Chapter 4

R Parity Violation and Neutrino Mass

4.1 Introduction

We have discussed the minimal supersymmetric standard model in chapter 1. The superpotential of the minimal supersymmetric standard model, which conserves a discrete $Z_2$ symmetry R-parity is

$$W_{MSSM} = Y_e \hat{H}_d \hat{L}^c + Y_d \hat{H}_d \hat{Q} \hat{D}^c - Y_u \hat{H}_u \hat{Q} \hat{U}^c + \mu \hat{H}_u \hat{H}_d. \quad (4.1)$$

R-parity or matter parity is defined as $R_p = (-1)^{3(B-L)+2S}$. For the standard model particles the charge is $+1$ and for the superpartners of the standard model particle it is $-1$. Apart from the above superpotential of the R-parity conserving minimal supersymmetric standard model, the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge invariance also allows the following R-parity violating superpotential

$$W_{\not R_p - MSSM} = -\epsilon \hat{H}_u \hat{L} + \lambda \hat{L} \hat{E}^c + \lambda' \hat{L} \hat{Q} \hat{D}^c + \lambda'' \hat{U} \hat{D}^c \hat{D}^c. \quad (4.2)$$

The 1st, 2nd and 3rd term of the above superpotential violate lepton number whereas the 4th term breaks baryon number conservation. The Majorana masses of the standard model neutrinos can be explained by the effective dimension-5 operator $\frac{LLHH}{M}$, which violate lepton number by two-units. Hence, the lepton number violation in the superpotential $W_{\not R_p - MSSM}$ opens up the possibility to generate non-zero neutrino mass [1–10]. This can be done through one loop [4,5] and two loop [6] diagrams generated via the lepton number breaking trilinear couplings $\lambda$ and $\lambda'$ (see Eq. (4.2)). Small neutrino masses can also be generated by the R-parity violating bilinear coupling $\hat{H}_u \hat{L}$ [1,2], through the neutrino-higgsino $\nu - \tilde{h}_u^0$ mixing. However these lepton and baryon number violating couplings of the superpotential $W_{\not R_p - MSSM}$ are severely constrained by non-observation of proton decay and data on heavy flavor physics from Belle and Babar [11]. In particular, the simultaneous presence of the lepton number violating $\lambda$, $\lambda'$ couplings and the baryon number
violating $\lambda''$ coupling are constrained as $\lambda''_{11k} \lambda''_{11k} \leq 10^{-24} (m_k/100\text{GeV})$. For other $\lambda'$ and $\lambda''$ couplings the bound is $\lambda'_{1} \lambda''_{1} \leq 10^{-9}$ [11]. Comparable bound exist on the product of $\lambda$ and $\lambda''$ couplings [12]. Once R-parity conservation is implemented, all the terms of $W_{R-MSSM}$ are forbidden and hence the minimal supersymmetric standard model does not suffer any proton decay constraint. However, only the absence of $\lambda''$ term is sufficient and minimalistic choice to avoid the proton decay constraint. Sticking to a renormalizable perturbation theory if R-parity is violated spontaneously, one can easily justify the absence of the baryon number violating $\lambda''$ operator in the superpotential $W_{R-MSSM}$, while at the same time the bilinear R-parity violating operator and trilinear R-parity violating operators are possible to generate. In this scenario R-parity is conserved in the superpotential. Once the sneutrino fields acquire vacuum expectation values, R-parity breaking terms are generated spontaneously [9, 10, 13–17]. In presence of additional singlet or triplet matter chiral fields, this provides a natural explanation for the origin of the R-parity and lepton number violating bilinear term $\epsilon$, without generating the baryon number violating $\lambda''$ term in the superpotential. Therefore, one can generate neutrino masses without running into problems with proton decay in this class of supersymmetric models.

There have been earlier attempts to construct spontaneous R-parity violating models within the MSSM gauge group [9, 10, 13, 14] and with the MSSM particle contents. In all these models the R-parity is broken spontaneously when the sneutrino acquires a VEV. This automatically breaks lepton number spontaneously. If lepton number is a global symmetry, this will give rise to a massless Goldstone called the Majoron [18]. All models which predict presence of Majoron are severely constrained. The phenomenology of the models with singlet Majoron has been studied in detail in [10, 14].

A possible way of avoiding the Majoron is by gauging the U(1) symmetry associated with lepton number such that the spontaneous R-parity and lepton number violation comes with the new gauge symmetry breaking. This gives rise to an additional neutral gauge boson and the phenomenology of these models have also been studied extensively in the literature [15, 16]. This idea has been used in a series of recent papers [17].

In this work we study spontaneous R-parity violation in the presence of a SU(2)$_L$ triplet $Y = 0$ matter chiral superfield, where we stick to the gauge group of the minimal supersymmetric standard model. Lepton number is broken explicitly in our model by the Majorana mass term of the heavy fermionic triplet, thereby circumventing the problem of the Majoron. We break R-parity spontaneously by giving vacuum expectation values to the 3 MSSM sneutrinos and the one additional sneutrino associated with the triplet which leads to the lepton-higgsino and lepton-gaugino mixing in addition to the conventional Yukawa driven neutrino-triplet neutrino mixing. This opens up the possibility of generating neutrino mass from a combination of the conventional type-III seesaw and the

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1It is possible to realize the other terms $\lambda$ and $\lambda'$ from the R-parity conserving MSSM superpotential only after redefinition of basis [1].
gaugino seesaw. We restrict ourself to just one additional SU(2)_{L} triplet Y = 0 matter chiral superfield, and explore the possibility of getting viable neutrino mass splitting and the mixing angles at tree level. With one generation of heavy triplet we get two massive neutrinos while the third state remains massless. Like the neutralino sector we also have R-parity conserving mixing between the standard model charged leptons and the heavy triplet charged lepton states. The spontaneous R-parity breaking brings about mixing between the charginos and the charged leptons, and hence modifies the chargino mass spectrum. In addition to the usual charged leptons, our model contains a pair of heavy charged fermions coming from the fermionic component of the triplet superfield. Another novel feature of our model comes from the fact that the additional triplet fermions and sfermions have direct gauge interactions. Hence they offer a much richer collider phenomenology. We discuss in brief about the possibility of detecting our model at colliders and the predicted R-parity violating signatures.

The main aspects of our spontaneous R-parity violating model are the following:

• We introduce one chiral superfield containing the triplets of SU(2)_{L} and with Y = 0.

• We have an explicit breaking of the lepton number due to the presence of the mass term of this chiral superfield in the superpotential. Therefore unlike as in [9, 10, 13, 14], the spontaneous breaking of R-parity does not create any Majoron in our model. Since we do not have any additional gauge symmetry, we also do not have any additional neutral gauge boson as in [15–17].

• Since we have only one additional triplet chiral superfield we have two massive neutrinos, with the lightest one remaining massless. One of the neutrinos get mass due to type-III seesaw and another due to gaugino seesaw. Combination of both gives rise to a neutrino mass matrix which is consistent with the current data.

• The triplet chiral superfield in our model modifies not only the neutrino-neutralino mass matrix, but also the charged lepton – chargino mass matrix. Being a triplet, it contains one neutral Majorana fermion Σ^{0}, two charged fermions Σ^{±}, one sneutrino $\tilde{\Sigma}^{0}$, and two charged sfermions $\tilde{\Sigma}^{±}$. Therefore, our neutral fermion mass matrix is a 8 × 8 matrix, giving mixing between the gauginos, higgsinos as well as the new TeV-scale neutral fermion $\Sigma^{0}$. Likewise, the new charged fermions $\Sigma^{±}$ will mix with the charginos and the charged leptons.

• There are thus new TeV-mass neutral and charged leptons and charged scalars in our model, which will have mixing with other MSSM particles. These could be probed at future colliders and could lead to a rich phenomenology. We give a very brief outline of the collider signatures in this work.

The chapter is organized as follows. In section 4.2 we describe the model and in section 4.3 we present the symmetry breaking analysis. In section 4.4, we discuss the
neutralino-neutrino mass matrix and in section 4.5 we discuss the chargino-charged lepton mass matrix. We concentrate on the neutrino phenomenology in section 4.6 and discuss the possibility of getting correct mass splittings and mixings even with one generation of SU(2)\textsubscript{L} triplet matter chiral supermultiplet. In section 4.7 we discuss about detecting our model in colliders and finally in section 4.8 we present our conclusion. Discussion on soft-supersymmetry breaking terms, gaugino-lepton-slepton mixing and the analytic expression of the low energy neutrino mass matrix have been presented in Appendix A, Appendix B and in Appendix C, respectively. In Appendix C we also analyze the constraints on the different sneutrino vacuum expectation values coming from the neutrino mass scale.

