CHAPTER 5

PLASMA CLOSURE VELOCITY MEASUREMENT IN HIGH POWER ELECTRON BEAM DIODE

5.1 SPACE CHARGE LIMITED ELECTRON FLOW IN A PLANAR DIODE

For shorter pulse duration <100 ns and at the comparatively low current density ~ 10 A/cm$^2$ electron flow remains unipolar [14]. However at the higher current density greater than few hundreds of A/cm$^2$ electron flow becomes bipolar [15]. Our observation can be described by a model for the bipolar space-charge limited flow in the presence of the plasma expanding from the cathode and anode surfaces. The anode plasma could be generated by either melting and subsequent evaporation of the anode material or by electron stimulated desorption of the contaminants on the anode surface [30]. The charge neutralizations of the electrons by the ions allow ~ 1.86 times the current to flow as compared to single species Child–Langmuir with the limiting electron current independent of the ion mass [6]. The combination of a very few ions and of a secondary emission of 1/42.8 for hydrogen causes the diode to be in space-charge limited bipolar flow [30]. Both the anode and cathode plasmas are assumed to expand with equal velocity $v$.

The diode current density $j$ at time $t$ during the pulse, and the total current $I$ are given by the Child Langmuir law. For a plane parallel diode consisting of a cathode of radius ‘$r$’ and a AK gap ‘$d$’ the $j$ and $I$ are given by [6],

$$j(t) = 1.86 \times 4 \varepsilon \frac{e}{m_e} \left( \frac{2e}{m_e} \right)^{1/2} \frac{V(t)^{1/2}}{(d-2vt)^2} = 1.86 \times 2.33 \times 10^{-6} \frac{V(t)^{1/2}}{(d-2vt)^2},$$

and

$$I(t) = \int_0^t j(t) \, dt.$$
\[ I(t) = j(t)\pi r^2 = 1.86 \times 2.33 \times 10^{-6} \frac{\pi r^2 V(t)^{1/2}}{(d - 2vt)^2}, \quad (5.2) \]

where \( V \) is the applied voltage, \( t \) is the time during pulse at which \( j \) and \( I \) are measured; \( e \) and \( m_e \) are electron charge and mass and \( \varepsilon_0 \) is the free space permittivity.

In fact, it could be some time delay in the anode plasma formation with respect to the cathode plasma generation [11]. A delay of 10 ns has been measured experimentally between cathode and anode plasma generation [11]. In that case Eqs. (5.1) and (5.2) should be modified with respect to the value of \((d - 2vt)\).

The diode impedance \( Z_d \) is given by,

\[ Z_d(t) = \frac{V(t)}{I(t)} = 7.3 \left( \frac{d - 2vt}{r^2} \right)^2, \quad (V \text{ in } MV). \quad (5.3) \]

The perveance expression for the electron flow in the planner region of the diode can be defined by

\[ P_{\text{planar}} = \frac{I(t)}{V(t)^{1/2}} = 1.86 \times 2.33 \times 10^{-6} \frac{\pi r^3}{(d - 2vt)^2}. \quad (5.4) \]

A perveance expression for the cathode surface must also include the effect of electron flow from the cathode circumferential edge. Contribution due to edge can be accounted for by using Langmuir Compton equation [2, 40] for cylindrically symmetric space charge limited electron flow. Edge effects in finite area diodes may significantly increase the value of space-charge limited current relative to the prediction of 1D Child–Langmuir Law [40, 33]. Edge contribution is particularly important for \( r/d < 1 \) and can be neglected for \( r/d \gg 1 \).

\[ P_{\text{edge}} = \frac{14.66 \times 10^{-6} \pi r}{8 \frac{d}{\alpha r^2}} \quad (5.5) \]

where \( \alpha = \ln(d/\nu r) - 0.4[\ln(d/\nu r)]^2 + 0.0917[\ln(d/\nu r)]^3 - 0.0142[\ln(d/\nu r)]^4 + \ldots \).
The perveance of the total bipolar electron flow from the cathode edge and face is equal to the sum of two components. Thus the diode perveance in the generalized form can be expressed as

\[ P_{\text{diode}} = P_{\text{planar}} + P_{\text{edge}}. \]  

(5.6)

