Chapter II

CONCEPTS AND ANALYTICAL FRAME FOR THE
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Introduction:

Increasing concern with the incidence of inequalities and the related issues of welfare has become the ubiquitous theme of current debates on economic development. This is evident in the various studies which explored the link between economic growth and the incidence of inequality in countries at various levels of development. These studies, theoretical as well as empirical, have often relied on various summary measures of inequality such as the coefficient of variation, Gini concentration ratio, standard deviation of logarithms, Kuznets' index etc. in gauging the relationship between development and inequality with no weighty reason explicitly spelt out for choosing one measure or the other. Most of the early literature was "concerned with the problem of choosing between the different summary measures, and such properties were discussed as ease of computation, ease of interpretation, the range of variation.

and whether they required information about the entire distribution."  2/ But this neglect of the deeper problems bearing on the inequality welfare relation was not universally shared as these issues were raised as early as 1920s by Alyn A. Young 3/ and Dalton. 4/ The latter argued that some concept of social welfare must animate any measure of inequality and that the form of the social welfare function must be explicitly taken into account in approaching the question. Though other explorations had intervened, it is only in the early 1970s that the thread has been effectively picked up by Atkinson when he addressed himself to the conceptual problems involved in formulating his normative index of inequality, the "equally distributed equivalent measure". 5/


Another problem contingent on the adoption of these measures in quantitative studies is worthy of attention. These indexes of inequality, often employed as yardsticks of judgement, exhibit disparate movements in direction and magnitude leading to equivocal conclusions. The answer to this paradox is to be sought in the technical nature of these measures. Focussing on the technical aspects of the problems one must take into account the implicit bias of these measures in their treatment of the different segments of the distribution with varying emphasis. This differential weightage to transfers at different parts of the distribution curve is reflected in the movement and magnitude of these summary measures. Thus when we try to rank distributions on the basis of these indexes no unique ordering may result. As Amartya Sen has aptly observed: "There are reasons to believe that our idea of inequality as a ranking relation may indeed be inherently incomplete."


2/ See A.B. Atkinson, op. cit.
If so, to find a measure of inequality that involves complete ordering may produce artificial problems because a measure can hardly be more precise than the concept it represents.\textsuperscript{2}

This chapter focuses on the conceptual frame for the study of inequalities by examining the various measures, positive and normative with their welfare content and limitations.

**Conceptual Frame:** It is customary to categorize measures of inequality into two classes, the positive and normative. The former "try to catch the extent of inequality in some objective sense usually employing some statistical measure of relative variation....\textsuperscript{2}" while the latter "try to measure inequality in terms of some normative notion of social welfare so that a high degree of inequality corresponds to a lower level of social welfare for a given total of income."\textsuperscript{10}" Of the measures adopted in the study, the coefficient of variation, the standard derivation of logarithms, the Gini concentration ratio and Kuznets index fall under the former category while Atkinson's measure belongs to the normative group.


\textsuperscript{2} Amartya Sen, *Ibid*, p.3.

The Coefficient of Variation: One of the commonly used measures of inequality is the coefficient of variation. This relative measure is defined as the ratio of the standard deviation of the variable under consideration (say \( X \)) to its arithmetic mean.

\[ C = \frac{\sigma_X}{\mu_X} \]

This measure is sensitive to income transfers at all income levels and attaches equal weightage to them at different income levels. It satisfies Piggott-Talton criterion of transfers.\(^{11/}\)

There are two counts on which this index is criticised; one is methodological, bearing on the deviations it effects from the mean. In the study of variation it may be pointed out whether it would not be better to capture income differences of each and everyone from the rest and not from the mean which might be nobody's income. Of course, this measure is not unique in this respect as it shares this limitation with the standard deviation of logarithms also. The other

\(^{11/}\) To borrow the terminology of Champernowne, this criterion requires that if a distribution is modified by altering two incomes only so as to have their total unaltered then the index concerned must be increased, unchanged or decreased according as the absolute difference between the two incomes is increased, unchanged or decreased. See D.C. Champernowne: "A Comparison of Measures of Inequality of Income Distribution", Economic Journal, Vol. 64, December 74, pp. 797-816.
criticism bears on its neutrality towards the treatment of income transfers at different points of the income distribution curve. This criticism is grounded in the acceptance of diminishing marginal of the principle utility; it focuses on the fact whether income transfers towards the farther ends of the income ladder should not be treated with differential weightage as the bottom segments would be marked by greater privation and misery while the other extremes are free from such mundane preoccupations. The limiting values of this measure are zero when there is perfect equality and

\[ 1 + \sqrt{n-1} \]  

