Chapter 5

Prediction of multiplicity distribution in p+p collisions at LHC energies

5.1 Introduction

The Large Hadron Collider (LHC) at CERN is designed for carrying out p+p collisions at √s = 14 TeV [1, 2, 3, 4]. Collisions at these unprecedented high energies will provide opportunities for new physics [1, 2, 3, 4]. In order to fully exploit the enormous physics potential it is important to have a complete understanding of the reaction mechanism. The particle multiplicity distributions, one of the first measurements to be made at LHC, will be used to test various particle production models based on different physics mechanism and also provide constrains on model features. Some of these models are based on string fragmentation mechanism [5] and some are based on Pomeron exchange [6].

In this chapter, we first make a compilation of the existing data on the average charged particle multiplicity (at midrapidity and full rapidity range) and charged particle pseudorapidity distribution as a function of √s. Then we judiciously extrapolate the measurements to obtain prediction of ⟨Nch⟩ (for midrapidity and full rapidity range), charged particle multiplicity distributions and dNch/dη distributions at LHC energies of √s = 10 and 14 TeV. These results are also compared to calculations from PYTHIA [5] and PHOJET [6] Monte Carlo models. Energy dependence of the average number of particles produced in p+p(¯p) collisions can be used to distinguish various models [7]. A statistical model [8] and some hydrodynamical models [9] predict a dependence of ⟨N⟩ ∼ s^{1/4}, whereas multiperipheral models [10] and Feynman’s scaling [11] lead to ⟨N⟩ ∼ ln(s). Such a dependence on ln(s) and its higher powers is also predicted by Regge-Mueller model [12]. Arguments based on simple phase space
considerations [13] however predict a power law dependence where $\langle N \rangle \sim s^{1/3}$. In this work we show that for the $\langle N_{ch} \rangle$ data available at various values of $\sqrt{s}$ below 2 TeV, one cannot distinguish between the logarithmic and power law dependence. However the measurements at the LHC energies will provide a clear answer. The pseudorapidity distributions on the other hand is found to be described by a form which resembles a generalized Fermi distribution. Such distributions have been used to explain the pseudorapidity distributions of produced particles in hadronic collisions at ISR [14] and heavy-ion collisions at RHIC [15]. At present already some data are available on $dN_{ch}/d\eta$ [16] through the lower energy LHC runs. These will be compared with present predictions based on the extrapolation scheme proposed.

The compiled experimental data presented in this chapter corresponds to Non-Single Diffractive (NSD) events for minimum bias triggers. The charged particle data were corrected for secondary interactions, gamma conversions, short lived decays ($K^0_S$, $\Lambda$), reconstruction efficiency and acceptance effects by the experiments. To match the experimental conditions, the model simulations presented are also corrected for short lived decays. A transverse momentum ($p_T$) cut of greater than 100 MeV/c are usually used in realistic experimental conditions, as for example in ALICE experiment at LHC. At LHC, while most of the experiments will have mid-rapidity measurements of charged particle multiplicity, ALICE experiment has the possibility to measure the distributions over $-5.0 < \eta < 3.5$ range [1]. The CMS and ATLAS experiments have a more limited coverage of $|\eta| < 2.5$ units [2, 3].

The model results presented here are from PYTHIA using version 6.4 (ATLAS tuned) and those from PHOJET with version 1.12 (with default settings). Simulations using PYTHIA and PHOJET suggest, a $6 \pm 2\%$ change on the charged hadron multiplicity due to a 100 MeV/c cut-off in $p_T$ at $\sqrt{s} = 10-14$ TeV. The extra 2% comes from the difference in results from PYTHIA and PHOJET models. The PHOJET model combines the ideas based on a dual parton model [17] on soft process of particle production in addition to using the lowest-order perturbative QCD for hard process. Regge phenomenology is used to parametrize the total, elastic and inelastic cross-sections. The initial and final state parton shower are generated in leading log-approximation. PYTHIA, on the other hand uses string fragmentation as a process of hadronization and tends to use the perturbative parton-parton scattering for low to high $p_T$ particle production. Although there are several other theoretical predictions on total cross-section expected at LHC energies [18], current work focuses on how a judicious extrapolation from existing multiplicity data compares with the calculations from some of the available models [5, 6].
5.2 Multiplicity distribution