Thus, the cathode emission area, the effective diode separation, and the beam envelope are the only parameters that can affect the diode perveance. These interpretations are strictly valid only in the nonrelativistic limit. But the error associated will be small if the electron kinetic energy is less than 500 keV [40]

5.2 PLASMA EXPANSION VELOCITY MEASUREMENT IN A HIGH POWER PLANAR DIODE

To measure the electrode plasma expansion velocity of the IREB planar diode, beam generation experiments were carried out at 18 mm, 25 mm and 31 mm AK gap respectively. For 18 mm AK gap a Tantalum disk has been used as an anode. Also for this case an inductance has been added to the charging circuit of the Blumlein line. In the case of the added inductance to the Blumlein circuit, the slower rise time reduces the prepulse voltage from 32% to \( \leq 10\% \). For other experiments beam generation studies has been carried out in the presence of prepulse and a SS plate has been used as an anode.

Fig. 5.1 (a) shows the diode voltage and current waveform for 18 mm AK gap. The diode peak voltage and current obtained in this experiment were 270 kV and 33 kA, respectively. Electron beam diode time varying impedance and perveance values were calculated using the voltage and current waveforms. The starting point or the zero time for perveance calculations was taken when the main voltage pulse is zero. Fig 5.1 (b) shows corresponding impedance and perveance derived from the diode voltage and current waveform. One can see that the diode impedance decreases with time showing impedance collapse. The diode perveance increases
FIG. 5.1 (a) Diode voltage and current waveform for 18 mm AK gap. (b) The temporal behavior of the diode impedance and perveance for 18 mm AK gap. Continuous line represents theoretical perveance.
FIG. 5.2 (a) Diode voltage and current waveform for 25 mm AK gap. (b) The temporal behavior of the diode impedance and perveance for 25 mm AK gap. Continuous line represents theoretical perveance.
FIG. 5.3 (a) Diode voltage and current waveform for 31 mm AK gap. (b) The temporal behavior of the diode impedance and perveance for 31 mm AK gap. Continuous line represents theoretical perveance.
rapidly after ~ 110 ns showing gap closure. The best fit for the theoretical model [Eq. (5.6)]
was obtained assuming the plasma expansion velocity to be 6.5 cm/μs. The peak current
density was \( j = 936 \text{ A/cm}^2 \).

The diode voltage and current waveforms for 25 mm AK gap is shown in Fig. 5.2 (a). The
diode peak voltage and current obtained in this experiment were 280 kV and 18 kA,
respectively. The experimental impedance and perveance derived from the diode voltage and
current is shown in Fig. 5.2 (b). The diode perveance increases rapidly after ~ 130 ns showing
gap closure. In this case the best fit for the theoretical model [Eq. (5.6)] was obtained
assuming the plasma expansion velocity to be 7.3 cm/μs. The peak current density was \( j = 515 \text{ A/cm}^2 \).

The diode voltage and current waveforms for 31 mm AK gap is shown in Fig. 5.3 (a). The
diode peak voltage and current obtained were 270 kV and 14 kA, respectively. Fig 5.3 (b)
displays the time varying impedance and perveance. As can be seen in Fig 5.3 that the diode
impedance decreases and perveance increases with time. The best fit for the theoretical model
was obtained assuming the plasma expansion velocity to be 9.5 cm/μs. The peak current
density in this case was \( j = 401 \text{ A/cm}^2 \). Such a high plasma expansion velocity has also been
observed with carbon nanotube cathode [112] with a high current density \( j = 309 \text{ A/cm}^2 \) (the
plasma velocity reported \( v = 9.1 \text{ cm/μs} \)). The plasma expansion velocity for 31 mm AK gap is
higher than the 25 mm AK gap, even though the current density is less for 31 mm AK gap.
The peak current density for 31 mm AK gap has been calculated using the actual cathode
diameter. But usually a large percentage of the cathode fails to take part in the emission
process. So if we account for the actual emission area, the current density for 31 mm AK gap
could be higher than the 25 mm AK gap, resulting in higher plasma expansion velocity. But
the increase in the plasma velocity with the AK gap cannot be explained only by the surface ratio argument. It may also be possible that the higher electric field at lower AK gap is slowing one of the two plasmas.

