CHAPTER - VII

ECONOMIES OF SCALE AND TECHNICAL CHANGE UNDER COBB-DOUGLAS PRODUCTION FUNCTION FOR THE INDUSTRIAL SECTORS OF GUJARAT STATE AS COMPARED TO ALL INDIA

7.1. Introduction:

The main purpose of studying a production function is to estimate the co-efficient of inputs, their marginal productivities and share in output and the degree of returns to scale. The aggregate production function (Solow, 1957) has been proved extremely useful in the empirical work and as a consequence, it is very widely favoured. The concept of production function is closely related to the concept of "technology". Production function corresponds to a specific level of technological knowledge and any change in this leads to a shift in production function. Cobb-Douglas (1928) production function was discussed for the first time in the theory of production. Solow (1957), ACS (1961), Walters (1963), Uzawa (1962) etc. have contributed greatly in this area. A lucid exposition for this subject is also described in the works of Allen (1938) and (1967), Ragnar Frisch (1965), Chiang (1967),
Henderson and wuants (1980) etc.

Some studies on production function using time-variable have been undertaken by Mehta S. S. (1980), Bhasin V. K. and Seth V. K. (1977), Goldar (1986), Singh and Singhal (1986), Jaiswal (1986) etc.

In this chapter, an attempt is made to estimate the parameters of Cobb-Douglas production function with and without technical progress for the entire industrial sector of Gujarat State as compared to India as a whole. The regression equations are estimated and the results obtained are interpreted.

7.2. Methodology:

Let \( V = f(K, L) \) denote the form of the production function where \( K \) and \( L \) are fixed capital investment and amount of labour employed respectively and \( V \) is the value added. There are many interesting forms of this production function. Let us consider here the unrestricted Cobb-Douglas production function which can be specified as

\[
V_t = A \cdot L_t^a \cdot K_t^b \cdot U_t \tag{7.1}
\]

where \( V_t \) = 'Value added by manufacturers in the industrial sector during the year \( t \).

\( L_t \) = Total No. of persons employed in the industrial sector during the year \( t \).
\[ K_t = \text{Fixed capital invested in the industrial sector during the year } t. \]

and \[ U_t = \text{disturbance term} \]

Output elasticities provide us with a normalised measure of the importance of a particular input. The parameters \( \alpha \) and \( \beta \) measure the partial elasticities of response for output to labour input and capital input respectively and \( A \) is the scale (or technology) parameter. The sum of these co-efficients, i.e. \( \alpha \) and \( \beta \), indicates the extent of economies or diseconomies of scale. If \( \alpha + \beta > 1 \), it gives increasing returns to scale, \( \alpha + \beta = 1 \) gives constant returns to scale and \( \alpha + \beta < 1 \) gives decreasing returns to scale which may change according to the changes in the scale of operations as well as technology.

The logarithmic per-capita form of equation (7.1) can be written as:

\[
\log \left( \frac{V_t}{L_t} \right) = \log A + \beta \log \left( \frac{K_t}{L_t} \right) + \log U_t \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (7.2)
\]

The above linear regression equation gives the relationship between labour productivity and capital-
intensity under constant returns to scale \( \text{(CRTS)} \).

For a given production function, the capital-labour ratio is bound to change rapidly either through substitution of capital for labour or due to the application of Labour saving techniques under new production functions. This type of change will affect the labour share not only over time but also makes the variation in capital-labour ratio. Therefore, we should consider a type of technological change to examine the direction of changes over time \( t \).

Hence equation (7.1) is rewritten under the neutral technical progress as:

\[
V_t = A \cdot e^{\lambda t} \cdot \frac{K_t}{L_t} \quad (7.3)
\]

The above form (7.3) can be written in the logarithm per-capita form of regression as under:

\[
\log \left( \frac{V_t}{L_t} \right) = \log A + \beta \log \left( \frac{K_t}{L_t} \right) + \lambda t + \log U_t \quad (7.4)
\]

where the technology parameter \( \lambda \) determines the neutral technical progress.

* Equation (7.2) under varying returns to scale is given by

\[
\log \left( \frac{V_t}{L_t} \right) = \log A + \beta \log \left( \frac{K_t}{L_t} \right) + \gamma \log L_t + \log U_t \quad (7.2.1)
\]

where \( \gamma = \lambda + \beta - 1 \).
The equations (7.2) and (7.4) given above assume that the degree of returns to scale is constant. Thus the elasticity of output with respect to capital is given by the co-efficient $\rho$ and therefore $1-\rho$ gives the implied elasticity of output with respect to labour. However to allow for the suitable type of changing return to scale with time variable, the co-efficient of log $L$ is included in the equation (7.4) and this can be written as:

$$\log\left(\frac{V_t}{L_t}\right) = \log A + \beta \log \left(\frac{K_t}{L_t}\right) + \gamma \log L + \lambda t + \log U_t$$

(7.5)

where $\gamma = \alpha + \beta - 1$

(Note that $\gamma = 0$ reduces equation (7.5) into equation (7.4) which is for constant returns to scale)

For the purpose of the estimation of the parameters $A$, $\beta$, $\gamma$ and $\lambda$, equation (7.5) is utilized.