The measured charged particle multiplicity distributions have been found to be well described by negative binomial distribution (NBD) at midrapidity and also for the full rapidity region in $p+\bar{p}$ [19, 20]. As mentioned earlier, the NBD distribution has a form,

$$P_{NBD}(\langle N_{ch} \rangle, k; n) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)} \cdot \left(\frac{\langle N_{ch} \rangle/k}{\langle N_{ch} \rangle/k+1}\right)^n \left(\frac{\langle N_{ch} \rangle/k}{\langle N_{ch} \rangle/k+1}\right)^{n+k},$$

(5.1)

It is characterised by two parameters, $\langle N_{ch} \rangle$ and $k$. The parameter $k$ is an interesting quantity. When $1/k \to 0$, the distribution goes over to a Poisson distribution (independent particle production) and the case $k = 1$ corresponds to a Geometric distribution. Under the limit of large multiplicity ($N_{ch} \to \text{Large}$), the NBD distribution goes over to a Gamma distribution.

The charged particle multiplicity distributions for $p+\bar{p}$ collisions as measured by the UA5 collaboration, at midrapidity ($|\eta| < 0.5$) and over full pseudorapidity range ($|\eta| < 5.0$), are shown in Fig. 5.1. The results correspond to $\sqrt{s}$ in the range of 200 to 900 GeV. NBDs have been fitted to the data shown, for both the rapidity intervals. One can see, the data are well described by NBDs in both the $\eta$ ranges.

However, there are certain points that need to be mentioned regarding the results presented in the Fig. 5.1. For the three values of $\sqrt{s}$ considered, the mid-rapidity distributions are quite narrow and closer to each other as compared to the data for the full rapidity range ($|\eta| < 5$). On the other hand with increase in energy the data for full rapidity window show increasing widths. We believe, this increase in width is mostly due to increase in multi-parton interactions resulting in higher particle production. This effect is not seen so clearly in the mid-rapidity data. As will be shown in the next chapter where we present forward-backward correlations in particle production, this effect has a clear bearing on the correlation strength. It must be mentioned here that the CDF experiment also carried out similar measurements of $N_{ch}$ at 1.8 TeV for $|\eta| < 1$ [21]. However, in that experiment a $p_T$ cut of 400 MeV/c has been used. We have carried out PYTHIA and PHOJET model calculations for such a case suggest that there is more than 50% loss in the average number of charged particles due to a $p_T$ cut-off of 400 MeV/c at midrapidity. Since all other results presented (Fig. 5.1) have a much smaller $p_T$ cut-off $\sim 100$ MeV/c, inclusion of CDF results would make this comparative study heavily dependent on the model based extrapolation to lower $p_T$ regions. Because of this, the CDF results are not included in the present discussions.

The NBD parameters $\langle N_{ch} \rangle$ and $k$ extracted from the UA5 data are shown in Fig. 5.2 in the top and bottom panels respectively. The data shown are plotted against $\sqrt{s}$. In the figures we have also included the same parameters as extracted from ISR (Intersecting Storage Ring) data [14] obtained at CERN. It is observed from
Figure 5.1: Multiplicity distribution for charged particles in $p + \bar{p}$ collisions at various center of mass energies at midrapidity (top panel) and full rapidity (bottom panel) ranges [19, 20]. The errors are statistical. The solid lines are NBD fit to the data points using the function given in Eqn. 5.1.
Figure 5.2: NBD fit parameters $\langle N_{ch} \rangle$ (top panel) and $1/k$ (bottom panel) as obtained from $p+p$ collision data of UA5 together with those from ISR experiment at various center of mass energies [19]. The lines are fits to the data points using functional forms as discussed in the text. Also shown for comparison are the results from PYTHIA and PHOJET models at $\sqrt{s} = 200, 540$ and $900$ GeV.
Fig. 5.2 that both at midrapidity and full rapidity range the $\langle N_{ch} \rangle$ increases with $\sqrt{s}$ while the $k$ value decreases with $\sqrt{s}$. Over the region of $\sqrt{s}$ for which measurements exists, the dependence of $\langle N_{ch} \rangle$ can be described reasonably well by both the following expressions (as shown in Fig. 5.2),

$$\langle N_{ch} \rangle = a + b \ln(\sqrt{s}) + c (\ln(\sqrt{s}))^2$$  (5.2)