So the electron beam emission mechanism in a high power planar diode can be explained by the bipolar space-charge limited flow model. Summary of the beam generation experiments for three different AK gaps are shown in Table I.

Table I. Results of the experiment with 67 mm diameter cathode showing diode voltage and current for various AK gaps.

<table>
<thead>
<tr>
<th>Anode-Cathode Gap (mm)</th>
<th>Diode Voltage (kV)</th>
<th>Diode Current (kA)</th>
<th>Electrode Plasma Expansion Velocity (cm/μs)</th>
<th>Current Density (A/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>270</td>
<td>33</td>
<td>6.5</td>
<td>936</td>
</tr>
<tr>
<td>25</td>
<td>280</td>
<td>18</td>
<td>7.3</td>
<td>515</td>
</tr>
<tr>
<td>31</td>
<td>270</td>
<td>14</td>
<td>9.5</td>
<td>401</td>
</tr>
</tbody>
</table>

One can see from the Table I that as we increase the AK gap the diode current and the current density reduces but the electrode plasma expansion velocity increases.

5.3 PLASMA EXPANSION VELOCITY MEASUREMENT IN A HIGH POWER PLANAR DIODE WITH A DIELECTRIC CATHODE HOLDER

Plasma closure velocity has been measured when the planar electron beam diode has been operated with a dielectric cathode holder. Experimental setup used has been described in the Section 4.5.2. Electron beam diode time varying impedance and perveance values were calculated using the voltage and current waveforms. The starting point or the zero time for
perveance calculations was taken when the main voltage pulse is zero. The experimental impedance and perveance derived from the diode voltage and current for 18-mm AK gap and 35-mm Perspex insulator are shown in Fig. 5.4. The best fit for the theoretical model [Equation (5.6)] was obtained assuming the plasma expansion velocity to be 5.1 cm/µs. The peak current density was $j = 760$ A/cm². The plasma expansion velocity deduced from the fit is valid over only one part of the shot and really approximate. It is clear from the Fig. 5.4 that, till 70 ns, the experimental and the theoretical perveance are not matching but after that perveance match nicely. With the increase of the Marx generator voltage diode current increases upto ~40 kA but the diode voltage remains almost same. Fig 5.5 shows impedance and perveance with a higher Marx generator voltage. In this case the best fit for the theoretical model [Equation (5.6)] was obtained assuming the plasma expansion velocity to be 5.7 cm/µs. The peak current density was $j = 1$ kA/cm². The plasma velocity increases with the increase of the Marx generator voltage because of the increase in the current density.

FIG. 5.4 The temporal behavior of the diode impedance and perveance for 18 mm AK gap and 35 mm Perspex insulator. Continuous line represents theoretical perveance.
The temporal behavior of the diode impedance and perveance for 25 mm AK gap were shown in Fig. 5.6. It is clear from the Fig. 5.6 that, till 70 ns, the experimental and the theoretical perveance are not matching, but after that it matches nicely. In this case the plasma expand at a higher speed of 7.9 cm/μs with \( j = 439 \text{ A/cm}^2 \).

With the increase of the Marx generator voltage both the diode current and voltage increases. Fig 5.7 shows corresponding impedance and perveance derived from the diode voltage and current waveform for 25 mm AK gap with a higher Marx generator voltage. In this case the best fit for the theoretical model [Equation (5.6)] was obtained assuming the plasma expansion velocity to be 8.1 cm/μs. The peak current density was \( j = 573 \text{ A/cm}^2 \). One can see that for all the cases, the plasma expansion velocity increases with the AK gap. It may be possible that the higher electric field at lower AK gap is slowing one of the two electrode plasmas.
FIG. 5.6 The temporal behavior of the diode impedance and perveance for 25 mm AK gap and 40 mm Perspex insulator. Continuous line represents theoretical perveance.