### 4.2 The Model

In this section, we discuss about our model. The superpotential of the supersymmetric standard model has given in Eq. (4.1) and in Eq. (4.2). In this work we will explore spontaneous R-parity violation in the presence of SU(2)\textsubscript{L} triplet \( Y = 0 \) matter chiral superfield. The matter chiral supermultiplets of the model are:

\[ \hat{Q} = \begin{pmatrix} \hat{U} \\ \hat{D} \end{pmatrix}, \hat{L} = \begin{pmatrix} \hat{\nu} \\ \hat{E} \end{pmatrix}, \hat{U}^c, \hat{D}^c \text{ and } \hat{E}^c \]

and the Higgs chiral supermultiplets are:

\[ \hat{H}_u = \begin{pmatrix} \hat{H}_u^+ \\ \hat{H}_u^0 \end{pmatrix}, \hat{H}_d = \begin{pmatrix} \hat{H}_d^- \\ \hat{H}_d^0 \end{pmatrix}. \]

In addition to the standard supermultiplet contents of the MSSM we introduce one SU(2)\textsubscript{L} triplet matter chiral supermultiplet \( \hat{\Sigma}_R \) with \( U(1)_Y \) hypercharge \( Y = 0 \). We represent \( \hat{\Sigma}_R \) as

\[ \hat{\Sigma}_R = \frac{1}{\sqrt{2}} \begin{pmatrix} \hat{\Sigma}_{0c}^R \\ \sqrt{2}\hat{\Sigma}_{+c}^R \\ \sqrt{2}\hat{\Sigma}_{-c}^R \end{pmatrix} \]

The three different chiral superfields in this multiplet are

\[ \hat{\Sigma}_{+c}^R = \hat{\Sigma}_R + \sqrt{2}\theta\hat{\Sigma}_{+c}^R + \theta\theta F_{\Sigma_{+c}}, \]

\[ \hat{\Sigma}_{-c}^R = \hat{\Sigma}_R - \sqrt{2}\theta\hat{\Sigma}_{-c}^R + \theta\theta F_{\Sigma_{-c}}, \]

\[ \hat{\Sigma}_{0c}^R = \hat{\Sigma}_R + \sqrt{2}\theta\hat{\Sigma}_{0c}^R + \theta\theta F_{\Sigma_{0c}}. \]
The SU(2) triplet fermions are \( \Sigma^+ c_R \), \( \Sigma^- c_R \) and \( \Sigma^0 c_R \) and their scalar superpartners are represented as \( \hat{\Sigma}^+ c_R \), \( \hat{\Sigma}^- c_R \) and \( \hat{\Sigma}^0 c_R \) respectively. \( F_{\Sigma^+ c_R}, F_{\Sigma^- c_R}, F_{\Sigma^0 c_R} \) represent the different auxiliary fields of the above mentioned multiplet. R-parity of \( \Sigma_R \) is \(-1\) where componentwise the fermions \( \Sigma^+ c_R \), \( \Sigma^- c_R \) and \( \Sigma^0 c_R \) have R-parity \(+1\) and their scalar superpartners have R-parity \(-1\). With these field contents, the R-parity conserving superpotential \( W \) of our model will be

\[
W = W_{\text{MSSM}} + W_\Sigma, \tag{4.7}
\]

where \( W_{\text{MSSM}} \) has already been written in Eq. (4.1) and \( W_\Sigma \) is given by

\[
W_\Sigma = -Y_\Sigma \bar{H}_u \sigma_2 \Sigma^+ c_R \nu_{L_i} + \frac{M}{2} Tr[\Sigma^+ c_R \Sigma^- c_R]. \tag{4.8}
\]

\( W_\Sigma \) is clearly R-parity conserving. The scalar fields \( \hat{\Sigma}^0 c_R \) and \( \nu_{L_i} \) are odd under R-parity. Hence in this model, R-parity would be spontaneously broken by the vacuum expectation values of these sneutrino fields. We will analyze the potential and spontaneous R-parity violation in the next section. On writing explicitly, one will get these following few terms from the above superpotential \( W_\Sigma \),

\[
\begin{align*}
W_\Sigma &= \frac{Y_\Sigma}{\sqrt{2}} \bar{H}_u \Sigma^0 c_R \nu_{L_i} + \frac{Y_\Sigma}{\sqrt{2}} \bar{H}_u \Sigma^0 c_R \nu_{L_i} - Y_\Sigma \bar{H}_u \Sigma^0 c_R \nu_{L_i} \\
&+ \frac{M}{2} \Sigma^0 c_R \Sigma^0 c_R + M \Sigma^+ c_R \Sigma^- c_R. \tag{4.9}
\end{align*}
\]

The kinetic terms of the \( \Sigma^0 c_R \) field is

\[
L^k_\Sigma = \int d^4 \theta Tr[\Sigma^0 c_R \Sigma^0 c_R]. \tag{4.10}
\]

The soft supersymmetry breaking Lagrangian of this model is

\[
L^{\text{soft}} = L^{\text{soft}}_{\text{MSSM}} + L^{\text{soft}}_\Sigma. \tag{4.11}
\]

For completeness we write the \( L^{\text{soft}}_{\text{MSSM}} \) Lagrangian in the Appendix A. \( L^{\text{soft}}_\Sigma \) contains the supersymmetry breaking terms corresponding to scalar \( \Sigma^0 c_R \) fields and is given by

\[
L^{\text{soft}}_\Sigma = -m^2_\Sigma Tr[\Sigma^0 c_R \Sigma^0 c_R] - (\tilde{m}^2 Tr[\Sigma^0 c_R \Sigma^0 c_R] + h.c) - (A_\Sigma H_u^T i \sigma_2 \Sigma^0 c_R \nu_{L_i} + h.c), \tag{4.12}
\]

where

\[
\Sigma^0 c_R = \frac{1}{\sqrt{2}} \begin{pmatrix} \Sigma^0 c_R \\ \sqrt{2} \Sigma^0 c_R \end{pmatrix}. \tag{4.13}
\]

We explicitly show in the Appendix A all the possible trilinear terms which will be generated from Eq. (4.9) and the interaction terms between gauginos and SU(2) triplet fermion and sfermion coming from Eq. (4.10). In the next section we analyze the neutral component of the potential and discuss spontaneous R-parity violation through \( \hat{\Sigma}^0 c_R \) and \( \nu_{L_i} \) vacuum expectation values.
4.3 Symmetry Breaking

In this section we write down the scalar potential which will be relevant to analyze the symmetry breaking of the Lagrangian. The potential is

$$V = V_F + V_D + V_{soft},$$

(4.14)

where $V_F$ and $V_D$, the contributions from different auxiliary components of the chiral superfield and different vector supermultiplets are given by

$$V_F = \sum_k F_k^* F_k$$

(4.15)

$$V_D = \frac{1}{2} \sum_a D^a D^a$$

(4.16)

respectively. Here the index $k$ denotes all possible auxiliary components of the matter chiral superfields whereas the index $a$ is the gauge index. The contribution from the soft supersymmetry breaking Lagrangian is given by $V_{soft}$. Below we write down the neutral component of the potential which would be relevant for our symmetry breaking analysis. The neutral component of the potential is given by

$$V_{\text{neutral}} = V^n_F + V^n_D + V^n_{soft},$$

(4.17)

where

$$V^n_F = |F^a_H^0|^2 + |F^a_H^0|^2 + |F^a_{\tilde{\nu}_L_i}|^2 + |F^a_{\tilde{\Sigma}_{cR}}|^2.$$  

(4.18)

The different $F_k$ are given by

$$F^a_{H^0_u} = \mu H^0_d - \sum_i Y_{\Sigma_i} \frac{\tilde{\Sigma}_{cR}}{\sqrt{2}} \tilde{\nu}_{L_i} + ..., $$

(4.19)

$$F^a_{H^0_d} = \mu H^0_u + ..., $$

(4.20)

$$F^a_{\tilde{\Sigma}_{cR}} = - \sum_i Y_{\Sigma_i} H^0_u \tilde{\nu}_{L_i} - M \tilde{\Sigma}_{cR} + ..., $$

(4.21)

$$F^a_{\tilde{\nu}_{L_i}} = - \frac{Y_{\Sigma_i}}{\sqrt{2}} H^0_u \tilde{\Sigma}_{cR} + ....$$  

(4.22)
In the above equations ... represents other terms which will not contribute to the neutral component of the potential. With all these $F_i$’s, $V^0_n$ is given by

$$V^0_n = |\mu H_0| + |\mu H_u| + \frac{1}{2} \left| \sum_i Y_{\Sigma_i} \sqrt{\Sigma_R} \tilde{\nu}_L \right|^2 + \frac{1}{2} \left| \sum_i Y_{\Sigma_i} H_u \tilde{\nu}_L \right|^2 + \frac{1}{2} \left| \sum_i Y_{\Sigma_i} H_u \tilde{\nu}_L \right|^2$$

$$- [\mu H_0' \left( \sum_i Y_{\Sigma_i} \tilde{\Sigma}_R \tilde{\nu}_L \right) + c.c] + |M|^2 \tilde{\Sigma}_R \tilde{\Sigma}_R + \left| \sum_i Y_{\Sigma_i} H_0 \tilde{\nu}_L (M \tilde{\Sigma}_R)^* + c.c \right| (4.23)$$

As we have three generation of leptons hence the generation index $i$ runs as $i=1,2,3$. The D term contribution of $V_{\text{neutral}}$ is given as

$$V_D = \frac{1}{8} (g^2 + g'^2)(|H_0|^2 - |H_u|^2 + \sum_i |\tilde{\nu}_L|^2)^2. \quad (4.25)$$

The component $\tilde{\Sigma}_R$ which has $Y = 0$ and the third component of the isospin $T_3 = 0$ does not contributes to $V^0_n$. The soft supersymmetry breaking contributions to the neutral part of the potential is given by $V^0_{\text{soft}}$ where

$$V^0_{\text{soft}} = -(bH_0' H_u + c.c) + m^2_{H_u} |H_u|^2 + m^2_{H_u} |H_u|^2$$

$$+ m^2 \tilde{\Sigma}_R \tilde{\Sigma}_R + \left[ \tilde{\Sigma}_R \tilde{H}_R \tilde{\Sigma}_R + c.c \right]$$

$$+ \sum_i m^2_{\tilde{\nu}_L} \tilde{\nu}_L + \left[ \sum_i \frac{A_{\Sigma_i}}{\sqrt{2}} H_0 \tilde{\Sigma}_R \tilde{\nu}_L + c.c \right]. \quad (4.26)$$

We represent the vacuum expectation values of $H_0$, $H_u$, $\tilde{\nu}_L$ and $\tilde{\Sigma}_R$ as $\langle H_0 \rangle = v_2$, $\langle H_u \rangle = v_1$, $\langle \tilde{\nu}_L \rangle = u_i$ and $\langle \tilde{\Sigma}_R \rangle = \tilde{u}$. We have considered a diagonal $m^2_{\tilde{\nu}_L}$ matrix. In terms of these vacuum expectation values the neutral component of the potential is

$$\langle V_{\text{neutral}} \rangle = (|\mu|^2 + m^2_{H_u}) |v_2|^2 + |\mu|^2 + m^2_{H_u}) |v_1|^2 - (bv_1 v_2 + c.c)$$

$$+ \frac{1}{8} (g^2 + g'^2)(|v_1|^2 - |v_2|^2 + \sum_i |u_i|^2)^2 + (|M|^2 + m^2_{\Sigma_i}) |\tilde{u}|^2$$

$$+ \sum_i m^2_{\tilde{\nu}_L} |u_i|^2 + m^2_{\tilde{\nu}_L} |u_i|^2 + \frac{1}{2} \sum_i Y_{\Sigma_i} |\tilde{\nu}_L|^2$$

$$+ \frac{1}{2} \sum_i Y_{\Sigma_i} |u_i|^2 + \left( \sum_i \frac{A_{\Sigma_i}}{\sqrt{2}} v_2 u_i \tilde{u} + c.c \right) - (\mu v_1 (\sum_i \frac{A_{\Sigma_i}}{\sqrt{2}} u_i)^* + c.c)$$

$$+ \left[ \sum_i \frac{Y_{\Sigma_i}}{\sqrt{2}} v_2 u_i (M \tilde{u})^* + c.c \right]. \quad (4.27)$$