and

$$\langle N_{ch} \rangle = a' + b' (\sqrt{s})^c'$$,  (5.3)

where $a, b, c, a', b'$ and $c'$ are fit parameters. The values of $a, b, c, a', b'$ and $c'$ at midrapidity are $3.8 \pm 0.1, 1.5 \pm 0.2, 0.45 \pm 0.1, 2.3 \pm 0.14, 1.5 \pm 0.13$ and $1.2 \pm 0.26$ respectively. For full rapidity coverage the same parameters have values $1.3 \pm 0.3, 0.62 \pm 0.17, 0.59 \pm 0.02, -10.5 \pm 0.86, 9.9 \pm 0.69$ and $0.22 \pm 0.01$ respectively. It is noted that an extrapolation of the power law function to LHC energies leads to a sudden increase in $\langle N_{ch} \rangle$ at midrapidity. Such a large jump in the multiplicity at midrapidity already seems to put some constraints on applicability of such a functional form.

As will be shown later such a functional form will lead to a probability distribution of charged particle multiplicity at midrapidity showing a very small drop at higher multiplicities. At higher multiplicities, a much faster drop to lower values is seen in case of PYTHIA, PHOJET models as seen for available experimental data shown in Fig. 5.1.

In Fig. 5.2 the experimental values of $\langle N_{ch} \rangle$ at $\sqrt{s} = 200, 540$ and $900$ GeV are compared to corresponding results from PYTHIA and PHOJET models. Both the models seem to be in reasonable agreement with the measurements at midrapidity as well as in the full rapidity window covered. As shown earlier [20], the $\sqrt{s}$ dependence of $1/k$ has a form $\alpha + \beta \ln(\sqrt{s})$, where $\alpha$ and $\beta$ are fit parameters. For the full rapidity window, the values of $\alpha$ and $\beta$ are found to be $-0.11 \pm 0.006$ and $0.06 \pm 0.001$ respectively. For midrapidity the values of $\alpha$ and $\beta$ are found to be $0.65 \pm 0.03$ and $0.07 \pm 0.03$ respectively. The results from PYTHIA and PHOJET models are also shown in the same figure. At higher energies the results from PHOJET are in better agreement with the measured values compared to those from PYTHIA. For the midrapidity measurements PHOJET model calculations fail to match the data for $\sqrt{s} = 200$ GeV.

We have examined the charged particle multiplicity distribution from the data reported by E735 Collaboration for $p+\bar{p}$ collisions at $\sqrt{s} = 1800$ GeV [22]. The multiplicity distribution in this case is found to be well explained using the sum of two NBD functions unlike the earlier data where a single NBD fit worked well. In this case the full distribution is given as,

$$F = \omega P_{NBD}^1(\langle N_{ch} \rangle_1, k_1; n) + (1 - \omega) P_{NBD}^2(\langle N_{ch} \rangle_2, k_2; n)$$  (5.4)
Figure 5.3: Relative cross-section of charged particles produced in \( p + \bar{p} \) collisions at \( \sqrt{s} = 1800 \text{ GeV} \) [22] fitted to a sum of two NBD functions. See text for more details.

where \( P_{NBD}^1 \) and \( P_{NBD}^2 \) have the same form as in Eqn. 5.1, \( \omega \) is a weight factor, \( \langle N_{ch} \rangle_1 \) and \( \langle N_{ch} \rangle_2 \), \( k_1 \) and \( k_2 \) are the respective NBD parameters. The fit to the E735 data by this function is shown in Fig. 5.3. The values of various parameters obtained from the fit are, \( \langle N_{ch} \rangle_1 = 36.5 \pm 1.7, \langle N_{ch} \rangle_2 = 86.9 \pm 2.6, k_1 = 2.8 \pm 0.2, k_2 = 10.3 \pm 0.8 \) and \( \omega = 0.18 \pm 0.03 \). The overall average value of the \( N_{ch} \) is found to be 44.4.

Since this data could not be fitted to a single NBD, in contrast to the data from other energies discussed in this paper, we have not included this measurement in our extrapolation studies.

