9.1 Introduction:

Cobb-Douglas production function assumes unitary value of the elasticity of substitution $\sigma$. Due to such a restrictive nature of this production function, ACMS (1961) has first proposed CES production function which assumes that elasticity of substitution ($\sigma$) can take any positive value between zero to infinity (other than one). ACMS production function assumes constant returns to scale. Brown and De cani (1963) and Soskies (1968) have modified CES production function to allow for varying returns to scale. A quite significant work has been carried out by many researchers to apply for CES production function in different areas of application using Indian data. A few of them are given by Goldar (1986), Mehta (1980), Singh and Singhal (1986), Jaiswal etc. (1981) and (1986), Baneraji (1975), Gujarati (1967),...
In this chapter, an attempt is made to fit CES production function with and without technical progress. For the purpose of studying CES production function relation, time-series data for the industrial sectors of Gujarat State and India as a whole (All industries) from the period 1960-1961 to 1980-1981 are considered. For the comparison at constant prices, the data are converted into real values by deflating the given series by means of the appropriate indices.

9.2 Methodology:

CES production function can be specified as

\[ V_t = A \left[ \theta K_t^\theta + (1-\theta) L_t^{\theta} \right]^{-1/\theta} \]

(9.1)

Where \( V_t \) = Value added by manufacturer in the industrial sector during the year \( t \).

\( K_t \) = Fixed capital invested in the industrial sector during the year \( t \).

\( L_t \) = Number of persons employed in the industrial sector during the year \( t \).
and $A$, $\delta$ and $\varphi$ are the efficiency, distribution and substitution parameters respectively. The elasticity of substitution ($\sigma$) for this production function is given by

$$\sigma = \frac{1}{1 + \varphi}$$

.........(9.2)

The above formula (9.1) assumes constant returns to scale. Brown and De cani (1963) have introduced a scale parameter $\mu$ to allow for the changing returns to scale which can characterise any degree of returns to scale. This can be written as under:

$$V_t = A \left[ \delta K_t - (1 - \delta) L_t \right]^{-\mu/\varphi}$$

.........(9.3)

which is the generalised form of equation (9.1).

In the above equation (9.3), an exponential type Hicks-neutral technological progress may be introduced by putting $A = A_0 \cdot e^{\lambda t}$. 
So that under neutral technical progress, the equation is given by

\[ \nu_t = A_0 \lambda t \left[ \lambda K_t^{\beta} + (1-\lambda) L_t^{1-\beta} \right]^{-\mu/\beta} \]

\[ \ldots \ldots \ldots \ldots \ldots \ldots \ldots (9.4) \]

Where \( \lambda \) indicates the constant rate of neutral technical progress which implies that the efficiency of both the inputs (viz. capital and labour) changes simultaneously.

Cobb-Douglas production function is a log-linear production function, but CES production function is not expressible in the log-linear form. So the estimation of CES production function cannot be obtained directly. There are several methods in the econometric research work which help to estimate this production function model. One of the methods usually applied is the side-relation between labour productivity and wage rate, which can be derived from CES production function under the assumption of perfect competition and profit maximisation. This approach was first proposed by
Fergusom (1965) which can be stated as under:

\[
\log \left( \frac{v_t}{l_t} \right) = \left( \frac{-\mu}{\mu + \phi} \right) \log (s^H) + \frac{\mu}{\mu + \phi} \log \omega \\
+ \frac{\phi(H-1)}{H + \phi} \log L + \frac{\lambda \phi}{H + \phi}(t)
\]

.........(9.5)

so that

\[
\log \left( \frac{v_t}{l_t} \right) = \beta_0 + \beta_1 \log \omega + \beta_2 \log L + \beta_3 \cdot t
\]

.........(9.6)

Where

\[
\beta_0 = \left( \frac{-\mu}{\mu + \phi} \right) \log (s^H)
\]

.........(9.7)

\[
\beta_1 = \frac{\mu}{\mu + \phi}
\]

.........(9.8)

\[
\beta_2 = \frac{\phi(H-1)}{H + \phi}
\]

.........(9.9)

and

\[
\beta_3 = \frac{\lambda \phi}{H + \phi}
\]

.........(9.10)

The co-efficients of the above equation (9.6) can be estimated by applying the OLS method. This approach provides estimates of our important parameters of the CES production function without using capital data.
The estimate of $\beta_1$ is indicative of the elasticity of substitution. $\beta_1$ equals to one when $s = 0$ and will be greater than one when $s < 0$. The estimate of $\beta_2$ gives an idea of the economies of scale.