(where \( n \) is the number of units or persons in the distribution) when perfect inequality rules the distribution.12/  

The Standard Deviation of Logarithms: In computing this measure it is customary in income distribution studies to take deviations from the arithmetic mean while geometric mean is employed in statistical literature. It is defined as

\[ \bar{y} = \left[ \frac{1}{n} \sum_{i=1}^{n} \left( \log \mu - \log y_i \right)^2 / n \right]^{1/2} \]

where \( \mu = \) Arithmetic mean, \( y \) the variable under study (income/expenditure)

\[ n = \text{size of the sample}, i = 1, 2, \ldots n \]

12/ For the limit values of the various measures of inequality see Thomas Stark: "The Distribution of Personal Income in United Kingdom, 1949-1963", Cambridge University Press, p. 139.
This measure is also sensitive to income transfers at all income levels but highlights the differences at the lower end of the distribution by attaching greater weightage to those transfers. It violates Piesou-Balton criterion as it ceases to be concave at higher income levels because they suffer severe contraction in logarithmic transformation.\(^{13/}\)

While this index has the advantage of eliminating the arbitrary nature of units in computation, it shares the limitation with the earlier measure in taking mean as the point of departure. Further it shares the arbitrary squaring procedure in common with the coefficient of variation. Another limitation of this index is that can only be computed when all the variates are positive and different from zero.

The Gini Concentration Ratio: As a measure of inequality this index is extensively used and gained almost universal support in studies on income distribution.\(^{15/}\) It has the theoretical attraction of not being dependent on any measure of the central tendency\(^{15/}\) and circumvents the methodological

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\(^{13/}\) Amartya Sen, *An Inequality*, p.29.


limitation shared by those cited earlier by taking into account differences between every pair of income in the computation. One way of computing this index is to rely on the cumulative percentage chart and use grouped observations though it involves errors of approximation; the other way is to compute this measure directly from individual observations. The former has the attraction of simplicity and elegance in presentation. We shall examine this in detail presently.

This method is a graphical device in which the index is viewed in terms of the Lorenz Curve. The cumulative percentages of population sharing income/expenditure,

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12/ "Although Pareto curves are probably the most commonly used for the description of income distribution in the upper income ranges, the Lorenz curve is undoubtedly the technique most commonly used to indicate differences in the degree of inequality of different income distributions. The same idea was introduced almost simultaneously by Ciri, Chatolain and Scallies... Lorenz himself recognised the possible ambiguity in comparison of income distributions when the curves intersect... Ciri turned his attention to this problem (as distinct from his logarithmic formula for the description of income distribution) in 1914. He then invented the "concentration ratio" a measure that is based on the areas outlined on a Lorenz diagram". See Mary Jean Fowia: "A Graphical Analysis of personal Income Distribution in the United States", The *Quarterly Economic Review*, Vol. XXXIV, No.1, September 1945, pp.567-628.
are plotted arithmetically against the cumulative shares of income or expenditure enjoyed.

In the diagram the Lorenz curve LCO coincides with the diagonal OL, the line of equal distribution, if income/expenditure is equally shared by everyone, that is if 10 per cent of income/expenditure is shared by 10 per cent of the population, 20 per cent of the same is shared by the 20 per cent of the population and so on.

The greater the degree of departure of LCO from OL and the greater the area enclosed between the chord and the

curve as a result of the latter sagging, the higher will be the degree of inequality in the distribution. The other extreme limit occurs when the curve LCO collapses into the axes and assumes the shape OXL. This occurs when one individual appropriates all the income/expenditure while the rest has none.

The Gini ratio of concentration is the ratio of the area of concentration (the difference between the line of absolute equality and the Lorenz curve) to the area of maximum possible concentration\(^*\) (the triangular area under the diagonal).

The Gini concentration

\[
\text{Ratio} = \frac{\text{Area LCO}}{\text{Area LKO}}
\]

The two extreme limits are evident, one when the numerator vanishes and the other unity\(^*\) when the area LCO is equal to the area LKO.

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\(^*\) The area of maximum concentration is the area that would be circumscribed by the Lorenz curve under the extreme condition that the total amount of income was possessed by one individual. Gini later elaborated his 'ratio of concentration' to apply to cases in which the limit of maximum concentration was defined in other ways and something less than complete equality was taken as the egalitarian limit...."; See H.J. Towne, op. cit., p.649.

\(^*\) See Thomas Stark, op. cit., p.139 for a statement of the invariability of this measure to attribute size of a distribution in contrast with other measures.
This method of presentation with Lorenz curves provides a convenient means of comparing the degrees of inequality in distributions as long as the curves do not cross. For if the Lorenz curve of one distribution lies everywhere above that of another then the former may be said to unambiguously more equal than the latter.