For simplicity we assume all the vacuum expectation values and all the parameters are real. Hence $\langle V_{\text{neutral}} \rangle$ simplifies to

$$\langle V_{\text{neutral}} \rangle = (\mu^2 + m^2_{H_u}) v_2^2 + (\mu^2 + m^2_{H_u}) v_2^2 - 2b v_1 v_2 + \frac{1}{8} (g^2 + g'^2)(v_1^2 - v_2^2 + \sum_i u_i^2)^2$$

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\[(M^2 + m_{\Sigma_i}^2)\tilde{u}^2 + \sum_i m_i^2 v_i^2 + 2\tilde{b}^2 \tilde{u}^2 + \frac{1}{2} \left( \sum_i Y_{\Sigma_i} u_i \right) \tilde{u}^2 + \frac{1}{2} \sum_i (Y_{\Sigma_i})^2 \tilde{u}^2 v_i^2 + \frac{1}{2} \left( \sum_i Y_{\Sigma_i} u_i \right)^2 v_i^2 + \sqrt{2} \sum_i (A_{\Sigma_i} v_2 u_i \tilde{u} - \mu v_1 Y_{\Sigma_i} \tilde{u} u_i + Y_{\Sigma_i} M v_2 u_i \tilde{u}). \quad (4.28)\]

Minimizing \(V_{\text{neutral}}\) with respect to \(v_1, v_2, \tilde{u}\) and \(u_i\) we get the following four equations,

\[2(\mu^2 + m_{\tilde{H}_d}^2)v_1 - 2b v_2 + \frac{v_1}{2} (g^2 + g'^2)(v_1^2 - v_2^2 + \Sigma_i u_i^2) - \sqrt{2}\mu \tilde{u} \sum_i Y_{\Sigma_i} u_i = 0, \quad (4.29)\]

\[2(\mu^2 + m_{\tilde{H}_u}^2)v_2 - 2b v_1 - \frac{v_2}{2} (g^2 + g'^2)(v_1^2 - v_2^2 + \sum_i u_i^2) + v_2 \left( \sum_i Y_{\Sigma_i} u_i \right)^2 + \sqrt{2} \sum_i (A_{\Sigma_i} + Y_{\Sigma_i} M) u_i \tilde{u} = 0, \quad (4.30)\]

\[2(m_{\Sigma}^2 + 2M^2 + 2\tilde{b}^2)\tilde{u} + \left( \sum_i Y_{\Sigma_i} u_i \right) \tilde{u} + \sqrt{2} \sum_i (A_{\Sigma_i} v_2 u_i - \mu Y_{\Sigma_i} v_1 u_i + Y_{\Sigma_i} M v_2 u_i) + \sum_i (Y_{\Sigma_i})^2 \tilde{u} v_i^2 = 0 \quad (4.31)\]

\[\frac{u_i}{2} (g^2 + g'^2)(v_1^2 - v_2^2 + \sum_j u_j^2) + (v_2^2 + \tilde{u}^2)[Y_{\Sigma_i} v_2 u_i + Y_{\Sigma_i} \sum_{j \neq i} Y_{\Sigma_j} u_j] + \sqrt{2} A_{\Sigma_i} v_2 \tilde{u} + \sqrt{2}[Y_{\Sigma_i} M v_2 \tilde{u} - \mu Y_{\Sigma_i} v_1 \tilde{u}] + 2m_{\tilde{b}_i}^2 u_i = 0. \quad (4.32)\]

respectively. In the last equation the index \(i\) is not summed over. As mentioned before, since \(\tilde{\nu}_{L_i}\) and \(\tilde{\Sigma}_{R_i}\) have nontrivial R-parity, hence R-parity is spontaneously broken when \(\tilde{\nu}_{L_i}\) and \(\tilde{\Sigma}_{R_i}\) take vacuum expectation values. As a result of this spontaneous R-parity violation, the bilinear term \(LH_u\) which will contribute to the neutrino mass matrix is generated. We will discuss in detail about the neutralino-neutrino and chargino-charged lepton mass matrix in the next section.

The minimization conditions given in Eqs. (4.29)-(4.32) can be used to give constraints on the vacuum expectation values \(u_i\) and \(\tilde{u}\). In order to get such relations we drop the generation indices for the moment and consider \(u_i = u\) for simplicity. In this case \(Y_{\Sigma_i}, A_{\Sigma_i}\) and \(m_{\tilde{b}_i}^2\) contain no generation index and would be just three numbers. From the simplified version of the last two equations Eq. (4.31) and Eq. (4.32) it can then be shown that in the limit that \(u\) is small, the two R-parity breaking vacuum expectation values \(u\) and \(\tilde{u}\) are proportional to each other. Combining these two equations one gets

\[u^2 = \frac{u^2}{Y_{\Sigma}^2 v_2^2 + 2(m_{\Sigma}^2 + 2\tilde{b}^2 + 2M^2) \left[ \frac{1}{2} (g^2 + g'^2)(v_1^2 - v_2^2 + u^2) + 2m_{\tilde{b}_i}^2 + Y_{\Sigma}^2 v_2^2 \right]} \quad (4.33)\]
Hence for small $u$ which is demanded from the smallness of neutrino mass (discussed in the next section and in Appendix C), $\bar{u}$ will also be of the same order as $u$ unless there is a cancellation between the terms in the denominator. In this work we will stick to the possibility of small $u$ and $\bar{u}$. We will show in the next section that one needs $u \sim 10^{-3}$ GeV to explain the neutrino data. Hence $\bar{u}$ will also have to be $10^{-3}$ GeV. In the $u = 0$ limit, $\bar{u}$ would also be 0 and this is our usual R-parity conserving scenario.

### 4.4 Neutralino-Neutrino Mass Matrix

In this section we discuss the consequence of R-parity violation through the neutralino-neutrino mixing. It is well known [19] that R-parity violation results in mixing between the neutrino-neutralino states. In our model the neutrino sector is enlarged and includes both the standard model neutrino $\nu_L$, as well as the heavier neutrino state $\Sigma_R^c$, which is a component of SU(2) triplet superfield. Since R-parity is violated we get higgsino-neutralino mixing terms generated from the Kähler potential of the $\hat{\Sigma}$, in addition to the conventional R-parity conserving Dirac mass term $\frac{Y_{\Sigma_L}}{\sqrt{2}} v_u \Sigma^c_R \nu_L$. The R-parity breaking former two terms originated from the term $\frac{Y_{\Sigma_L}}{\sqrt{2}} H_u^c \Sigma^c_R \nu_L$, in Eq. (4.9), once the sneutrino fields $\tilde{\nu}_L$, and $\tilde{\Sigma}_R^c$ get vacuum expectation values. The third term also has the same origin and it is the conventional Dirac mass term in type I or type-III seesaw. In addition to the higgsino-neutrino mixing terms generated from the superpotential $W_{\Sigma}$, there would also be gaugino-neutralino mixing terms generated from the Kähler potential of the $L_i$ and $\Sigma_R^c$.

In the Appendix B we show explicitly the contributions coming from $W_{\Sigma}$ and the neutrino-sneutrino-gaugino terms originating from the kinetic term of the triplet superfield $\Sigma_R^c$ written down in Eq. (4.10). Here we write the color singlet neutral-fermion mass matrix of this model in the basis $\psi = (\lambda^0, \tilde{\lambda}^3, H_u^0, H_d^0, \Sigma_R^c, \nu_L, \nu_{L_2}, \nu_{L_3})^T$ where with one generation of $\Sigma_R^c$, the neutral fermion mass matrix is a $8 \times 8$ matrix. The mass term is given by

$$L_n = -\frac{1}{2} \psi^T M_n \psi + h.c. \quad (4.34)$$

where

$$M_n = \frac{1}{\sqrt{2}} \begin{pmatrix}
\sqrt{2} M^1 & 0 & -g' v_1 & g' v_2 & 0 & -g' u_1 & -g' u_2 & -g' u_3 \\
0 & \sqrt{2} M^2 & g v_1 & -g v_2 & 0 & g u_1 & g u_2 & g u_3 \\
-g' v_1 & g v_1 & 0 & -\sqrt{2} \mu & 0 & 0 & 0 & 0 \\
g' v_2 & -g v_2 & -\sqrt{2} \mu & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sum_{i} Y_{\Sigma_i} v_i & \sqrt{2} M & Y_{\Sigma_1} \tilde{u} & Y_{\Sigma_2} \tilde{u} & Y_{\Sigma_3} \tilde{u} \\
-g' u_1 & g u_1 & 0 & Y_{\Sigma_1} \tilde{u} & Y_{\Sigma_1} v_2 & 0 & 0 & 0 \\
-g' u_2 & g u_2 & 0 & Y_{\Sigma_2} \tilde{u} & Y_{\Sigma_2} v_2 & 0 & 0 & 0 \\
-g' u_3 & g u_3 & 0 & Y_{\Sigma_3} \tilde{u} & Y_{\Sigma_3} v_2 & 0 & 0 & 0 
\end{pmatrix} \quad (4.35)$$
Here $M^{1,2}$ are the soft supersymmetry breaking gaugino mass parameters (see Appendix B), whereas $M$ corresponds to the triplet-fermion bilinear term. We define the $3 \times 5$ matrix $m_D$ as

$$m_T^D = \frac{1}{\sqrt{2}} \begin{pmatrix} -g'u_1 & gu_1 & 0 & Y_{\Sigma_1} \hat{u} & Y_{\Sigma_1} v_2 \\ -g'u_2 & gu_2 & 0 & Y_{\Sigma_2} \hat{u} & Y_{\Sigma_2} v_2 \\ -g'u_3 & gu_3 & 0 & Y_{\Sigma_3} \hat{u} & Y_{\Sigma_3} v_2 \end{pmatrix}.$$  (4.36)

