CHAPTER XI

NUMERICAL ABILITY - A CONSTRUCT

In the previous chapter the need for the present test was described in the context of secondary schools in the State of Gujarat. The utility and limitations of the present work were also indicated.

In this chapter the nature-nurture functions of mental abilities, models of 'mental structure' and components of 'numerical ability complex' are described in turn.

Nature-nurture interactions

Mental measurements, at their best, are indirect and largely hypothetical in nature. Whether the approach to test construction is rational or empirical, the nature of the 'entity' being measured is not discovered completely. The existence and extent of the measurement variable is just inferred experimentally and expressed statistically. The ultimate reality of measurement variables, their origin, development, interactions and contribution of heredity and environment are issues of great interest but they are beyond the scope of the present work. Only a summary of generalizations from Gagne, (p.90) would suffice to provide a frame of reference.

Human beings can display various functions such as discrimination and conceptualization. These functions develop
within a framework of anatomical structures, which is unique for each individual. These functions are also shaped by learning that occurs in varying social and cultural environments. The inherent set of sensory effectors and neural structures set limit to the development of most of these functions for each human being. And therefore learning is also a matter of individual differences. To discover such inter-personal and intra-personal variations is a field of differential psychology.

The mental functions are facilitated through the use of concrete, abstract and verbal imagery. Imagery depends on experiences available as well as receptivity and acuity of the sensory systems.

In this sense the verbal, numerical and mechanical abilities can be conceived to be highly cultural. They are largely developed by informal and formal experiences within the environment. Such abilities represent general potentialities, which can facilitate the learning of a variety of skills or job elements, in educational and vocational set-ups. Learning play a major role in influencing development of abilities, within the boundaries set by heridity and environmental factors.

According to Gagne, "the numerical ability continues to grow up to the age 22, remains almost stable up to 52 and then declines." (Gagne, p. 96)
According to Guilford the average child reaches eighty per cent of maturity in Numerical facility at the age of sixteen. (Guilford, 3, p.420)

In both the references referred to above Numerical Facility is implied by the term numerical ability.

The central idea in his works on Primary Mental Abilities has been conceptualized by L. L. Thurston by stating that, "the human intellect consists of a finite number of distinguishable functions which are called primary factors or abilities". (Barnette (Ed.), p. 123)

Models of mental structure

Efforts are being made, continuously, to discover and describe the nature and organization of mental functions from various approaches. Two of the models presented by psychometricians are briefly summarised here, to the extent that they are relevant to the design of the present work, as well as the rationale behind the designs of similar tests of repute. The description of the two models indicates how the hypothetical construct of numerical ability can be viewed as a component of 'general intelligence models'.

A hierarchical model. The organizational hierarchical model of the British School has been well presented by Humphreys.
At the top of the hierarchy is "g" the general factor depending on numerous overlapping bonds rather than a unitary force or power within the person. There are at least two major group factors: the verbal, numerical educational factor and the mechanical spatial, practical factor, beneath "g". Below the broad group factors are the Thurstone's primaries. Guilford's more numerous primaries are presumably still further down in the hierarchy. Specific factors are at the bottom of the hierarchy.

It is argued that the hierarchical model defines all factors in terms of original measurement variables. The model also indicates that the test can be constructed to measure a factor at any level of the hierarchy. One can move up or down the hierarchy to make the required kind of prediction. All ability tests-aptitude, ability and achievement—are subsumed under the same model. (Gronlund (Ed.), p. 260)

A morphological model. - The morphological or popularly called the "Structure of Intellect " model, by Guilford, with its three parameters - operation, content and product - strives to develop unique definition of each 'cellular' specific intellectual ability in terms of one of the five kinds of operations, combined with one of the four kinds of content and with one of the six kinds of products. (Guilford, 3, p. 465) The three parameters or dimensions of Guilford's conception of the structure of Intellect are
The semantic contents have to do with language. The symbolic contents are dealt with as numerical ideas and concepts. The figural contents are related to visual, auditory and other types of configurations, patterns and shapes, while the behavioural contents are related to the way in which persons behave in social context.

Guilford's proposal, though considered to be "ranked heresay" by Guilford himself, strives to provide a meaningful and synthetic classification of forty odd factors already recognised. (Gronlund (Ed.), p. 275) It also suggests areas in which new factors might be found. Subsequent researches have verified the existence of some of these additional factors. Helmtadter has recognised the tremendous value of Guilford's taxonomic work in the area of testing, as well as the usefulness of the factorial approach to content validity. (Helmstadter, p. 111)

A taxonomical hierarchy can be viewed even in Guilford's model. The dimension of operation can be assumed to be ranging from simple to complex and specific to general as one moves from cognition to evaluation. The dimension of products can be viewed from units to implications and the dimension of content from figural to behaviours. Such hierarchy can be tested by administering a large number of items on each aspect of the same dimension on a single sample of examinees.
The mental functions covered by some of the items of numerical ability tests can be properly accommodated in the lattice of the convergent production factors suggested in Guilford's Structure of Intellect model. The cells of the lattice are convergent symbolic units (NSU), - classes (NSC), - relations (NSR), - systems (NSS), - transformations (NST), and - implications (NSI). (Guilford, 1967, p.171)

There is scope for six factors for each of the other operations namely, cognition, memory, divergent thinking and evaluation.