Diagram 1

Diagram 2

In the diagram 1 the distribution represented by A is more equal unequivocally than the distribution portrayed by B as
the Lorenz curve A lies above Lorenz B at every point and is
closer to the line of absolute equality. When the Lorenz
curves intersect as shown in diagram 2, no definite pronouncement regarding the inequality of the distribution can be
made; up to F the curve B is nearer to the diagonal and above
the curve A, indicating lesser inequality than that of A.
Beyond F, A moves nearer the diagonal while B swings away from
the line indicating greater inequality in the distribution.
In such situations one cannot comment unambiguously on the
distributions since without introducing values one cannot
weigh the lesser of distribution A at the top against the
greater inequality at the bottom. Moreover, when one examines
the values implicit in the Gini coefficient, there is no
reason to believe that they correspond to any values that we
should like to hold regarding equity — indeed there is
little reason to expect that they should, since the coefficient
was put forward as a purely statistical measure rather than
as representing any consideration of social welfare.21/
This kind of introducing values explicitly into the measure-
ment of inequalities has been attempted by Atkinson whose
measure we shall subsequently present in detail.

21/ A.F. Atkinson "Poverty and Income Inequality in Britain",
in Poverty, Inequality & Class Structure (Ed.)
Dorothy Widdershurn, Cambridge University Press, 1974,
p.51.
When the Gini index is computed from individual observations it is defined as the arithmetic average of the absolute values of differences between all pairs of incomes.\(^{22}\)

\[ G = \frac{1}{2n^2 \mu} \sum_{i=1}^{n} \sum_{j=1}^{n} |\gamma_i - \gamma_j| \]

where \(\gamma_i\) and \(\gamma_j\) are the per capita income/expenditure of the \(i\)th and \(j\)th households, \(\mu\) the mean of the observations and \(n\) the sample size. The Gini index satisfies Pigou-Dalton criterion of transfers\(^{23}\) like the coefficient of variation. Further it scores over others in avoiding the arbitrary procedure of squaring. Another "appeal of the Gini coefficient or of the relative mean difference, lies in the fact that it is a very direct measure of income difference taking note of differences between every pair of incomes."\(^{24}\)

With regard to the relative sensitivity to transfers of incomes at different levels of the income distribution, "Gini coefficient depends not on the size of the income levels but on the number of people in between them...it implies a welfare function which is just a weighted sum of different people's..."\(^{25}\)

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\(^{22}\) Cf. Amartya Sen, \textit{op.cit.}, p.31.

\(^{23}\) If it is computed from the original ungrouped data, see Yankov Kondor, "Value Judgements implied in the use of Various Measures of Income Inequality", \textit{The Review of Income and Wealth}, September 1975, p.398.

\(^{24}\) Amartya Sen, \textit{op.cit.}, p.31.
income levels with weights being determined by the rank
order position of the person in the ranking by income
level. It should be noted that though the Gini index
does not imply a strictly concave group welfare function
"any transfer from the poor to the rich or vice versa is
strictly recorded in the Gini measure in the appropriate
direction". One major limitation of this measure is
that unlike Theil's measure and coefficient of variations,
it cannot be decomposed into a Gini measure of inequality
within groups and a Gini measure of inequality between
group means.

Kuznet's Total Disparity Measure: This aggregate index
of inequality is derived from differences between actual
and egalitarian shares in total number and total income
of groups of population under consideration. In mathematical
terms it is defined as the "sum of differences, signs
disregarded, between the percentage shares in number and the
percentage shares in total income for the classes

25/ Martya Sen, Ibid, p.32.
26/ Martya Sen, Ibid, p.34.
27/ See Graham, Pyatt: "On the Interpretation and Disaggre-
gation of Gini coefficients", The Economic Journal,
Vol. 86, June 76, pp.223-255 and V.H. Rao, "Two
Decomposition of Concentration Ratio", Journal of the
Royal Statistical Society, Series A, Vol. 132, part 6,
1969.
The components of this index may be illustrated by means of Lorenz curve although the curve itself may not be estimated to calculate the index.20/ One advantage of this measure is that "the differences in shares for the classes can be observed in their original form, without being obscured in cumulative arrays, partition values .... it helps to reveal those parts of the distribution in which the share differentials are particularly large and contribute heavily to the magnitude of the summary measures.30/

This measure has several limitations. Like other summary measures, this index also expresses inequality as a single number which may conceal more than it reveals. Moreover, expressing inequality by any single number "may be too drastic an abbreviation in the sense that it is liable to prevent meaningful discussion and decision on the income distribution by individuals whose values differ."31/


31/ Naaikov Kondor, op.cit., p.309.
Secondly, it is not as sensitive as the Cini Coefficient to income inequalities. Palto-Figou criterion is violated and this measure is "insensitive to rank preserving equalization on the same side of the mean, whereas the Cini coefficient is sensitive to such transfers." Another major limitation of the total disparity measure is that "like the Cini coefficient, it lacks the property of additivity of variance found only in normal or near normal distributions".