FIG. 5.7 The temporal behavior of the diode impedance and perveance for 25 mm AK gap and 40 mm Perspex insulator. Continuous line represents theoretical perveance.
5.4 SPACE CHARGE LIMITED ELECTRON FLOW IN A CYLINDRICAL DIODE

For cylindrical diode in the idealized case of 1D electron flow between long coaxial cylinders
the space charge limited electron current per unit length \( j_{1D} \) at some radial position \( r \) between
two coaxial cylinders is given by the Langmuir-Blodgett law [94].

\[
\begin{align*}
  j_{1D} &= \frac{8\pi \varepsilon_0}{9} \left( \frac{2e}{m_e} \right)^{1/2} \frac{V^{3/2}}{r \beta^2}, \\
  \beta &= \mu - 2\frac{\mu^3}{5} + \frac{11\mu^4}{120} - \frac{47\mu^5}{3300} + \cdots 
\end{align*}
\]

where \( V \) is the potential, and \( \beta = f\left(\frac{r}{r_c}\right) \) is a function expressed by the infinite series,

\[
\beta = \mu - 2\frac{\mu^3}{5} + \frac{11\mu^4}{120} - \frac{47\mu^5}{3300} + \cdots 
\]

with \( \mu = \ln \frac{r}{r_c} \).

Here \( r_c \) is the cathode radius, \( e \) is the electron charge, \( m_e \) is the electron mass, and \( \varepsilon_0 \) is the free
space permittivity.

The Langmuir-Blodgett law has been extended to two dimensions by performing 2D particle
in cell simulations [95]. But the results obtained using these simulations were limited to low
voltage and current regime (few kilovolts and few Amps). Approximate analytical solutions
for the space-charge limited current in 1D and 2D cylindrical diodes are also have been
calculated by various authors [96, 97].

Our observation can be described by a simple model for the electron space-charge limited
current in the presence of the plasma expanding from the cathode and anode surfaces. The
anode plasma is formed from the desorbed and ionized gases [11]. Also during the positive
half of the prepulse voltage, the explosive emission plasma can be generated at the anode
surface. Both the anode and cathode plasmas are assumed to expand with equal velocity \( v_r \). In
this case the time-dependent cathode and anode radius becomes $r_c - v_r t$ and $r_a + v_r t$ respectively. Thus, the value of $\mu$ in Eq. (5.8) is replaced by $\mu = \ln \frac{r_a + v_r t}{r_c - v_r t}$ and $\beta$ becomes time dependent parameter. Then the time dependent Langmuir-Blodgett law can be written as

$$j_{di}(t) = \frac{8\pi e_o}{9} \left( \frac{2e}{m_e} \right)^{1/2} \frac{V^{3/2}}{(r_c + v_r t)\beta(t)^2},$$  \hspace{1cm} (5.9)$$

if the cathode width is $h$ then the time dependent $P(t)$ of the cylindrical diode reads:

$$P(t) = \frac{8\pi e_o}{9} h \left( \frac{2e}{m_e} \right)^{1/2} \frac{1}{(r_c + v_r t)\beta(t)^2},$$  \hspace{1cm} (5.10)$$

In fact, it could be some time delay in the anode plasma formation with respect to the cathode plasma generation [11]. In that case Eqs. (5.9) and (5.10) should be modified with respect to the value of $r_c$.

It was shown that in the Space Charge Limited (SCL) regime many of the results from planar diodes provide reasonably good estimates for cylindrical diodes [98]. Also when the AK gap distance $d_c$ is very small compared with the cathode radius, $r_c \gg d_c$ the approximate analytical solutions for SCL current density [96] becomes proportional to $\frac{V^{3/2}}{d^2}$. So if $d(t)$ is the time dependent AK gap the approximate value of $P(t)$ reads:

$$P(t) \propto \frac{1}{d(t)^2},$$  \hspace{1cm} (5.11)$$

The proportionality constant is determined by the initial AK gap and the measured perveance at the time of the initial rise in the diode voltage [65].
5.5 PLASMA EXPANSION VELOCITY MEASUREMENT IN A HIGH POWER CYLINDRICAL DIODE

To study the plasma expansion and impedance collapse in a high power cylindrical diode, the experiments were carried out in three phases. The experimental setup used is described in the Section 3.3. In phase I the cathode inner diameter was set to 12.3 cm with the radial AK gap of 1.85 cm. The diode chamber was evacuated using a diffusion pump backed by a rotary pump. The vacuum in the diode chamber was $\leq 6 \times 10^{-5}$ mbar. In the phase II experiment, the cathode inner diameter was set to 11.9 cm with the radial AK gap of 1.65 cm. In phase III experiment, the cathode inner diameter was set to 11 cm with the radial AK gap of 1.2 cm.