As usual, from the regression of equation (7.2) and (7.5) the standard errors of the regression estimates as well as the co-efficient of determination can be obtained to judge the appropriateness of the fitted models.
7.3 Application

To illustrate the application of the above models, ASI data for the industrial sectors of Gujarat State and All India are considered from the year 1960-'61 to 1980-'81. The original series are converted at constant prices by deflating the basic variables appropriately.

Table - 7.1
† Regression of Labour Productivity \( \left( \frac{V}{L} \right) \) on Capital Intensity \( \left( \frac{K}{L} \right) \)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( \hat{\alpha} )</th>
<th>( \hat{\beta} )</th>
<th>( R^2 )</th>
<th>( \bar{R}^2 )</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gujarat State</td>
<td>4.8006 (0.8194)</td>
<td>0.3762 (0.0922)</td>
<td>0.47</td>
<td>0.44</td>
<td>16.67</td>
</tr>
<tr>
<td>All India</td>
<td>5.7440 (0.7436)</td>
<td>0.2632 (0.0825)</td>
<td>0.35</td>
<td>0.31</td>
<td>10.17</td>
</tr>
</tbody>
</table>

Note: (i) The values in the bracket ( ) show S.E. of estimates
(ii) The values in the bracket \( \sqrt{ } \) show t - statistics
** Significant at 1% of significance level
* Significant at 5% of significance level
† This regression analysis is based upon equation (7.2)
Table 7.1 gives the results of the regression analysis done for both the industrial sectors. A two variable structural relationship between the labour productivity and the capital-intensity has been assumed and the fitted regression coefficients are found to be highly significant for both the industrial sectors. The values of $R^2$ and $R^2$ indicates positive relationship between labour productivity and capital-intensity. It may be noted that for a unit change in capital-intensity, there is an increase of about 38% and 26% in labour productivity, for the industrial sector at state and national levels respectively. Thus it may be worthwhile to conclude that an increase in capital-intensity also brings in a corresponding increase in labour productivity.
Table 7.2

Cobb-Douglas Production Function
without Technical Progress

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( \hat{\lambda} )</th>
<th>( \hat{\beta} )</th>
<th>( \gamma )</th>
<th>( R^2 )</th>
<th>( \bar{R}^2 )</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gujarat State</td>
<td>7.4169 (0.6975)</td>
<td>0.0504 (0.0963)</td>
<td>0.8793 (0.1590)</td>
<td>0.80</td>
<td>0.78</td>
<td>36.59</td>
</tr>
<tr>
<td>All India</td>
<td>6.4188 (0.5899)</td>
<td>0.0070 (0.0936)</td>
<td>0.4650 (0.1198)</td>
<td>0.65</td>
<td>0.61</td>
<td>16.37</td>
</tr>
</tbody>
</table>

Note: (i) The values in the ( ) show S.E. of estimates
(ii) The values in the [ ] show t - statistics
** Significant at 1% of significance level
* Significant at 5% of significance level
† This regression analysis is based upon equation (7.2.1)

Table 7.2 gives the results of the fitted Cobb-Douglas production function model without technical progress. There are increasing returns to scale for both the industrial sectors of the economy. The regression co-efficient \( \gamma \) is highly significant for both the industrial sectors of the economy. The values of \( R^2 \) and \( \bar{R}^2 \) are also significant.
It may be concluded from the fitted model that about 65% to 80% of the variation is explained for both the industrial sectors.

Table - 7.3

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Constant</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\lambda$</th>
<th>$R^2$</th>
<th>$\bar{R}^2$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gujarat State</td>
<td>10.0898 (1.5590)</td>
<td>0.2510 (0.1393)</td>
<td>-0.1224 (0.5510)</td>
<td>0.0407 (0.0206)</td>
<td>0.84</td>
<td>0.81</td>
<td>29.06</td>
</tr>
<tr>
<td>All India</td>
<td>10.7442 (1.6197)</td>
<td>0.0258 (0.0799)</td>
<td>-0.7649 (0.4496)</td>
<td>0.0450 (0.0160)</td>
<td>0.76</td>
<td>0.72</td>
<td>17.72</td>
</tr>
</tbody>
</table>

Note: (i) The values in the ( ) show S.E. of estimates (ii) The values in the $J$ show t - statistics ** Significant at 1% of significance level * Significant at 5% of significance level † This regression analysis is based upon equation (7.5)

Table - 7.3 gives the analysis done using Cobb-Douglas production function with technical progress. It is apparent that $R^2$ and $\bar{R}^2$ are highly significant for both the industrial sectors. The neutral technological progress
is also found significant for both the industrial sectors at state and at national levels. About 76% to 84% of the variation is explained by the fitted model. It may be worthwhile to conclude that the model fitted under neutral technical progress is a better fit for both the industrial sectors.

6.4 Concluding Remarks:

From the above analysis for both the industrial sectors of the economy, it may be worthwhile to conclude that an increase in capital intensity in any industrial sector brings about a corresponding increase in the labour productivity, thus the growth in output has been visualised by means of the neutral technical progress as explained by the technological parameter. There are varying returns to scale and the growth oriented innovative technological developments may lead to further progress in the industries.