$s$ and $\mu$ can be estimated by solving the equations (9.8) and (9.9) so that we get

$$\hat{\mu} = \frac{\hat{\beta}_2}{(1 - \hat{\beta}_1) + 1} \quad \text{...............(9.11)}$$

and

$$\hat{s} = \frac{1 - \hat{\beta}_1}{\hat{\beta}_1} (\hat{\mu}) \quad \text{...............(9.12)}$$

then by substituting the estimated value of $\hat{\mu}$ into (9.12), we get

$$\hat{s} = \frac{\hat{\beta}_2 - \hat{\beta}_1 + 1}{\hat{\beta}_1} \quad \text{...............(9.13)}$$

since the elasticity of substitution we get,

$$\sigma = \frac{\hat{\beta}_2}{1 + \hat{\beta}_2} \quad \text{...............(9.14)}$$
The parameter $\delta$ can be estimated from the relation

$$\beta_0 = \left( \frac{-\mu}{\mu + \delta} \right) \log \delta \mu$$

Hence

$$\delta = \frac{\mu}{\mu - \beta_0} \left( 1 + \frac{\delta}{\mu} \right)$$

..........(9.15)

and finally

$$\lambda = \frac{\beta_3}{1 - \beta_1}$$

..........(9.16)

Note that when $\hat{\mu} = 1$, $\beta_1$ is equal to $\sigma$ for constant returns to scale, then the above form (9.5) reduces to

$$\log \left( \frac{v_t}{u_t} \right) = \beta_0 + \sigma \log \omega + \lambda (1 - \sigma) t$$

..........(9.17)
9.3 Results:

Tabled - 9.1.1 and 9.1.2 give the results of the fitted CES production function model and their parameters respectively without technical progress while Tables - 9.2.1 and 9.2.2. give the respective results of the fitted CES production function with technical progress.

Table - 9.1.1

Estimates of the CES production function
without time-variable.

<table>
<thead>
<tr>
<th>Regression Co-efficient</th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$R^2$</th>
<th>$-2R^2$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gujarat State</td>
<td>2.9269 (2.8378)</td>
<td>0.5535 (0.3799)</td>
<td>0.7827 (0.0930)</td>
<td>0.82</td>
<td>0.80</td>
<td>41.20</td>
</tr>
<tr>
<td>All India</td>
<td>1.3809 (1.7731)</td>
<td>0.7689 (0.2702)</td>
<td>0.2460 (0.1003)</td>
<td>0.76</td>
<td>0.73</td>
<td>28.02</td>
</tr>
</tbody>
</table>

Note: (i) The values in the bracket ( ) show S.E. of estimates
(ii) The values in the bracket $/\prime$ $/\prime$ show t-statistics
** Significant at 1% of significance level
* Significant at 5% of significance level
From the above table - 9.1.1, it can be seen that the regression co-efficients for both the industrial sectors are found to be significant. The values of $R^2$ and $\bar{R}^2$ for the fitted model are also significant for both the industrial sectors. About 76% to 82% of the variation is explained by the fitted model. $R^2$ and $\bar{R}^2$ also indicate the positive correlation between the variables. It may be concluded that a unit increase in the wage rate (under given fixed employment) brings about 55% increase in labour productivity for the industrial sector of Gujarat State and about 77% increase in labour productivity for the industrial sector of all India at constant prices. Similarly, for per unit increase in employment (when the wage rate is unaltered) there is about 78% and 25% increase in the labour productivity at constant prices respectively for the industrial sector at state and at national levels.
Table-9.1.2

Estimation of the parameters for CES production function without technical progress

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$h$</th>
<th>$\vartheta$</th>
<th>$\delta$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gujarat State</td>
<td>2.7530</td>
<td>2.2208</td>
<td>0.0018</td>
<td>0.3105</td>
</tr>
<tr>
<td>All India</td>
<td>2.0645</td>
<td>0.6205</td>
<td>0.0801</td>
<td>0.6171</td>
</tr>
</tbody>
</table>

Table-9.1.2 gives the estimated values of the parameters for both the industrial sectors based upon the regression run in table-9.1.1. It may be visualised that the scale parameter $h$ is about 2.75 and 2.07 for the industrial sector of Gujarat State and that for all India respectively at constant prices. Since $h > 1$, it may be concluded that there are increasing returns to scale for both the industrial sectors. The elasticity of substitution ($\vartheta$) is about 0.31 and 0.62 for the industrial sector of Gujarat State and all India respectively. Since $\vartheta < 1$, it may be worthwhile to conclude that the amount of substitution that is permissible between the input factors of production is relatively lower.