The extrapolated values of \( \langle N_{ch} \rangle \) and \( k \) at both midrapidity and full rapidity regions using the functional forms as in Eqns. 5.2 and 5.3 for \( \sqrt{s} = 10 \) and 14 TeV are given in the Table 5.1. Note the difference in values of \( \langle N_{ch} \rangle \) depending on the dependence on \( \sqrt{s} \) as per Eqn. 5.2 (logarithmic dependence) or Eqn. 5.3 (power law dependence). Knowing these values (parameters of NBD function) we can predict the multiplicity distributions for both midrapidity and full rapidity ranges at \( \sqrt{s} = 10 \) and 14 TeV using Eqn. 5.1. These distributions for both midrapidity regions and full rapidity regions are shown in Fig. 5.4 and Fig. 5.5 respectively. The top panel is for \( \sqrt{s} = 10 \text{ TeV} \) and the bottom panel is for \( \sqrt{s} = 14 \text{ TeV} \). The results of the extrapolation of \( \langle N_{ch} \rangle \) using Eqn. 5.2, for midrapidity, are seen to be closer to those obtained from PYTHIA. These data lie above those for PHOJET. On the other hand the extrapolation using the power law (Eqn. 5.3) is seen to produce much higher multiplicity at midrapidity compared to both PYTHIA and PHOJET. For the full rapidity window (\( |\eta| < 5 \)), the extrapolated results are seen to be closer to PHOJET data. Here, as compared to the power law based extrapolation(Eqn. 5.3), the one based on Eqn. 5.2 is found to yield results closer to the PHOJET predictions. In both full and mid-rapidity windows, PYTHIA results are found to be higher compared to...
Table 5.1: Extrapolated NBD parameters $\langle N_{ch} \rangle$ and $k$ for different $\sqrt{s}$ at midrapidity and full rapidity range for $p+p$ collisions at $\sqrt{s} = 10$ and 14 TeV.

<table>
<thead>
<tr>
<th>Eqn.</th>
<th>$\sqrt{s}$ (TeV)</th>
<th>$\langle N_{ch} \rangle$</th>
<th>$k$</th>
<th>$\eta$ range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eqn. 5.2</td>
<td>10</td>
<td>$9.6 \pm 1.1$</td>
<td>$1.2 \pm 0.14$</td>
<td>$</td>
</tr>
<tr>
<td>Eqn. 5.2</td>
<td>10</td>
<td>$56.7 \pm 3.8$</td>
<td>$2.2 \pm 0.09$</td>
<td>$</td>
</tr>
<tr>
<td>Eqn. 5.3</td>
<td>10</td>
<td>$25.0 \pm 11.5$</td>
<td>$1.24 \pm 0.14$</td>
<td>$</td>
</tr>
<tr>
<td>Eqn. 5.3</td>
<td>10</td>
<td>$63.7 \pm 11.3$</td>
<td>$2.2 \pm 0.09$</td>
<td>$</td>
</tr>
<tr>
<td>Eqn. 5.2</td>
<td>14</td>
<td>$10.8 \pm 1.3$</td>
<td>$1.20 \pm 0.14$</td>
<td>$</td>
</tr>
<tr>
<td>Eqn. 5.2</td>
<td>14</td>
<td>$60.7 \pm 4.0$</td>
<td>$2.14 \pm 0.09$</td>
<td>$</td>
</tr>
<tr>
<td>Eqn. 5.3</td>
<td>14</td>
<td>$36.2 \pm 18.5$</td>
<td>$1.20 \pm 0.14$</td>
<td>$</td>
</tr>
<tr>
<td>Eqn. 5.3</td>
<td>14</td>
<td>$69.4 \pm 12.2$</td>
<td>$2.14 \pm 0.09$</td>
<td>$</td>
</tr>
</tbody>
</table>

those from PHOJET calculations. As mentioned earlier this is due to higher multiparton interactions included in PYTHIA. Actual experimental measurements at LHC will confirm the preferred $\sqrt{s}$ dependence of $\langle N_{ch} \rangle$.