Defined in this way, the $8 \times 8$ neutral fermion mass matrix can be written as

$$M_n = \begin{pmatrix} M' & m_D \\ m_D^T & 0 \end{pmatrix},$$  (4.37)

where $M'$ represents the $5 \times 5$ matrix

$$M' = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} M^1 & 0 & -g'v_1 & g'v_2 & 0 \\ 0 & \sqrt{2} M^2 & gv_1 & -gv_2 & 0 \\ -g'v_1 & gv_1 & 0 & -\sqrt{2} \mu & 0 \\ g'v_2 & -gv_2 & -\sqrt{2} \mu & 0 & \sum_i Y_{\Sigma_i} u_i \\ 0 & 0 & 0 & 0 & \sqrt{2} M \end{pmatrix}.$$  (4.38)

The low energy neutrino mass would be generated once the neutralino and exotic triplet fermions get integrated out. Hence the low energy neutrino mass matrix $m_\nu$ is

$$m_\nu \sim -m_T^D M'^{-1} m_D.$$  (4.39)

For $M'$ in the TeV range, $m_\nu \sim 1$ eV demands that $m_D$ should be $10^{-3}$ GeV. If one takes $v_2 \sim 100$ GeV then this sets $Y_{\Sigma} \sim 10^{-5}$ and the scale of $u$ to be $10^{-3}$ GeV. Since in our model for small value of $u$, the $\hat{u}$ and $u$ are proportional to each other, hence we naturally get $\hat{u} \sim u \sim 10^{-3}$ GeV. We have discussed in more detail in Appendix C how the smallness of neutrino mass can restrict the vacuum expectation values $u_i$, $\hat{u}$ and the Yukawas $Y_{\Sigma_i}$. One can clearly see from Eq. (4.35) that in the $u = 0$ and $\hat{u} = 0$ limit, the gaugino-higgsino sector completely decouples from the standard model neutrino-exotic neutrino sector and the low energy neutrino mass would be governed via the usual type-III seesaw only. In the work presented in the previous chapter, we have taken a large Yukawa coupling $Y_{\Sigma} \sim 1$. In the present work since we do not extend the Higgs sector than the minimal supersymmetric standard model and in addition we choose a TeV scale triplet fermion mass parameter $M$, hence from the neutrino mass constraint the Yukawa is bounded to take such a small value $Y_{\Sigma} < 10^{-5}$. However, as of the two Higgs doublet type-III seesaw model presented in the previous chapter, one can further extend the Higgs sector of this model by two SU(2) doublet, so that one of the new Higgs contributes to the neutrino mass generation. In that case, one can choose a small VEV of the new Higgs doublet and take the large Yukawa.
4.5 Chargino-Charged Lepton Mass Matrix

Like the neutralino-neutrino mixing as discussed in the previous section, R-parity violation will also result in chargino-charged lepton mixing, which in our model is significantly different compared to the other existing models of spontaneous and explicit R-parity violation, because of the presence of extra heavy triplet charged fermionic states in our model. Like the enlarged neutrino sector ($\nu_L, \Sigma^0_c$) we have also an extended charged lepton sector. With one generation of $\Sigma^0_c$ we have two additional heavier triplet charged leptons $\Sigma^+_c$ and $\Sigma^-_c$ in our model, in addition to the standard model charged leptons. Hence we get mixing between the charginos and the standard model charged leptons as well as the heavier triplet charged leptons. The possible contributions to the different mixing terms would come from the superpotential as well as from the kinetic terms of the different superfields. Since we have written down explicitly the charginos-charged lepton-sneutrino interaction terms in Appendix, it is straightforward to see the contribution to the mass matrix coming from Eq. (B5), Eq. (B6) and Eq. (B8) once the $\tilde{\nu}_L$ states get vacuum expectation values. The chargino-charged lepton mass matrix in the basis

$$ L_c = -\psi_1^T M_c \psi_2 + h.c, $$

is

$$ M_c = \frac{1}{\sqrt{2}} \begin{pmatrix}
\sqrt{2}M^2 & \sqrt{2}gv_1 & \sqrt{2}gu_1 & \sqrt{2}gu_2 & \sqrt{2}gu_3 & \sqrt{2}g\tilde{u} \\
\sqrt{2}gv_2 & \sqrt{2}\mu & Y_{\Sigma^+_c} \tilde{u} & Y_{\Sigma^+_c} \tilde{u} & Y_{\Sigma^+_c} \tilde{u} & \sum_i \sqrt{2}Y_{\Sigma^+_i} u_i \\
0 & -\sqrt{2}Y_{e_1} u_1 & \sqrt{2}Y_{e_1} v_1 & 0 & 0 & 0 \\
0 & -\sqrt{2}Y_{e_2} u_2 & 0 & \sqrt{2}Y_{e_2} v_1 & 0 & 0 \\
0 & -\sqrt{2}Y_{e_3} u_3 & 0 & 0 & \sqrt{2}Y_{e_3} v_1 & 0 \\
-gu & 0 & \sqrt{2}Y_{\Sigma^+_1} v_2 & \sqrt{2}Y_{\Sigma^+_2} v_2 & \sqrt{2}Y_{\Sigma^+_3} v_2 & \sqrt{2}M
\end{pmatrix} \tag{4.41} $$

The chargino-charged lepton mass matrix would be diagonalized by bi-unitary transformation $M_c = T M^d_c S^T$.

4.6 Neutrino Mass and Mixing

R-parity violation can contribute significantly to the $3 \times 3$ standard model light neutrino mass matrix. In this section we concentrate on determining the neutrino mass square differences and the appropriate mixings. With only one generation of singlet/triplet heavy Majorana neutrino it is not possible to get viable neutrino mass square differences and mixings in the R-parity conserving type-I or type-III seesaw scenario. Since R-parity
is violated, we have neutrino-neutralino mixing apart from the conventional standard model neutrino-heavy neutrino mixing, which has significant effect in determining the low energy neutrino mass square differences and mixing angles of PMNS mixing matrix, through the gaugino and higgsino mass parameters $M^{1,2}$, $\mu$ and the R-parity violating sneutrino vacuum expectation values $u_i$ and $\tilde{u}$. Below we write the approximate $3 \times 3$ standard model neutrino mass matrix. Since $Y_{\Sigma_i}$, $u_i$ and $\tilde{u}$ are very small, all the terms which are proportional to $Y_{\Sigma_i}^2u_i^2$ and the terms $Y_{\Sigma_i}^3u_i\tilde{u}$ are neglected and we write down only the leading order terms. The exact analytical expression of the low energy neutrino mass matrix for our model has been given in the Appendix C. The approximate light neutrino mass matrix has the following form,

$$m_\nu \sim \frac{v_2^2}{2M} A + \frac{\alpha \mu}{2} B + \frac{\alpha \tilde{\mu} v_1}{2\sqrt{2}} C,$$  \hspace{1cm} (4.42)

where the matrix $A$, $B$ and $C$ respectively are,

$$A = \begin{pmatrix}
Y_{\Sigma_1}^2 & Y_{\Sigma_1}Y_{\Sigma_2} & Y_{\Sigma_1}Y_{\Sigma_3} \\
Y_{\Sigma_1}Y_{\Sigma_2} & Y_{\Sigma_2}^2 & Y_{\Sigma_2}Y_{\Sigma_3} \\
Y_{\Sigma_1}Y_{\Sigma_3} & Y_{\Sigma_2}Y_{\Sigma_3} & Y_{\Sigma_3}^2
\end{pmatrix},$$  \hspace{1cm} (4.43)

$$B = \begin{pmatrix}
u_1^2 & u_1 & u_1u_3 \\
u_1u_2 & u_2 & u_2u_3 \\
u_1u_3 & u_2u_3 & u_3
\end{pmatrix},$$  \hspace{1cm} (4.44)

$$C = \begin{pmatrix}
u_1Y_{\Sigma_1} & u_1Y_{\Sigma_2} + u_2Y_{\Sigma_1} & u_1Y_{\Sigma_3} + u_2Y_{\Sigma_2} + u_3Y_{\Sigma_1} \\
u_1Y_{\Sigma_2} + u_2Y_{\Sigma_1} & 2u_1Y_{\Sigma_2} & u_2Y_{\Sigma_3} + u_3Y_{\Sigma_2} \\
u_1Y_{\Sigma_3} + u_3Y_{\Sigma_1} & u_2Y_{\Sigma_3} + u_3Y_{\Sigma_2} & 2u_3Y_{\Sigma_3}
\end{pmatrix}.$$  \hspace{1cm} (4.45)

The parameter $\alpha$ depends on gaugino masses $M^{1,2}$, the higgsino mass parameter $\mu$ and two vacuum expectation values $v_{1,2}$ as follows

$$\alpha = \frac{(M_1g^2 + M_2g'^2)}{M_1M_2\mu - (M_1g^2 + M_2g'^2)v_1v_2}. \hspace{1cm} (4.46)$$

The 1st term in Eq. (4.42) which depends only on the Yukawa couplings $Y_{\Sigma_i}$, triplet fermion mass parameter $M$ and the vacuum expectation value $v_2$, is the conventional R-parity conserving type-I or type-III seesaw term. The 2nd and 3rd terms involve the gaugino mass parameters $M^{1,2}$, the higgsino mass parameter $\mu$ and R-parity violating vacuum expectation values $u_i$ and $\tilde{u}$. Hence the appearance of these two terms are undoubtedly the artifact of R-parity violation.