The hierarchical model accommodates numerical ability as a member of "the verbal, numerical educational group factor" below "g" and above the primary and specific factors in the organization of mental abilities.

A synthesis. - One has to strike a balance between the structural and functional approaches to achieve optimum validity and flexibility to serve the purpose of the test. The expectation that factoring will yield the capacities or unitary powers in absolute terms is farfetched due to lack of experimental controls on factor-analysis studies.

The morphological model helps to decide the medium, level and form of the measurement tasks. The present test of Numerical Ability falls in the symbolic media in essence. The Language Ability Test falls in semantic and the Pictorial Test of Mechanical comprehension in the figural
media. From the hierarchical model point of view the three tests can be accommodated as members of the major group factors under "g". The model indicates the hierarchical relations amongst categories of factors but it is silent on the products and operations of the mental task, induced by the test items. The multiple choice objective test items are considered suitable for convergent thinking in any medium. The numerical test items generally lead to the transormations at the end of the numerical operations.

In short, numerical ability is an important component of the so called general mental ability. Structurally it can be further analysed into specific factors. Functionally it is a part of scholastic ability. Before the Numerical Ability is defined the taxonomical relations of ability with aptitude and achievement are worth considering.

Ability continuum

In the context of progress at the level of education, training or vocation, a variety of characteristics are assumed to be operating. These characteristics are inconsistently referred to as aptitudes, abilities, factors and traits.

The definition of terms aptitude, ability and achievement can be better explained if they are thought to be on the same variable continuum of maximum performance. It is considered inappropriate to think of tests of achievement,
ability and aptitude as discrete categories of tools, distinguishable by knowing or supposing whether they measure innate ability or influence of environment or a mixture of both.

In general, aptitude tests can be thought of as those which measure mental functions that improve little with practice; ability tests as those which show some relationship to general enrichment of the environment; and achievement tests as those constructed specifically to measure the degree of accomplishment in specific training or subject area. (Helmstadter, p. 11)

Numerical ability defined by others

The test constructors are not unanimous on the definition of the term, Numerical Ability. They further vary on functional contents of tests purporting to measure the factor 'N' under any name. Seven test batteries, all accommodating factor 'N', are presented by their authors and reviewed by Super in a reprint series from the Personnel and Guidance Journal. (Super, 2, pp. 9, 22, 32, 38, 58, 59, 69, 81)

Naming the tests. - Bennett, Crowder and Thurstone have preferred to label their subtests as tests of Numerical Ability. Dvorak has given a common label of Numerical Aptitude to his subjects II and VI. Guilford has called his subtest number III of Numerical operations measuring Number
Factor. Segel has tagged his subtest number V arithmetic reasoning and subtest number VI arithmetic computation under Numerical Reasoning. Flanagan has referred to each of his subtests as measuring some job element. His subtest number IX measures arithmetic skill and subtest number XVI measures reasoning.

Contents of the tests. So far as the functional contents of the tests are concerned, Crowder, Guilford and Thurstone have preferred to include the items of fundamental arithmetic operations or numerical facility alone. Dvorak, Segel and Flanagan have preferred to include items of numerical facility and numerical problem solving, in two subtests timed separately. The problems are cast in verbal sentences therein. Bennett has avoided the use of language in his test of Numerical Ability, which is a mixture of a few items on numerical facility and the rest on the handling of numerical concepts.

The two test batteries, namely, Differential Aptitude Tests and The Academic Promise Tests, designed by Bennett and others, seem to be popular amongst school counsellors. The analytical study of their subtests can lend some useful hints regarding the suitability of subtests and the type of items to be considered for inclusion in the tests for educational guidance.
The Differential Aptitude Tests Battery, which is adjudged by Super as ready for use in educational counseling for grades 8 to 12, has adopted the definition of numerical ability as "understanding of numerical relationships and facility in handling numerical concepts". (Super, 2, pp. 9, 89)

The subtest Numerical ability in the Academic Promise Tests by Bennette and others, is a mixture of items on number series (15) and straightforward arithmetic computations and arithmetic reasoning items (45), on the lines of Otis and Thorndike. (Buros (Ed.), 2, p. 765)

Factorial studies. - From a factor analytic study by Adkins, the Numerical Ability can be viewed as a group of related factors such as Arithmetic, Number series, Numerical operations I (+, x), Numerical operations II (−, ÷) and Numerical puzzles, which are coded as 2, 32, 33, 34 and 35 in the report. The high inter-correlations among these factors indicate a few things. It is difficult to construct factorially pure tests. Secondly the numerical ability test could be constructed with closely related but identifiable subfactors. Thirdly Numerical operations II has larger overlapping than the Numerical operations I. (Adkins, p. 108)