### 5.3 Pseudorapidity distribution

The pseudorapidity distributions of charged particles, $dN_{ch}/d\eta$, as obtained from existing experimental data at $\sqrt{s} = 53$, 200, 546 and 900 GeV [23] can be described by the following functional form.

$$
\frac{dN_{ch}}{d\eta} = \frac{C + \eta}{1 + \exp\left(\frac{\eta - \eta_0}{\delta}\right)}
$$

(5.5)

This formula is chosen to describe the central plateau and the fall off in the fragmentation region of the distribution by means of the parameters $\eta_0$ and $\delta$ respectively. In the above equation $C$ is a fit parameter, describing the magnitude of the distribution and the dip at the $\eta = 0$. A similar form has been used to describe the $p+p$ data at ISR energies [14] and heavy-ion collisions at RHIC [15]. This distribution corresponds to a generalization of the Fermi distribution. Some recent work even suggest a relation of this functional form to be valid for $\eta$-distribution coming from a string percolation model [24]. The values of the parameters $C$, $\eta_0$ and $\delta$ obtained by fitting the experimental data Eqn. 5.5 are given in Table 5.2. Fits to data are shown in Fig. 5.6 (a). The value of parameters $C$ and $\eta_0$ are found to increase with increasing $\sqrt{s}$. The value of the parameter $\delta$ is found to be approximately independent of $\sqrt{s}$ within errors. The constancy of $\delta$ is another way of demonstrating the concept of limiting fragmentation [25]. In such a scenario, $dN_{ch}/d\eta$ when plotted as a function of pseudorapidity shifted by beam rapidity ($\eta - y_{beam}$) is expected to be
Figure 5.4: Estimated multiplicity distribution for charged particles in $p+p$ collisions at $\sqrt{s} = 10$ (top panel) and $14$ TeV (bottom panel) in midrapidity. Solid and dashed lines are distributions obtained from $\langle N_{ch} \rangle$ extrapolation using Eqns. 5.2 and 5.3 respectively. The dotted lines reflect errors in multiplicity distributions due to extrapolation of the parameters $\langle N_{ch} \rangle$ and $k$. The results are compared to corresponding calculations from PYTHIA and PHOJET.
Figure 5.5: Same as Fig. 5.4 for full rapidity region.

Table 5.2: Parameters $C$, $\eta_0$ and $\delta$ for different $\sqrt{s}$.

<table>
<thead>
<tr>
<th>Collision</th>
<th>$\sqrt{s}$ (GeV)</th>
<th>$C$</th>
<th>$\eta_0$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p+\bar{p}$</td>
<td>53</td>
<td>$2.4 \pm 0.23$</td>
<td>$1.5 \pm 0.17$</td>
<td>$1.12 \pm 0.1$</td>
</tr>
<tr>
<td>$p+\bar{p}$</td>
<td>200</td>
<td>$2.5 \pm 0.07$</td>
<td>$2.5 \pm 0.05$</td>
<td>$1.10 \pm 0.04$</td>
</tr>
<tr>
<td>$p+\bar{p}$</td>
<td>546</td>
<td>$3.0 \pm 0.10$</td>
<td>$2.9 \pm 0.07$</td>
<td>$1.15 \pm 0.04$</td>
</tr>
<tr>
<td>$p+\bar{p}$</td>
<td>900</td>
<td>$3.6 \pm 0.10$</td>
<td>$3.0 \pm 0.05$</td>
<td>$1.36 \pm 0.05$</td>
</tr>
</tbody>
</table>
Figure 5.6: (a) Pseudorapidity distribution for charged particles in $p+\bar{p}$ collisions at various center of mass energies [23]. The solid lines are fit to the data points using the function given in Eqn. 5.5. (b) Comparison of the pseudorapidity distributions for charged particles at various $\sqrt{s}$ to PYTHIA and PHOJET model calculations.
independent of $\sqrt{s}$ at forward rapidities [15]. In Fig. 5.6 (b) we have shown the experimentally measured charged particle pseudorapidity distributions along with those from the PYTHIA and the PHOJET simulations. The CDF data [26] are not used for the above parametrization for making predictions at LHC energies because of the limited $\eta$ coverage of the experiment. It is observed that for the lowest beam energy studied, $\sqrt{s} = 53$ GeV, both PYTHIA and PHOJET simulation results are in good agreement with the experimental data. With increase in beam energy PYTHIA results are seen to be in better agreement with the data except at the top energy ($\sqrt{s} = 1800$ GeV [21]) studied. But the error bars are large which makes it difficult to make a conclusion regarding the choice of one model over the other. However, there is some systematic trend which shows that the value of $dN_{ch}/d\eta$ at $\eta = 0$ is higher for PHOJET simulations than for PYTHIA. The difference between the two sets is seen to increase with increasing energy.