\(^2\)The standard charged lepton mass matrix which is obtained from Eq. (4.41) turns out to be almost diagonal and therefore the PMNS mixing comes almost entirely from $M_{\nu}$. 100
We next discuss the neutrino oscillation parameters, the three angles in the $U_{PMNS}$ mixing matrix and two mass square differences $\Delta m^2_{21}$ and $\Delta m^2_{31}$. Note that with the mass matrix $m_\nu$ given in Eq. (4.43), i.e. taking only the effect of triplet Yukawa contribution into account one would get the three following mass eigenvalues for the three light standard model neutrinos,

$$m_1 = 0, \quad m_2 = 0, \quad m_3 = \frac{v^2}{2M}(Y_{\Sigma_1}^2 + Y_{\Sigma_2}^2 + Y_{\Sigma_3}^2).$$

and the eigenvectors

$$\frac{1}{\sqrt{Y_{\Sigma_1}^2 + Y_{\Sigma_3}^2}} \begin{pmatrix} -Y_{\Sigma_3} \\ 0 \\ Y_{\Sigma_1} \end{pmatrix}, \quad \frac{1}{\sqrt{Y_{\Sigma_1}^2 + Y_{\Sigma_2}^2}} \begin{pmatrix} -Y_{\Sigma_2} \\ Y_{\Sigma_1} \\ 0 \end{pmatrix}, \quad \frac{1}{\sqrt{Y_{\Sigma_1}^2 + Y_{\Sigma_3}^2}} \begin{pmatrix} Y_{\Sigma_1} \\ Y_{\Sigma_2} \\ Y_{\Sigma_3} \end{pmatrix},$$

respectively. Clearly, two of the light neutrinos emerge as massless while the third one gets mass, which is in conflict with the low energy neutrino data. Similarly if one has only matrix $B$ which comes as a consequence of $R$-parity violation then one also would obtain only one light neutrino to be massive, in general determining only the largest atmospheric mass scale [20, 21]. However the simultaneous presence of the matrix $A$, $B$ and $C$ in Eq. (4.42) make a second eigenvalue non-zero, while the third one remains zero. With the choice $\tilde{u}$ as of the same order of $u$, the third term in Eq. (4.42) would be suppressed compared to the first two terms. Hence the simultaneous presence of the matrix $A$ and $B$ are very crucial to get both the solar and atmospheric mass splitting and the allowed oscillation parameters. Eigenvalues of the full $M_\nu$ given in Eq. (4.42) are

$$m_1 = 0, \quad m_{2,3} \sim \frac{1}{2}[W \mp \sqrt{W^2 - V}],$$

where

$$W = \frac{v^2}{2M} \sum_i Y_{\Sigma_i}^2 + \frac{\mu \alpha}{2} \sum_i u_i^2 + \frac{\tilde{u}v\alpha}{\sqrt{2}} \sum_i u_i Y_{\Sigma_i},$$

and

$$V = 4(\frac{v^2 \mu \alpha}{4M} - \frac{\tilde{u}^2 v^2 \alpha^2}{8})[Y_{\Sigma_1}^2 (u_2^2 + u_3^2) + Y_{\Sigma_2}^2 (u_3^2 + u_1^2) + Y_{\Sigma_3}^2 (u_1^2 + u_2^2) - 2(u_1 u_2 Y_{\Sigma_1} Y_{\Sigma_2} + u_1 u_3 Y_{\Sigma_1} Y_{\Sigma_3} + u_2 u_3 Y_{\Sigma_2} Y_{\Sigma_3})].$$

Similarly we have obtained approximate analytic expressions for the mixing matrix, however the expressions obtained are too complicated and hence we do not present them here. Instead we show in Tables 4.1 and Table 4.2 an example set of model parameters
\((M^{1,2}, M, \mu, Y_\Sigma, v_{1,2}, \tilde{u}, u_i)\) which give the experimentally allowed mass-square differences \(\Delta m_{21}^2\) and \(\Delta m_{31}^2\) and mixing angles \(\sin^2 \theta_{12}, \sin^2 \theta_{23}\) and \(\sin^2 \theta_{13}\), as well as the total neutrino mass \(m_t\). The values obtained for these neutrino parameters for the model points given in Tables 4.1 and 4.2 is shown in Table 4.3. We have presented these results assuming \(\Delta m_{31}^2 > 0\) (normal hierarchy).

<table>
<thead>
<tr>
<th>(M^1) (GeV)</th>
<th>(M^2) (GeV)</th>
<th>(M) (GeV)</th>
<th>(\mu) (GeV)</th>
<th>(Y_{\Sigma_1})</th>
<th>(Y_{\Sigma_2})</th>
<th>(Y_{\Sigma_3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>600</td>
<td>353.24</td>
<td>88.31</td>
<td>(5.62 \times 10^{-7})</td>
<td>(8.72 \times 10^{-7})</td>
<td>(3.84 \times 10^{-8})</td>
</tr>
</tbody>
</table>

Table 4.1: Sample point in the parameter space for the case of normal hierarchy. \(M^{1,2}\) is the gaugino mass parameter, \(\mu\) is higgsino mass parameter, \(M\) is triplet fermion mass parameter, \(Y_{\Sigma_1}, Y_{\Sigma_2}\) and \(Y_{\Sigma_3}\) correspond to the superpotential coupling between the standard model Lepton superfields \(\hat{L}_i\), SU(2) triplet superfield \(\hat{\Sigma}_0^c\) and Higgs superfield \(\hat{H}_u\).

<table>
<thead>
<tr>
<th>(v_1) (GeV)</th>
<th>(v_2) (GeV)</th>
<th>(\tilde{u}) (GeV)</th>
<th>(u_1) (GeV)</th>
<th>(u_2) (GeV)</th>
<th>(u_3) (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0</td>
<td>100.0</td>
<td>(5.74 \times 10^{-5})</td>
<td>(1.69 \times 10^{-5})</td>
<td>(9.55 \times 10^{-5})</td>
<td>(1.26 \times 10^{-4})</td>
</tr>
</tbody>
</table>

Table 4.2: Sample point in the parameter space for the case of normal hierarchy. \(v_{1,2}\) are the vacuum expectation values of \(H^0_{d,u}\) fields respectively, \(\langle \nu_{\mu_i} \rangle = u_i\) for \(i=1,2,3\) and \(\tilde{u}\) is the vacuum expectation value of triplet sneutrino state \(\tilde{\Sigma}_0^c\).

<table>
<thead>
<tr>
<th>(\Delta m_{21}^2) (eV(^2))</th>
<th>(\Delta m_{31}^2) (eV(^2))</th>
<th>(\sin^2 \theta_{12})</th>
<th>(\sin^2 \theta_{23})</th>
<th>(\sin^2 \theta_{13})</th>
<th>(m_t) (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(7.44 \times 10^{-5}) eV(^2)</td>
<td>(2.60 \times 10^{-3}) eV(^2)</td>
<td>0.33</td>
<td>0.507</td>
<td>4.34 \times 10^{-2}</td>
<td>10^{-2}</td>
</tr>
</tbody>
</table>

Table 4.3: Values for neutrino oscillation parameters for the input parameters specified in Table 4.1 and in Table 4.2.

At this point we would like to comment on the possibility of radiatively-induced neutrino mass generation in our model. In a generic R-parity violating MSSM, both the lepton number and baryon number violating operators are present. Apart from the tree level bilinear term \(\epsilon \hat{L} \hat{H}_u\) which mixes neutrino with higgsino, the trilinear lepton number violating operators \(\lambda \hat{L} \hat{E}^c, \lambda' \hat{L} \hat{Q} \hat{D}^c\) and also the bilinear operator \(\hat{L} \hat{H}_u\) contribute to the radiatively-induced neutrino mass generation [4–6]. In general there could be several loops governed by the \(\lambda, \lambda', \lambda''\) couplings. However in the spontaneous R-parity violating model, working in the weak basis we would only have \(\hat{L} \hat{H}_u\) operator coming from \(\hat{H}_u \hat{\Sigma}_R^c \hat{L}\) term in the superpotential, unless we do a basis redefinition. Similarly in the scalar potential we would obtain \(H_u \hat{L}\) coupling coming out from \(H_u \hat{\Sigma}_R^c \hat{L}\) term in Eq. (4.12). Hence we can have loops governed by \(A_{\Sigma} A_{\Sigma}\) couplings, like the \(BB\)
loop in Fig. 3 of [4]. For the sake of completeness we have presented the diagram in Fig. 4.1. The one loop corrected neutrino mass coming out from this diagram would be

\[ m_\nu \sim \frac{g^2 \tilde{\mu}^2}{64\pi^2 \cos^2 \beta} \frac{A_{\Sigma} A_{\Sigma'}}{m_\Sigma}. \]

In general for moderate values of \( \cos \beta \), if one chooses the average slepton mass \( \tilde{m} \) to be in the TeV order and the soft supersymmetry breaking coupling \( A_{\Sigma} \sim 10^2 \text{ GeV} \), then because \( \tilde{u} \sim 10^{-3} \text{ GeV} \), the contribution coming from this diagram would be roughly suppressed by a factor of \( 10^{-2} \) compared to the tree level neutrino mass. Similar conclusion can be drawn for the \( \epsilon A_{\Sigma} \) [4] loop induced neutrino mass, shown in Fig. 4.2. In our model the R-parity violating \( \lambda \) and \( \lambda' \) couplings do not get generated in the weak basis. However, from the R-parity conserving superpotential given in Eq. (4.1) it would be possible to generate \( \lambda \) and \( \lambda' \) couplings via mixings and only after transforming to a mass basis or after rotating away the bilinear R-parity violating term. Hence, in general for our model we would expect the contributions coming from the \( \lambda \lambda \) and \( \lambda \lambda' \) loops to be suppressed compared to the tree level neutrino masses. Apart from these different bilinear and trilinear radiatively induced neutrino mass generation, there could be another source of radiative neutrino mass, namely the non-universality in the slepton mass matrices. In the R-parity conserving limit the analysis would be same as of [22], only triplet fermions in our model are generating the Majorana sneutrino masses \( \tilde{\nu}_i \tilde{\nu}_j \). However, in this work we stick to the universality of the slepton masses, hence this kind of radiative neutrino mass generation will not play any non trivial role. Due to the RG running from the high scale to the low scale the universal soft supersymmetry breaking slepton masses could possibly get some off-diagonal contribution [23], which we do not address in this present work.