The diode voltage and current waveforms in the phase I is shown in Fig. 4.17 (a) and Fig. 4.17 (b). The diode peak voltage and current obtained in this experiment was 340 kV and 30 kA, respectively, at 340 kV Marx generator voltage. Electron beam diode time varying impedance and perveance values were calculated using the voltage and current waveforms. The starting point or the zero time for perveance calculations was taken when the main voltage pulse is zero. The experimental impedance and perveance derived from the diode voltage and current is shown in Fig. 5.8. The best fit for the theoretical model [Eq. (5.10)] was obtained assuming the plasma expansion velocity to be $5 \, cm/\mu s$.

In phase II the best fit for the theoretical model was obtained assuming the plasma expansion velocity to be $5.3 \, cm/\mu s$ [Fig. 4.18]. This plasma velocity measurement may not be very accurate because of the modification in the diode geometry by the expansion of the prepulse generated plasma prior to the main voltage pulse.

In phase III experiment the best fit for $\varphi_d = 250$ kV was obtained assuming the plasma expansion velocity to be $3.4 \, cm/\mu s$ [Fig. 4.19]. Where $\varphi_d$ is the Marx generator voltage. Again
this plasma velocity measurement may not be very accurate because of the expansion of the prepulse generated plasma prior to the main voltage pulse.

The time dependence of the diode gap is calculated using Eq. (5.11). The temporal behaviors of the diode gaps are shown in Fig. 5.9. One can see that in the initial stage of the Phase I and Phase III the plasma expansion velocity is very fast but after around 40 ns, the plasma expansion becomes relatively slow. In the case of Phase II experiment the plasma expansion velocity is very fast for the entire pulse duration [Fig. 5.9 (b)].
FIG. 5.9 (a) The temporal behavior of the diode gap in Phase I and Phase III experiment. (b) The temporal behavior of the diode gap in Phase II experiment.
The closure rate calculated from Eq. 5.11 overestimates the gap closure rate since any increase in measured current is attributed to a decrease in the AK gap due to plasma expansion across the gap and does not take into account the possibility of radial plasma expansion [65]. Equation (5.11) also neglects the effect of electron flow from the cathode circumferential edge and the cathode edge contribution.

5.6 CONCLUSIONS

Electron beam emission mechanism can be explained by the bipolar space-charge limited flow model. The time varying electron beam diode impedance and perveance were measured for 18, 25 and 31 mm AK gaps. It was found that the diode impedance collapse due to plasma expansion from the cathode and anode surfaces. For 31 mm AK gap the anode and cathode plasmas expand at 9.5 cm/μs toward each other. Plasma expansion velocity decreases for 25 mm AK gap.

Electrode plasma expansion velocity has been measured when the planar diode is operated with a dielectric cathode holder to suppress prepulse. It was found that for 18-mm AK gap, the anode and cathode plasmas expand at 5.1 cm/μs toward each other. Plasma velocity increases with the current density. Plasma expansion velocity increases for 25-mm AK gap.

Cylindrical electron beam diode perveance was measured for 1.85, 1.65, and 1.2 cm AK gaps. The anode and cathode plasma expansion velocities were measured using the perveance data. For 1.85 cm radial AK gap the anode and cathode plasmas expand at 5 cm/μs toward each other. Plasma expansion velocity decreases for 1.2 cm AK gap.

The time dependence of the diode gap is calculated for cylindrical diode. It was found that in the initial stage of the 1.85 cm and 1.2 cm accelerating gap the plasma expansion velocity is
very fast but after around 40 ns, the plasma expansion becomes relatively slow. In the case of 1.65 cm diode gap the plasma expansion velocity is very fast for the entire pulse duration. It was found that almost for all the cases, the plasma expansion velocity increases with the AK gap. It may be possible that the higher electric field at lower AK gap is slowing one of the two electrode plasmas.

The faster plasma expansion velocity obtained for larger anode cathode gaps is related to smaller number of the cathode plasma spots and respectively to larger current densities for larger anode-cathode gaps. This results in larger plasma density and temperature and, respectively, to larger plasma expansion velocity.