Using the average value of $\delta$ and extrapolating the values of $C$ and $\eta_0$ to $\sqrt{s} = 10$ and 14 TeV, one can estimate the full pseudorapidity distribution for charged particles. Such an extrapolation can be made by fitting the variation of $C$ with $\sqrt{s}$ with a functional form

$$C = 3.7 + 1.15 \ln(\sqrt{s}) + 0.25 (\ln(\sqrt{s}))^2 \quad (5.6)$$

and the variation of $\eta_0$ with $\sqrt{s}$ as

$$\eta_0 = 3.1 + 0.4 \ln(\sqrt{s}). \quad (5.7)$$

The values of $C$ obtained are $7.63 \pm 0.87$ and $8.43 \pm 1.04$ for $\sqrt{s} = 10$ and 14 TeV respectively. Those for $\eta_0$ are $4.01 \pm 0.14$ and $4.15 \pm 0.15$ for $\sqrt{s} = 10$ and 14 TeV respectively. Using the parameter values as obtained above, we have determined the pseudorapidity distribution of charged particles at $\sqrt{s} = 10$ and 14 TeV over a range $0 < \eta < 8$. These distributions for 10 and 14 TeV are shown in Fig. 5.7 (a) & (b) respectively, along with simulations results obtained from PYTHIA and PHOJET models. Also included are lower and higher limits, as determined from the uncertainties in the fitting parameter. The difference between the limits show an uncertainty band. It is observed that at both $\sqrt{s} = 10$ and 14 TeV, the predictions from a systematic extrapolation of existing data in general are above the model predictions. It is interesting to note that the PYTHIA results are now above the PHOJET results unlike what was seen at energies below 2 TeV. Further the PYTHIA results are seen to be close to the lower error band (dotted red lines) for $\eta < 4$. In general all the three distributions have almost similar shape. But there are differences. These differences between the models could arise due to several reasons. The event generation in PYTHIA is mainly designed to describe the possible hard interactions in $p+p(\bar{p})$ collisions. It also includes soft hadronic interactions [5]. However PHOJET
Figure 5.7: (a) Expected pseudorapidity distribution (solid lines) for charged particles in $p+p$ collisions at $\sqrt{s} = 10$ TeV. (b) Similar distribution (as in (a)) for $\sqrt{s} = 14$ TeV. These results are obtained from the extrapolation using the existing data from $p+\bar{p}$ collisions at lower energies. The dotted red lines indicate the uncertainties associated with the extrapolation. Also shown for comparison are the expected $dN_{ch}/d\eta$ from PYTHIA and PHOJET model calculations for the same two cases (as shown in (a) and (b))
Figure 5.8: Pseudorapidity distribution for charged particles in $p+p$ collisions at $\sqrt{s} = 2.36$ and 7 TeV as obtained from the extrapolation (solid lines). The dotted lines indicate the uncertainties associated with the extrapolation. Also shown for comparison are the $dN_{ch}/d\eta$ from ALICE and CMS measurements [16].

takes care of the soft component of hadron-hadron, photon-hadron interactions at high energies in an effective manner. To this, it adds the hard component calculated by perturbative QCD at the partonic level [6]. Another difference lies in the parametrization used to get the $p+p(\bar{p})$ cross-sections. PYTHIA uses those derived from the Pomeron exchange model while the PHOJET uses the optical theorem and cross-sections are corrected for high energies using the unitarity principle. PYTHIA, tuned to CMS experiment, is seen to generate results in better agreement with the present predicted results. Recently ALICE and ATLAS experiments have carried out $dN_{ch}/d\eta$ measurements at 2.36 and 7.0 TeV. Finally we have carried out extrapolations of our data to obtain results at these two energies. These data together with the available experimental results for NSD events are shown in Fig. 5.8. One can see there is a reasonably good agreement at 2.36 TeV. However at 7 TeV the data (only from CMS) are seen to lie just below the lower error band. This indicates that the present extrapolation method is not all that bad. Lack of experimental data over a wider $\eta$ range does not allow one to use our fitting and see the fitting parameters against those obtained for data at lower energies.
Bibliography