### 4.7 Collider Signature

In this section we discuss very briefly about the possibility of testing our model in collider experiments. Because R-parity is violated in our model there will be extra channels compared to the R-parity conserving minimal supersymmetric standard model, which carry the information about R-parity violation. As R-parity gets broken, the triplet neutral heavy lepton \( \Sigma^0_{Rc} \) and standard model neutrino \( \nu_{Li} \) mix with the neutral higgsino \( \tilde{H}^0_u \) and \( \tilde{H}^0_d \), with bino \( \lambda^0 \) and wino \( \lambda^3 \), in addition to the usual R-parity conserving Dirac mixing between them. Hence in the mass basis with one generation of heavy triplet fermion, there will be 5 neutralinos in our model. We adopt the convention where the neutralino state \( \tilde{\chi}^0_i \) are arranged according to the descending order of their mass and \( \tilde{\chi}^0_5 \) is the lightest neutralino. If one adopts gravity mediated supersymmetry breaking as the origin of soft supersymmetry breaking Lagrangian, then the lightest neutralino is in general the lightest supersymmetric particle (LSP). But as R-parity is broken, in any case LSP will not be stable [20,24]. For MSSM, the production mechanism of neutralinos and sneutrinos in colliders have been extensively discussed in [25,26]. Depending on the parameters, the lightest neutralino can be gaugino dominated or higgsino dominated.
In our model in addition to the standard model neutrinos we also have a heavy triplet neutral fermion $\Sigma^0_R$. Hence in this kind of model where heavy triplet fermions are present, the lightest neutralino can also be $\Sigma^0_R$ dominated. Here we present a very qualitative discussion on the possible neutralino, sneutrino, slepton and chargino decay modes.

- (A) Neutralino two body decay: As R-parity is violated, the lightest neutralino can decay through R-parity violating decay modes. It can decay via the two body decay mode $\tilde{\chi}_5^0 \to lW$ or $\tilde{\chi}_5^0 \to \nu Z$. Other heavier neutralinos can decay to the lighter neutralino state $\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 Z$, or through the decay modes $\tilde{\chi}_i^0 \to l^\pm W^\mp/\nu Z$. The gauge boson can decay leptonically or hadronically producing

$$\chi_i^0 \to l^\pm W^\mp \to l^\pm l^\mp + E_T,$$
$$\chi_i^0 \to l^\pm W^\mp \to l^\pm + 2j,$$
$$\chi_i^0 \to \nu Z \to l^\pm l^\mp + E_T,$$
$$\chi_i^0 \to \nu Z \to 2b + E_T.$$

- (B) Sneutrino and slepton decay: Because of R-parity violation the slepton can decay to a charged lepton and a neutrino $[27]$ via $\tilde{l} \to \nu l$. The sneutrino can also have the possible decay $\tilde{\nu} \to l^+ l^-$. In the explicit R-parity violating scenario, this interaction term between $\tilde{l} l^\pm l^\mp$ and $\bar{l} l \nu$ would have significant contribution from $\lambda LLE^C$. Here $l^\pm$, $\nu$ and $\tilde{l}$ all are in mass basis. In the spontaneous R-parity violating scenario these interactions are possible only after basis redefinition. The different contributions to the above mentioned interaction terms will come from the MSSM R-parity conserving term $\tilde{H}_d \tilde{L} \tilde{E}^c$ as well as from the kinetic terms of the different superfields after one goes to the mass basis.

- (C) Three body neutralino decay modes: The other possible decay modes for the neutralino are $\tilde{\chi}_i^0 \to \nu \tilde{\nu}$, $l^\pm l^\mp$, $\nu h$. If the lightest neutralino $\tilde{\chi}_i^0$ is the lightest supersymmetric particle, then the slepton or sneutrino would be virtual. The sneutrino or slepton can decay through the R-parity violating decay modes. Hence neutralino can have three body final states such as $\tilde{\chi}_i^0 \to \nu \tilde{\nu} \to \nu l^\pm l^\mp$, $\tilde{\chi}_i^0 \to l^\pm l^\mp \to l^\pm \nu l^\mp$.

Our special interest is in the case where the lightest neutralino state is significantly triplet fermion $\Sigma^0$ dominated. Besides the Yukawa interaction, the triplet fermion $\Sigma^\pm, \Sigma^0$ have gauge interactions. Hence they could be produced at a significant rate in a proton proton collider such as LHC through gauge interactions. The triplet fermions $\Sigma^\pm$ and $\Sigma^0$ could be produced via $pp \to W^\pm/Z \to \Sigma^\pm \Sigma^0/\Sigma^+$ channels apart from the Higgs mediated channels. In fact the production cross section of these triplet fermions should be more compared to the production cross section of the singlet neutrino dominated.

\footnote{By $\Sigma^\pm, \Sigma^0$ we mean chargino state $\tilde{\chi}^\pm$ or neutralino state $\tilde{\chi}^0$ significantly dominated by $\Sigma^\pm$ and $\Sigma^0$ respectively.}
neutralino states [7], as for the former case the triplet fermions have direct interactions with the gauge bosons. The decay channels for these triplet neutral fermions are as $\Sigma^0 \rightarrow \nu Z$, $\Sigma^0 \rightarrow l^\pm W^\mp$, $\Sigma^0 \rightarrow \nu h$ and also $\Sigma^0 \rightarrow \nu \tilde{\nu}^*$, $\tilde{l} \tilde{l}^*$ followed by R-parity violating subsequent decays of the slepton/sneutrino.

Apart from the neutralino sector in our model, the charged higgsino and charged winos mix with standard model leptons and heavy charged leptons $\Sigma^\pm$. Hence, just like in the neutralino sector, there will also be R-parity violating chargino decay. The chargino $\tilde{\chi}^\pm_i$ can decay into the following states, $\tilde{\chi}^\pm_i \rightarrow l^\pm Z$, $\nu W$, $\nu h^\pm$ and also to $\tilde{\chi}^\pm_i \rightarrow \tilde{l}\tilde{l} \rightarrow l^\pm l^\mp$. Just like as for the neutralinos, depending on the parameters, in this kind of model with heavy extra triplet fermions the lightest chargino could be as well $\Sigma^\pm$ dominated. Moreover, because of R-parity violation, there will be mixing between the slepton and charged Higgs and sneutrino and neutral Higgs. The sneutrino state $\tilde{\Sigma}^\pm$ can have the decays $\tilde{\Sigma}^\pm \rightarrow \nu l^\pm$, $\nu W^\pm$, $\tilde{l} Z$. Similarly, $\tilde{\Sigma}^0$ can decay into $\tilde{\Sigma}^0 \rightarrow l^+ l^-$. Apart from these above mentioned decay channels, some other possible decay channels are, $\tilde{\Sigma}^+ \rightarrow \tilde{d} u$, $\tilde{\Sigma}^0 \rightarrow \tilde{u} \bar{u}$, $\tilde{\Sigma}^+ \rightarrow \tilde{u} d$. For the R-parity conserving seesaw scenario, these decay modes will be totally absent, as there is no mixing between the triplet/singlet fermion and the higgsino/gauginos and mixing between sfermions and Higgs bosons.

We would also like to comment about the possibility of lepton flavor violation in our model. For the non-supersymmetric type-III seesaw, the reader can find detailed study on lepton flavor violation in [28]. Embedding the triplet fermions in a supersymmetric framework opens up many new diagrams which can contribute to lepton flavor violation, for example $\mu \rightarrow e\gamma$. In general the sneutrino, triplet sneutrino, different sleptons and the charginos or neutralinos can flow within the loop [23, 29–31]. In our model the R-parity violating effect comes very selectively. The main contribution comes via the bilinear R-parity violating terms which get only generated spontaneously. Hence as discussed before, we ignore $\lambda$ and $\lambda'$ dominated diagrams [30] and we expect that the lepton flavor violating contribution would be mainly governed by the soft supersymmetry breaking off-diagonal slepton mass contribution. Since we stick to the universal soft supersymmetry breaking slepton masses, only RG running might generate the off diagonal supersymmetry breaking slepton masses [23]. In the R-parity conserving limit the soft supersymmetry breaking slepton masses get a contribution $m^2_L \propto (3m^2_0 + A^2_0)(Y_\Sigma Y_\Sigma^T) \log(M_X/M_{\Sigma})^4$. For $m_0, A_0 \sim \text{TeV}$ and $Y_\Sigma \sim 10^{-6}$, $m^2_L \sim 10^{-6} \log(M_X/M_{\Sigma}) \text{GeV}^2$. The branching ratio would be roughly $\frac{\Gamma_{\mu \rightarrow e\gamma}}{\Gamma_{\mu \rightarrow e\gamma}} \sim \frac{m^2_L}{m^2} \tan^2 \beta$. However, because of extremely small Yukawa $Y_\Sigma \sim 10^{-6} - 10^{-8}$ our model would not violate the bound coming from $\mu \rightarrow e\gamma$.

\footnote{For simplicity we have taken $Y_\Sigma$ to be real.}
4.8 Conclusion

In this work we have explored the possibility of spontaneous R-parity violation in the context of basic MSSM gauge group and have explored its impact on the neutrino mass and mixing. Since the R-parity violating terms also break lepton number, spontaneous R-parity violation could potentially generate the massless mode Majoron. We avoid the problem of the Majoron by introducing explicit breaking of lepton number in the R-parity conserved part of the superpotential. We do this by adding a SU(2) triplet $Y = 0$ matter chiral superfield $\hat{\Sigma}_R$ in our model. The gauge invariant bilinear term in these triplet superfields provides explicit lepton number violation in our model. The superpartners of the standard model neutrino and the triplet heavy neutrino states acquire vacuum expectation values $u$ and $\tilde{u}$, thereby breaking R-parity spontaneously.