The substitution parameter $\vartheta$ is a simple function of the elasticity of substitution $\vartheta (\vartheta = \frac{1}{\vartheta} - 1)$. The distribution parameter $\delta (0 \leq \delta \leq 1)$, determines the division of the
factor income. The estimated value of $\beta$ for this model is about 0.002 and 0.08 for the industrial sector of Gujarat State and all India respectively. It may also be worthwhile to conclude that both the industrial sectors are labour intensive.

Table - 9.2.1

Estimates of the CES production function with time-variable

<table>
<thead>
<tr>
<th>Regression coefficients</th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$R^2$</th>
<th>$-R^2$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gujarat State</td>
<td>3.2939</td>
<td>0.5180</td>
<td>0.6814</td>
<td>0.0033</td>
<td>0.82</td>
<td>0.79</td>
<td>26.02</td>
</tr>
<tr>
<td></td>
<td>(3.4081)</td>
<td>(0.4262)</td>
<td>(0.4961)</td>
<td>(0.0159)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.9667</td>
<td>1.2157</td>
<td>1.3767</td>
<td>0.2087</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All India</td>
<td>5.7222</td>
<td>0.6385</td>
<td>0.7499</td>
<td>0.0372</td>
<td>0.83</td>
<td>0.80</td>
<td>27.87</td>
</tr>
<tr>
<td></td>
<td>(2.2003)</td>
<td>(0.2367)</td>
<td>(0.3747)</td>
<td>(0.0136)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.6017</td>
<td>2.6987</td>
<td>2.0017</td>
<td>2.7317</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note: (i) The values in the bracket ( ) show S.E. of estimates
(ii) The values in the bracket $/\_\_\_$ show t-statistics
** Significant at 1% of significance level
* Significant at 5% of significance level
Above table - 9.2.1 gives the estimates of the co-efficients for CBS production function with time variable. The values of $R^2$ and $\bar{R}^2$ for the fitted model are highly significant for both the industrial sectors of the economy. At constant prices, about 82% to 83% of the variation is explained by the fitted model. From the above table, it may be worthwhile to conclude that a unit increase in wage rate (when the other variables are fixed) brings about 52% and 64% increase in labour productivity respectively for the industrial sector at state and national levels.

Table - 9.2.2
Estimation of the parameters for CBS production function with technical progress

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$H$</th>
<th>$S$</th>
<th>$\delta$</th>
<th>$\lambda$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gujarat State</td>
<td>2.1437</td>
<td>2.2460</td>
<td>0.0008</td>
<td>0.0064</td>
<td>0.3081</td>
</tr>
<tr>
<td>All India</td>
<td>3.0744</td>
<td>1.7406</td>
<td>0.0004</td>
<td>0.1029</td>
<td>0.3649</td>
</tr>
</tbody>
</table>
From the above table, it can be seen that the degree of homogeneity $\rho$ is about 2.14 for the industrial sector of Gujarat State, while the same for the industrial sector of India as a whole is about 3.07. Thus there are increasing returns to scale prevailing in both the industrial sectors of the economy so far as such a model is considered. The elasticity of substitution ($\sigma$) is about 0.31 and 0.36 respectively for Gujarat State and all India at constant prices. The distribution parameter $\delta$ is about 0.0008 and 0.0004 respectively for the industrial sector of Gujarat State and all India. The technology parameter $\lambda$ has the value 0.006 and 0.10 respectively for the industrial sectors of Gujarat State and all India. This shows that the technological progress is almost negligible in the industrial sector of Gujarat State as compared to the same for India as a whole. It may also be concluded that the new technological innovations may have a sizable effect upon the industrial development for the entire industrial sector at national level as compared to the same at state level.
9.4 Concluding Remarks:

To summarise the above findings based upon the above analysis, it may be concluded that both the industrial sectors are labour intensive. Thus employment potentials can be increased to raise the output in both the sectors which is a substantial need of the economy as a whole. It is also interesting to note that technological change brings a remarkable change in the growth of output. A substantial change in wage rate can also induce more labour productivity in the industrial sectors of the economy.