In Thurstone's first FMA analysis his four numerical operations tests and four others were found to be loaded by
Number factor as follows:

<table>
<thead>
<tr>
<th>Operation</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplication</td>
<td>.81</td>
</tr>
<tr>
<td>Addition</td>
<td>.76</td>
</tr>
<tr>
<td>Subtraction</td>
<td>.67</td>
</tr>
<tr>
<td>Division</td>
<td>.62</td>
</tr>
<tr>
<td>Number code</td>
<td>.62</td>
</tr>
<tr>
<td>Num. judgement</td>
<td>.43</td>
</tr>
<tr>
<td>Tabular completion</td>
<td>.39</td>
</tr>
<tr>
<td>Arith. reasoning</td>
<td>.38</td>
</tr>
</tbody>
</table>

(Guilford, 3, p. 132)

The typical traditional numerical facility or number factor was supposed to occupy the cell N1 - convergent production of symbolic implications, but recent studies have raised doubts about the univocality of the factor 'N'. It has a large component of specific practice. (Guilford, 3, p. 134)

Thurstone also had doubts regarding the underlying nature of the number factor. He reports:

Another factor that was isolated early in the factorial experiments was the number factor "N". This factor represents facility in doing simple arithmetical tasks but it is rather narrow. The number factor is not heavily involved in arithmetic reasoning or in mathematics. In fact it would not be surprising to find good mathematicians who are relatively poor in this factor. On the other hand a cashier would soon quit this job if he were poor in the number factor.

We have been puzzled about the underlying nature of the number factor. If we are dealing with differences in native endowments, then it should be possible to describe the nature of this factor in terms of fundamental processes rather than in such terms as "number", which is a cultural concept. Eventually, when the nature of this factor is more completely understood, it may turn out to be merely a historical accident that the factor was identified in tests that happened to be
numerical in character. This is a question for the future to answer.

(Barnette (Ed.), p.116)

The review of the standard tests and factor studies referred to above indicate that numerical ability, popularly known as numerical facility, continues to grow up to the age of 22, requiring 16 years to reach 80 per cent of maturity.

The univocality of Number as a pure factor is seriously doubted by some of the eminent psychometricians.

Test items on numerical series, numerical concepts and numerical problems require mental operations related to a group factor of numerical ability but identifiable by experimental controls, as separate sub-factors.

Numerical ability is considered to be the proper term to designate the measurement variable for the present work. The author's construct of numerical ability and relevant terms are now defined at this stage.

A construct of Numerical Ability

The test author, on the basis of the survey of relevant literature referred to in this chapter could formulate a hypothetical construct of Numerical Ability for the design of the test. It was then decided to include the
following operational levels, say aspects, of numerical ability in his construct:

1. Numerical facility measured through four fundamental operations involving, primarily, number facts of addition and multiplication.

2. Numerical reasoning (easy, inductive, non-verbal) through number series, requiring the examinee to find relations, formulate the rule and supply the missing number in the series.

3. Use of basic (arithmetical) numerical concepts (relatively easy), through novel items of transforming one numerical form into another.

4. Use of numerical concepts (algebraic, relatively difficult), through learning-set items of transforming one numerical form into another.

5. Use of numerical concepts, (deductive, relatively difficult) through solving problems, cast in verbal form.

This hypothetical model was kept in view during the process of the test development. It was found to be working reasonably well, as revealed by data on item analysis and factor analysis, at a later stage.
More terms defined

The rest of the terms were adopted by the author from Good who is quoted:

**Ability** : the actual power present in an organism to carry to completion any given act.

(Good, 1, p. 1)

**Number concept** : (1) the abstract idea of any order of multiplicity grasped in terms of meaningful symbols which may be manipulated to solve problems; the generalization which the learner gains involves a mental picture based upon his experience and understanding of the interrelationships of himself, objects, and symbols; the number concept of the modern Hindu-Arabic number system would encompass, among other specifics, the zero as a place holder, the base 10 idea, the unitary symbol representing a multiplicity of objects, the decimal point, signed numbers, and the paired opposite idea; (2) a general idea developed in regard to some procedure or basic process in arithmetic and identified by a particular word or phrase; for example, the general idea expressed by the word "subtraction" is that of the division of a single group of objects into two smaller groups.

(Good, 1, p. 118)

**Conceptual thinking** : (1) the process of thinking in which abstract concepts are related without reference to concrete situations; (2) the manipulation of linguistic or mathematical symbols in terms of their inherent logical relations.

(Good, 1, p. 570)
Problem solving ability: (math.) (1) the degree of capability necessary to find the correct solution to problem situations; (2) an estimate of such ability derived by administering standardized tests.

(Good, 1, p. 2)

In this chapter, the contribution of nature and nurture in development of mental abilities was cursorily mentioned. It was followed by the presentation and synthesis of two models of mental organization. The concept of ability as a variable continuum and numerical ability complex in general were described. In the end the hypothetical construct of numerical ability adopted for the present work was described.

In the following chapter some test-literature is reviewed in brief.