From the minimization condition of the scalar potential, we showed that in our model, the vacuum expectation values of the superpartner of the triplet heavy neutrino and the standard model neutrinos turn out to be proportional to each other. For the supersymmetry breaking soft masses in the TeV range, smallness of neutrino mass ($\sim eV$) constraints the standard model sneutrino vacuum expectation value $u \sim 10^{-3}$ GeV. Since for small $u$ the sneutrino vacuum expectation values $u$ and $\tilde{u}$ are proportional, hence in the absence of any fine tuned cancellation between the different soft parameters, one can expect that $\tilde{u}$ will be of the same order as $u$, i.e $10^{-3}$ GeV. We have analyzed the neutralino-neutrino mass matrix and have shown that the R-parity violation can have significant effect in determining the correct neutrino mass and mixing. Since neutrino experiments demand at least two massive neutrinos, we restrict ourselves to only one generation of $\hat{\Sigma}_R$ superfield. While in R-parity conserving type-III susy seesaw with only one generation of triplet neutrino state $\Sigma_0^c$, two of the standard model neutrinos turn out to be massless. In addition, if one invokes R-Parity violation then one among the massless states can be made massive. Alongwith the neutralino-neutrino mixing, we have chargino-charged lepton mixing in our model.

Finally we discussed the neutralino, chargino, slepton and sneutrino decay decays and the possible collider signature of this model. Because of R-parity violation the neutralino and chargino can decay through a number of R-parity violating decay channels alongwith the possible R-parity violating decays of sneutrinos and sleptons. In the context of our model, we have listed few of these channels.
Appendix

A: Soft supersymmetry breaking lagrangian of MSSM

The soft supersymmetry breaking Lagrangian of this model is given by,

\[ L_{\text{soft}} = L_{\text{soft MSSM}} + L_{\text{soft } \Sigma}, \]

(A1)

where \( L_{\text{soft MSSM}} \) has been written in Eq. (4.12) and the MSSM soft supersymmetry breaking lagrangian has the following form,

\[ -L_{\text{soft MSSM}} = (m_{\tilde{Q}}^2)^{ij} \tilde{Q}_i^j \tilde{Q}_j + (m_{\tilde{u}_c}^2)^{ij} \tilde{u}_c^j \tilde{u}_c^j + (m_{\tilde{d}_c}^2)^{ij} \tilde{d}_c^j \tilde{d}_c^j + (m_{\tilde{L}}^2)^{ij} \tilde{L}_i^j \tilde{L}_j \]

\[ + (m_{\tilde{e}_c}^2)^{ij} \tilde{e}_c^j \tilde{e}_c^j + m_{H_u}^2 H_u^i H_u + m_{H_d}^2 H_d^i H_d + (bH_u H_d + \text{H.c.}) \]

\[ + \left[ -A_{ij}^u H_u \tilde{Q}_i \tilde{u}_c^j + A_{ij}^d H_d \tilde{Q}_i \tilde{d}_c^j + A_{ij}^e H_d \tilde{L}_i \tilde{e}_c^j + H.c. \right] \]

\[ + \frac{1}{2} \left( M^3 \tilde{g} \tilde{g} + M^2 \tilde{\lambda} \tilde{\lambda} + M^1 \tilde{\lambda}^0 \tilde{\lambda}^0 + \text{H.c.} \right). \]

(A2)

where \( i \) and \( j \) are generation indices, \( m_{\tilde{Q}}^2, m_{\tilde{L}}^2 \) and other terms in the first two lines of the above equation represent the mass-square of different squarks, slepton, sneutrino\(^5\) and Higgs fields. In the third line the trilinear interaction terms have been written down and in the fourth line \( M^3, M^2 \) and \( M^1 \) are respectively the masses of the gluinos \( \tilde{g} \), winos \( \tilde{\lambda}^{1,2,3} \) and bino \( \tilde{\lambda}^0 \).

B: Gaugino-lepton-slepton mixing

In this section we write down explicitly all the interaction terms generated from \( W_{\Sigma} \), as well as the gaugino-triplet leptons-triplet sleptons interaction terms originating from \( L_k^c \).

As has already been discussed in section 2, \( W_{\Sigma} \) is given by

\[ W_{\Sigma} = -Y_{\Sigma} \hat{H}_u^T (i\sigma_2) \tilde{\Sigma}^c_R \tilde{L}_i + \frac{M}{2} Tr[\tilde{\Sigma}^c_R \tilde{\Sigma}^c_R]. \]  

(B1)

\[ W_{\Sigma} = \frac{Y_{\Sigma}}{\sqrt{2}} \hat{H}_u \tilde{\Sigma}_R^{0c} \tilde{\nu}_L_i + Y_{\Sigma} \hat{H}_u \tilde{\Sigma}_R^{0c} \tilde{e}_L_i - \frac{Y_{\Sigma}}{\sqrt{2}} \hat{H}_u \tilde{\Sigma}_R^{0c} \tilde{\nu}_L_i + Y_{\Sigma} \hat{H}_u \tilde{\Sigma}_R^{0c} \tilde{e}_L_i \]

\[ + \frac{M}{2} \tilde{\Sigma}_R^{0c} \tilde{\Sigma}_R^{0c} + M \tilde{\Sigma}_R^c \tilde{\Sigma}_R^c. \]  

(B2)

In Table. 4.4 we show all the trilinear interaction terms generated from \( Y_{\Sigma} \hat{H}_u \tilde{\Sigma}_R^c \tilde{L}_i \).

\(^5\)\( m_{\tilde{\nu}}^2 \) represents the mass square of the superpartner of the standard model neutrino and \( m_{\tilde{\nu}}^2 = m_{\tilde{L}}^2 \).

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The Kähler potential of the $\hat{\Sigma}_R$ field is given by

$$L^k_\Sigma = \int d^4\theta Tr[\hat{\Sigma}_R^\dagger e^{2gV} \hat{\Sigma}_R].$$  \hspace{1cm} (B3)$$

where $V$ represents the SU(2) vector supermultiplets. From the above kinetic term one will get the following gaugino-triplet fermion-triplet sfermion interactions

$$-L_{\Sigma-\hat{\Sigma}_-\hat{\lambda}} = \frac{g}{\sqrt{2}} ((\hat{\Sigma}_R^0)^* \lambda^3 \Sigma_{-R} - (\hat{\Sigma}_R^+)^* \hat{\lambda}^3 \Sigma_{+R}) + h.c.,$$  \hspace{1cm} (B4)$$

$$-L_{\Sigma-\hat{\Sigma}_+\hat{\lambda}} = \frac{g}{\sqrt{2}} ((\hat{\Sigma}_R^0)^* \lambda^+ \Sigma_{+R} - (\hat{\Sigma}_R^0)^* \hat{\lambda}^\dagger \Sigma_{+R}) + h.c.,$$  \hspace{1cm} (B5)$$

$$-L_{\Sigma-\hat{\Sigma}_-\hat{\lambda}} = \frac{g}{\sqrt{2}} ((\hat{\Sigma}_R^0)^* \lambda^- \Sigma_{-R} - (\hat{\Sigma}_R^0)^* \hat{\lambda}^{-\dagger} \Sigma_{-R}) + h.c.$$  \hspace{1cm} (B6)$$

Note that the mixing terms between the gauginos and triplet fermions $\lambda^+ \Sigma_{+R}^0$ and $\hat{\lambda}^- \Sigma_{-R}^0$ contribute to the color singlet charged fermion mass matrix Eq. (4.41) and these mixing terms are generated from Eq. (B5) and Eq. (B6) respectively, once $\Sigma_{+R}^0$ gets vacuum expectation value. In addition to these interaction terms between gauginos, triplet leptons and triplet sleptons, we also explicitly write down the gaugino-standard model lepton-slepton interaction terms which will be generated from the kinetic term of the $L_i$ superfields

$$L^k_L = \int d^4\theta \hat{L}_i^\dagger e^{2gV+2g'V'} \hat{L}_i.$$  \hspace{1cm} (B7)$$

$$L^k_L = -\frac{g}{\sqrt{2}} \hat{\nu}_L \bar{\lambda^3} \nu_{L_i} - \frac{g}{\sqrt{2}} \hat{l_i^*} \bar{\lambda^3} l_i^- - g \hat{\nu}_L \hat{\lambda^+} l_i^- - g l_i^* \hat{\lambda^-} \nu_{L_i}$$

$$+ \frac{g'}{\sqrt{2}} \hat{\nu}_L \bar{\lambda^0} \nu_{L_i} + g' \hat{\nu}_L \bar{\lambda^0} l_i^- + h.c.$$  \hspace{1cm} (B8)$$

Table 4.4: Trilinear interaction terms between standard model leptons/sleptons, Higgs/higgsinos and the SU(2) triplet fermions/sfermions. These interaction terms originate from the superpotential $W_\Sigma.$
Similar interaction terms would be generated from $\hat{E}^c$ kinetic term and kinetic terms of other superfields like the Higgs $\hat{H}_{u,d}$ and other quarks. Looking at Eq. (B8) it is clear that once the sneutrino fields $\tilde{\nu}_{Li}$ get vacuum expectation values the first and fifth term of the above equation would contribute to gaugino-neutrino mixing while the third term would contribute in the chargino-charged lepton mixing, as have already been shown in Eq. (4.35) and in Eq. (4.41).

