2\) and \(Z_3\) subgroups. The symmetry group \(A_4\) can be used to produce the tribimaximal mixing and viable neutrino mass splitting by introducing additional standard model gauge singlet Higgs fields, which transform as three as well as one dimensional irreducible representation of \(A_4\) \cite{3}. These gauge singlet Higgs fields which are charged under the flavor symmetry group are denoted as flavon. In our model \cite{3} we have two flavon fields \(\phi_{S,T}\) which transform as three dimensional irreducible representation of the group \(A_4\). In addition, we also have three other flavons \(\xi, \xi', \xi''\) which transform as \(1, 1', 1''\) respectively. The Lagrangian describing the Yukawa interaction between the different standard model leptons, Higgs and the flavons follows the effective field theoretical description. The different flavon fields take the vacuum expectation values, thereby resulting in a spontaneous breaking of the symmetry group \(A_4\). The \(A_4\) triplet field \(\phi_S\) can alone generate the
tribimaximal mixing in the neutrino sector if all the vacuum expectation values $v_i$ of its component fields are equal. However it gives the atmospheric mass splitting $\Delta m_{31}^2 = 0$, and hence is clearly incompatible with the neutrino oscillation data. To generate viable neutrino mass splittings in association with tribimaximal mixing, the one dimensional representations has to be included. Although the representation $1$ is the minimalistic choice to recover the correct mass, this particular choice ends up with a severe fine-tuning between the different parameters of the theory. Other than this, the normal hierarchy ($\Delta m_{31}^2 > 0$) between the standard model neutrino masses is the only allowed possibility. The fine-tuning between the parameters can be reduced by introducing additional one dimensional flavon fields $\xi'$ and $\xi''$. Along with the triplet $\phi_S$, the combination of the one dimensional flavon fields ($\xi', \xi''$) and ($\xi, \xi', \xi''$), and with certain relations between the different vacuum expectation values and Yukawa couplings generate tribimaximal mixing as well as viable mass splittings. In this set up both normal ($\Delta m_{31}^2 > 0$) and inverted ($\Delta m_{31}^2 < 0$) hierarchies are possible. Deviation from the particular relations between the different Higgs vacuum expectation values and Yukawa couplings will lead to deviation from tribimaximal mixing. In the charged lepton sector the diagonal charged lepton mass matrix emerges as a consequence of an additional discrete symmetry $Z_3$, as well as the vacuum alignment of the flavon field $\phi_T$.

The symmetry group $S_3$ is a permutation group of three objects and is the smallest non-abelian symmetry group. This group has two distinct one dimensional and one two dimensional irreducible representations, along with one $Z_3$ and three $Z_2$ subgroups. In [4] we have constructed a flavor model based on the symmetry group $S_3$, which reproduces the observed neutrino mass and mixing, as well as the standard model charged lepton mass hierarchy. We use two $SU(2)$ Higgs triplets ($\Delta$) with hypercharge $Y = 2$, arranged in a doublet of $S_3$, and the standard model singlet Higgs ($\phi_e, \xi$) which are also put as doublets of $S_3$. Due to the appropriate charge assignment under additional discrete symmetry groups $Z_4$ and $Z_3$, the flavon $\phi_e$ enters only in the charged lepton Yukawa Lagrangian, whereas the other flavon $\xi$ enters both in the neutrino as well as in the charged lepton Yukawa interaction. The Higgs triplets $\Delta$ and the flavon field $\xi$ take vacuum expectation value, and generate standard model neutrino masses. To reproduce the observed lepton masses and mixings, the symmetry group $S_3$ has to be broken such that the neutrino sector contains the exact/approximate $Z_2$ symmetry along the $\nu_\mu - \nu_\tau$ direction, while it is broken down maximally in the charged lepton sector. This particular feature is achieved by the vacuum alignments of the different Higgs fields $\Delta$, $\phi_e$ and $\xi$. Exact $\mu - \tau$ symmetry in the neutrino mass matrix is achieved as a consequence of the vacuum alignments $\langle \Delta_1 \rangle = \langle \Delta_2 \rangle$ and $\langle \xi_1 \rangle = \langle \xi_2 \rangle$, otherwise resulting in mildly broken $\mu - \tau$ symmetry. These vacuum alignments have been discussed in the scalar potential. The mild breaking $\mu - \tau$ symmetry opens up the possibility of CP violation in the leptonic sector. The charged lepton sector offers very tiny contribution to the physically observed PMNS mixing matrix, while the main contribution comes from the neutrino mixing matrix. In the neutrino sector both normal and inverted hierarchy are allowed possibilities. Since the Higgs triplet $\Delta$
interacts with the gauge bosons via their kinetic terms, they can be produced at the LHC and then can be traced via their subsequent decays. The doubly charged Higgs can decay to different states such as dileptons, gauge bosons, singly charged Higgs $H^\pm$. In our model the mixing between the two doubly charged Higgs is very closely related with the extent of the $\mu - \tau$ symmetry in the neutrino sector. In the exact $\mu - \tau$ limit the mixing angle $\theta$ between the two doubly charged Higgs is $\theta = \frac{\pi}{4}$, whereas mild breaking of the $\mu - \tau$ symmetry results in a mild deviation $\theta \sim \frac{\pi}{4}$. This close connection between the $\mu - \tau$ symmetry and the doubly charged mixing angle significantly effects the doubly charged Higgs-dileptonic vertices. In the exact $\mu - \tau$ limit, the vertex factors $H_{2}^{++} - \mu - \tau$ and $H_{2}^{++} - e - e$ are zero, hence the doubly charged Higgs $H_{2}^{++}$ never decays to $\mu^+ + \tau^+$ or to $2e^+$ states. Other than this, the non observation of the $e\mu$ and $e\tau$ states in the dileptonic decay of the doubly charged Higgs would possibly disfavor the inverted mass hierarchy of the standard model neutrino. We have very briefly commented about lepton flavor violation in our model.
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