The value of this index varies in the range 0 to 200, attaining the former when there is complete equality and approximating to the latter when there is extreme inequality.

Atkinson's Index: Formative notions of social welfare are concealed in the conventional summary measures as they embody implicit judgements about the weight attached to inequality at different points on the income scale.

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Atkinson's measure, the "Equally Distributed Equivalent Measure" incorporates distributional objectives explicitly through a parameter (ε), 'Inequality aversion', is defined as "the level of income per head which if equally distributed would give the same level of social welfare as the present distribution". To illustrate in brief, let us consider a cardinal welfare function between income enjoyed per head (Y) and the resulting welfare (ω).

\[ \omega = f(Y) \quad f'(Y) > 0 \quad f''(Y) < 0 \]

Then the aggregate social welfare generated by the current distribution is

\[ W = \sum_{i=1}^{n} \omega_i = \sum_{i=1}^{n} f(Y_i) \]

where i stands for the individual. Since ω by assumption is a concave function, equitable distribution yields higher aggregate welfare \( W^* \) than \( W \). Dalton opted for the ratio

\[ W^* = \frac{W}{W^*} \]


37/ Dalton Conforming to a strictly concave utility function, suggests the ratio of actual level of Social Welfare to that which would be achieved if incomes were equally distributed as a measure of inequality. For details see Dalton U. "The Measurement of Inequality of Incomes", Economic Journal, Vol. XG, 1928.
of actual level of social welfare (\( W \)) to the maximum (\( W^* \)) which would result from equal distribution as a measure of inequality. As this measure is variant to positive linear transformation of the welfare function, Atkinson proposed a measure that is invariant\(^{28}\) with respect to the above transformation by introducing the concept of equally distributed equivalent level of income (\( Y_{DE} \)).

Let \( \mu \) be the per capita income of the community and \( v \) the aggregate social welfare as indicated. One can imagine a variant of the present income distribution yielding the same aggregate welfare \( v \) but in which every one enjoys the same per capita income \( Y_{DE} \). By assumption, since the welfare function is concave.

\[ Y_{DE} < \mu \]

The departure of \( Y_{DE} \) from \( \mu \) as a proportion of \( \mu \) is Atkinson's index of inequality:

\[ I = \frac{\mu - Y_{DE}}{\mu} = 1 - \frac{v + Y_{DE}}{\mu} \]

\(^{28}\) Amartya Sen does not concede this improvement as fundamental as the ordering revealed by Palton's measure is unaffected by positive linear transformation. For details of criticism and extended discussion of Atkinson's measure, see Amartya Sen, op. cit., pp. 37-39, and Chap. 3.
A fall in the magnitude of $I$ indicates greater equality in distribution and this calls for a reduction in the difference between $Y_{DBB}$ and $\mu$.

Operationally, Atkinson's measure $I$ is

$$I = 1 - \left[ \frac{1}{n} \sum_{i=1}^{n} \left( \frac{Y_i}{\bar{Y}} \right)^{1-e} \right]^{1/e}$$

where $X_i$ denotes the per capita income/expenditure of the $i$th household, $f_i$ the proportion of population in the $i$th household, $Y$ the mean per capita income/expenditure of all households and $\epsilon$ represents the weight attached to inequality in the distribution. "It ranges from zero, which means that society is indifferent about the distribution, to infinity, which means that society is concerned only with the position of the lowest income group. This latter position may be seen as corresponding to that developed by Rawls (1972) in contractual theory of justice where inequality is assessed in terms of the position of the least advantaged members of society; where lies between these extremes depends on the importance attached to redistribution towards the bottom."

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The magnitude of I lies between the limits 0 and 1, the former occurs when income is equally distributed and the latter extremity when complete inequality rules the distribution. The intuitive appeal of this measure is worth mentioning. "The value taken by I has a straightforward interpretation. A value of 30 per cent means that if incomes were equally distributed we would need only (100 p.c. - 30 p.c.) = 70 p.c. of the present national income to reach the same level of social welfare - or alternatively that the gains from redistribution to bring about equality is equivalent to raising national income by 30 per cent... In this way the measure I provides an index of the potential gains from redistribution."  

The measures of inequality thus far discussed in detail have been applied in the measurement of household expenditure inequalities in the next chapter.

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