C: The Neutrino Mass, $Y_\Sigma$ and $u$

In this section we discuss in detail how the smallness of neutrino mass restricts the order of magnitude of the Yukawa $Y_\Sigma$ and the sneutrino vacuum expectation value $u$. Below we write the analytical expression of the low energy neutrino mass of our model. The $3 \times 3$ neutrino mass matrix is

$$m_\nu \sim -m_D^T M'^{-1} m_D,$$

where $m_D$ and $M'$ have already been given in Eq. (4.36) and in Eq. (4.38) respectively. With the specified $m_D$ and $M'$, the $3 \times 3$ standard model neutrino mass matrix $m_\nu$ has the following form,

$$-m_\nu = 2v_2^2 \alpha_t A + 2M\mu^2 \alpha_t B + \sqrt{2} \alpha_t \tilde{\nu} v_1 M\mu C + M\alpha_v^2 \tilde{\nu}^2 A - \sqrt{2} \alpha_t \tilde{\nu} v_1 v_2 \sum_{i=1,2,3} u_i Y_\Sigma_i A - \alpha_t v_1 v_2 \mu F.$$

The matrix $A$, $B$ and $C$ have already been presented in Eq. (4.43), Eq. (4.44) and Eq. (4.45). The matrix $F$ has the following form,

$$F = \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{12} & F_{22} & F_{23} \\ F_{13} & F_{23} & F_{33} \end{pmatrix},$$

where the elements of $F$ are

$$F_{ij}(i \neq j) = Y_{\Sigma_i} Y_{\Sigma_j} (u_i^2 + u_j^2) + u_i u_j (Y_{\Sigma_i}^2 + Y_{\Sigma_j}^2) + u_k Y_{\Sigma_k} (u_i Y_{\Sigma_j} + u_j Y_{\Sigma_i}),$$

and

$$F_{ii} = 2Y_{\Sigma_i}^2 u_i^2 + 2u_i Y_{\Sigma_i} \sum_{k \neq i} Y_{\Sigma_k} u_k.$$

The indices $i, j, k$ in Eq. (C4) and the index $i$ in Eq. (C5) are not summed over. $\alpha_t$ has this following expression,

$$\alpha_t = \frac{(M_1 g^2 + M_2 g'^2)}{4MM_1 M_2 \mu^2 - 4M\mu(M_1 g^2 + M_2 g'^2)v_1 v_2 - (M_1 g^2 + M_2 g'^2)v_1^2 (\Sigma_i u_i Y_{\Sigma_i})^2},$$

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while the parameter $\alpha_1$ is

$$\alpha_1 = \frac{M_1 M_2 \mu^2 - (M_1 g^2 + M_2 g'^2)v_1 v_2 \mu}{4M M_1 M_2 \mu^2 - 4M \mu (M_1 g^2 + M_2 g'^2) v_1 v_2 - (M_1 g^2 + M_2 g'^2) v_1^2 (\Sigma_i u_i Y_i)^2}. \quad (C7)$$

Below we show how the choice of TeV scale gaugino masses and the triplet fermion mass $M$ dictate the sneutrino vacuum expectation value $u$ to be smaller than $10^{-3}$ GeV and the Yukawa $Y_i \leq 10^{-5}$ to have consistent small ($\leq eV$) standard model neutrino mass. We consider the following three cases and show that only Case (A) is viable.

Case (A): If one assumes $MM_1 M_2 \mu^2$ and $M(\mu (M_1 g^2 + M_2 g'^2) v_1 v_2 \gg (M_1 g^2 + M_2 g'^2) u_1^2 Y_1^2 v_1$, the first and second term dominate over the third one$^6$, in the denominator of Eq. (C6) and Eq. (C7). Then $\alpha_1$ and $\alpha_1$ simplify to

$$\alpha \sim \frac{(M_1 g^2 + M_2 g'^2)}{4M M_1 M_2 \mu^2} + \ldots \quad (C8)$$

and

$$\alpha_1 \sim \frac{1}{4M} + \ldots \quad (C9)$$

The neutrino mass matrix Eq. (C2) will have the following form,

$$m_\nu \sim \frac{v_1^2 Y_2^2}{2M} + \frac{u^2 (M_1 g^2 + M_2 g'^2)}{2M_1 M_2} + \sqrt{2} \bar{u} v_1 u Y_2 \frac{(M_1 g^2 + M_2 g'^2)}{4M_1 M_2 \mu^2} + \frac{v_1^2 \bar{u}^2 Y_2^2 (M_1 g^2 + M_2 g'^2)}{4M_1 M_2 \mu^2} - \sqrt{2} \bar{u} v_1^2 v_2 u Y_2 \frac{(M_1 g^2 + M_2 g'^2)}{4M M_1 M_2 \mu^2} - v_1 v_2 \mu u \frac{u^2 Y_2^2 (M_1 g^2 + M_2 g'^2)}{4M M_1 M_2 \mu^2}. \quad (C10)$$

The order of $Y_\Sigma$ and $u$ would be determined from the first two terms of the above equation respectively. The fourth, fifth and sixth terms of Eq. (C10) cannot determine the order of $Y_\Sigma$ and $u$. To show this with an example let us consider the contribution coming from the fourth term $\frac{u^2 (M_1 g^2 + M_2 g'^2) v_1^2 \bar{u}^2 Y_2^2}{MM_1 M_2 \mu^2}$. Contribution of the order of 1 eV from this term demands $Y_\Sigma^2 \bar{u}^2 \sim 10^{-4}$GeV$^2$ for $M^{1.2}$ in the TeV range, $\mu \sim 10^2$ GeV and $v_1$ in the GeV range$^7$. But if one assumes that $Y_\Sigma \sim 1$ and $\bar{u}^2 \sim 10^{-4}$GeV$^2$ then we get the contribution from the first term of Eq. (C10) $\frac{v_1^2 \bar{u}^2 Y_2^2}{M} \gg 1$eV, for $M \sim$ TeV and $v_2 \sim 100$ GeV. The choice $Y_\Sigma^2 \sim 10^{-4}$ and $\bar{u} \sim 1$ GeV also leads to $\frac{v_1^2 \bar{u}^2 Y_2^2}{M} \gg 1$eV for $v_2 \sim 10^2$ GeV. The other option is to have $Y_\Sigma^2 \sim 10^{-12}$ and $\bar{u}^2 \sim 10^8$GeV$^2$ so that $Y_\Sigma^2 \bar{u}^2 \sim 10^{-4}$GeV$^2$. But to satisfy this choice one needs $10^7$ order of magnitude hierarchy between the two vacuum expectation values $\bar{u}$ and $u$$^8$. This in turn demands acute hierarchy between the different soft supersymmetry

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$^6$Most likely between the first and second terms, the first term would have larger value for the choice of TeV scale gaugino mass.

$^7$As an example $v_1 \sim 10$ GeV.

$^8$As the scale of $u$ is fixed from the second term of Eq. (C10) and $u < 10^{-3}$ GeV.
breaking mass terms in Eq. (4.33). Similarly, $Y_S$ and $u$ could also not be fixed from fifth and sixth terms of Eq. (C10). Hence, we fix $Y_S$ and $u$ from the first two terms only. The third term is larger than the fourth, fifth and sixth terms, but will still be subdominant compared to the first two terms. The contribution from the first three terms to the low energy neutrino mass matrix is

$$m_\nu \sim \frac{v_3^2 Y_S^2}{2M} + \frac{u^2 (M_1 g^2 + M_2 g'^2)}{2M_1 M_2} + \sqrt{2} \bar{u} v_1 u Y_S (M_1 g^2 + M_2 g'^2).$$

(C11)

It is straightforward to see from the first term of this approximate expression, that $m_\nu \leq 1 \text{eV}$ demands $Y_S^2 \leq 10^{-10}$ for $M$ in the TeV range and $v_2 \sim 100 \text{ GeV}$. On the other hand for $M_{1,2}$ also in the TeV range, the bound on sneutrino vacuum expectation value $u$ comes from the second term as $u^2 \leq 10^{-6} \text{ GeV}^2$. For the choice of $\bar{u} \sim u$, which is a natural consequence of the scalar potential minimization conditions, one can check that the third term in Eq. (C11) would be much smaller compared to the second term because of the presence of an additional suppression factor $Y_S$. Hence, neutrino masses demand that $Y_S \leq 10^{-5}$ and $u$ and $\bar{u} \leq 10^{-3}$ GeV. One can explicitly check that for this above mentioned $Y_S$, $u$ and $\bar{u}$ the contributions coming from 4th, 5th and 6th terms of Eq. (C10) are smaller compared to the 1st three terms.

Case (B): If one assumes $M M_1 M_2 \mu^2 \ll (M_1 g^2 + M_2 g'^2) u^2 Y_S^2 v_1^2$, which is possible to achieve only for large value of the vacuum expectation value $u$ and Yukawa $Y_S$ 10, so that in the denominator of Eq. (C7) the third term dominates over the first two. Then the last term in Eq. (C2) contributes as $\sim \frac{v_1 v_2 \mu Y_S^2 \mu^2}{v_1^2 v_1^2} \sim \frac{\mu^2}{v_1^2} \gg 1 \text{eV}$ for $\mu$, $v_2$ and $v_1$ in the hundred GeV and GeV range. Therefore, this limit is not a viable option.

Case(C): We consider the last possibility $M M_1 M_2 \mu^2 \sim (M_1 g^2 + M_2 g'^2) u^2 Y_S^2 v_1^2$, which also demands large Yukawas and large vacuum expectation value $u$. If one considers $M_{1,2}$ and $M$ in the TeV range and $\mu$, $v_2$ in the hundred GeV range, then demanding that the first term in Eq. (C2) should be smaller than $\text{eV}$ results in the bound $Y_S^2 \leq 10^{-10}$. Hence, in order to satisfy $M M_1 M_2 \mu^2 \sim (M_1 g^2 + M_2 g'^2) u^2 Y_S^2 v_1^2$, one needs $u \geq 10^9 \text{ GeV}$ for $v_1$ in the GeV range. For such large values of $u$, the second term in Eq. (C2) will give a very large contribution to the neutrino mass. This case is therefore also ruled out by the neutrino data.

Hence Case (B) and Case (C) which demand large $u$ and $Y_S$ are clearly inconsistent with the neutrino mass scale, and the only allowed possibility is Case (A). This case requires $u \leq 10^{-3} \text{ GeV}$ and $Y_S \leq 10^{-5}$, for the gaugino and triplet fermion masses $M_{1,2}$ and $M$ in the TeV range. As for small $u$, $\bar{u}$ and $u$ are proportional to each other, one obtains $\bar{u} \sim 10^{-3} \text{ GeV}$ as well.

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9As the third term $\propto \bar{u} u Y_S$ and from the 1st two terms $Y_S$ and $u$ will turn out to be small.

10To satisfy this condition, the combination of $Y_S$ and $u$ has to be such that $u^2 Y_S^2 \gg 10^8 \text{ GeV}^2$ for $M_{1,2}, M \sim \text{TeV}$, $\mu$ in hundred GeV and $v_1$ in GeV range.
Bibliography


Figure 4.1: The $A_{\Sigma}A_{\Sigma}$ loop-generated neutrino mass. The cross on the internal solid line represents the majorana mass for the neutralino and the blob on the dashed line represents the mixing between sneutrino and the neutral Higgs.

Figure 4.2: Neutrino Majorana mass generated by $\epsilon A_{\Sigma}$ loop.